# Visualizing the dynamics of two-dimensional foams with FoamVis

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**Abstract**

We describe an interactive computer program FoamVis that provides techniques for visualization, exploration and analysis of time-dependent 2D foam simulation data generated by the Surface Evolver. The program takes a sequence of Surface Evolver files and animates them to show velocity vectors, local strain, the position of topological changes, and the forces on objects immersed in a foam flow, and enables the user to simultaneously compare different simulations. We use FoamVis to show the correlation between different attributes, such as bubble velocity and T1 density, in a foam flowing through a constriction, the relationship between orientation and foam-induced forces acting on an elliptical object falling through a foam under gravity, and the changes in bubble size that occur during diffusion-driven coarsening of a foam containing a mixture of gases.

*Key words:* Surface Evolver, foam flow, foam coarsening, visualization
*PACS:* 83.80.Iz Emulsions and foams; 47.50.-d Non-Newtonian fluid flows

## 1. Introduction

Foams are widely used in industrial processes and domestic applications [1, 2]. In rheological terms, they can be viewed as yield-stress, shear-thinning materials with a microstructure that satisfies precise local geometric rules, known as Plateau's laws, at equilibrium. It is the microstructure that gives rise to the bulk rheological response, and so it is of interest to understand how the microstructure changes during flow.

To simplify the task of simulating disordered flowing foams, we consider the dry limit, in which the proportion of liquid to gas in the foam structure becomes negligible. Then bubbles are polyhedra that pack together to fill space. Further, we make here the assumption that the foam is always at equilibrium and hops between equilibria in a quasi-static fashion.

The Surface Evolver [3] is freely-available software that minimizes the energy of a collection of surfaces. For foams, the relevant energy is the surface energy: the total surface area of the films in three dimensions (3D), or the total perimeter in two dimensions (2D), multiplied by the surface tension. The Surface Evolver allows the volume of each bubble to be constrained to a given target value. A foam structure is described as a list of vertices connected by edges which form closed loops (a face). Each bubble corresponds to one face in 2D, or a closed group of faces in 3D. A minimum of energy is found by gradient descent, and when edges or faces shrink to zero length or area, topological changes [4] are performed.

We use the Surface Evolver to simulate quasi-static flows of foams at the bubble-scale, since its precision gives access to the precise geometry of the bubbles and therefore quantities such as bubble pressures. Due to this level of

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complexity, a simulation of a few hundred bubbles, even in 2D, is too slow to view in real time, taking of the order of tens of minutes per quasi-static step. We therefore chose to store the foam geometry at each step for later analysis and visualization, and here we describe a second interactive software tool that helps in these tasks.

Our FoamVis software [5] aims to visualize the behaviour of (simulated) foams, removing the need for a user to do the processing required to access the Surface Evolver data. In particular, as we shall describe below, FoamVis infers attributes about the foam structure that are not even explicitly identified in the simulation itself. FoamVis allows the user to correlate different attributes and compare simulations and therefore, by simulating many different sets of parameters, to relate foam response to material properties such as bubble size and its distribution and liquid fraction.

Other tools to manipulate Evolver data include Brakke's *evmovie*, which is distributed with Evolver, and the *Surface Evolver – Fluid Interface Tool* (SE-FIT) [6]. The first scrolls through a sequence of evolver files, without adding any further visualization, while SE-FIT provides an interface for direct interaction with Evolver, displaying the results as well as importing and exporting data. Most work on visualization of foams is restricted to their static structure (see the review in [5]).

We first describe FoamVis in more detail (§ 2), before showing how it has allowed us to gain new insights into our simulations of foam coarsening (§ 3.1), foam flow through a constriction (§ 3.2), and sedimentation of an elliptical object through a foam (§ 3.3). All these examples are 2D, with constant surface tension.

## 2. Method

For any Surface Evolver simulation, a small change in the boundary conditions (for example an increase in strain, or a change in the position of an object moving through a foam) is followed by gradient descent to a local minimum of energy, including topological changes where necessary. At this point we record the geometry of the foam using Surface Evolver's "dump" command, which records all boundary constraints followed by a list of vertices, edges, faces and bubbles. We also define an array that stores all topological changes during the last step.

FoamVis takes as its argument a list of these geometry "dump" files, for example every step of a quasi-static coarsening simulation. It parses each in turn, interpreting the geometry and boundary constraints and calculating bounds on various attributes, including those specified explicitly in the Surface Evolver file, such as the area of a face or the pressure of a bubble, and those calculated immediately from the geometry, such as the bubble centres (given as an average of the positions of the vertices associated with each bubble) and the displacement of each centre from the previous file parsed (velocity). Further details are available in [5].

With the data loaded into FoamVis, the user then has the possibility to display various different attributes in a number of different views (see figure 1). Scalar attributes, such as pressure and velocity magnitude, are shown with colour; vector quantities like the velocity are shown with arrows; tensor quantities, such as measures of the local strain [7], are shown with ellipses (in 2D) [5]. All can be averaged (on a per-pixel basis using the graphics card) over a specified number of steps.

## 3. Results

### *3.1. Coarsening of a foam containing a mixture of gases*

Differences in gas pressure between neighbouring bubbles in a foam cause the gas to flow from one bubble to another through the films, which act as semi-permeable membranes. The gas diffuses from high pressure bubbles (which therefore shrink) to those with lower pressure (which grow). The diffusion constant for the process depends on the solubility of the gas in the liquid from which the foam is made. For example, carbon dioxide is more soluble than nitrogen in water, so that foams made with nitrogen last longer than those made with carbon dioxide. If the gas has more than one component, then it is the *partial pressure* of each gas [8], as well as its solubility, that affects the evolution, or coarsening, of the foam in time. (In principle, the osmotic pressure due to the mixing of the gases should also be taken into account [9, 10], but this is not necessary to illustrate the principle, and we reserve its inclusion for future simulations.)

Weaire and Pageron [8] showed that the addition of a small amount of an insoluble gas, such as $C_2F_6$, can arrest the coarsening process completely. We used FoamVis to illustrate this process, giving access to the geometry of the foam during this arrest. We first simulated the diffusion-driven coarsening of a 2D foam of 1000 bubbles in the Surface Evolver, with a gas consisting of two components. Only a small amount of insoluble gas is necessary; in the case simulated we added enough to fill only 0.5% of the initial area of each bubble.

Figure 1 shows the result of the simulation: no bubbles disappear, but instead small bubbles congregate at the vertices of the large ones. The visualization uses the same transformation applied to all images, showing four time-steps simultaneously. It is also possible to compare time-steps from different simulations, for example with different initial fractions of insoluble gas.

### *3.2. Flow through a constriction*

In the field of rheology, contraction flows provide a way to test many properties of a non-Newtonian fluid in the same experiment, for example both shear and extension. Since foam is certainly non-Newtonian, there have been a number of experiments on contraction flows of foams (see the review in [11]), and more recently quasi-static simulations have been performed [12], and compared with experiments in order to validate them in the limit of low flow-rates.

There are many quantities that one might wish to compare, including velocity profiles, stress profiles, the local deformation gradient and the position of the T1 topological changes, or neighbour-switching events, by which bubbles move past one another, as well as looking for any correlations between them in any given experiment of simulation. FoamVis makes the task of representing these quantities quicker and easier and, having saved all the necessary information from the simulation, it requires only a mouse click to change colour schemes, vector and tensor representations, magnification, and time-average.

Figure 2 shows various derived attributes for foam flow through a constriction: velocity and pressure together; the local strain, measured with the texture tensor [7], averaged over all time-steps; and the correlation between high bubble velocities and T1s.

FoamVis also allows us to compare two or more simulations in the same view. In figure 3 we directly compare the T1 distribution for flow of the same foam through two constriction geometries with different degrees of corner rounding [12]. Rather than individual points, we represent the cumulative position of T1s using a previously-proposed density estimate [13], which, in rough terms, smooths the T1 distribution by placing a Gaussian at each point at each time. The standard deviation of this Gaussian is a user-defined parameter, which is set to be five times the bubble size in figure 2. We see that the region in which most T1s occur moves upstream when the corners of the constriction become sharper, and the downstream band of topological changes separates into two regions.

### *3.3. Sedimentation in foams*

In applications such as ore-separation, a foam is generated to collect and selectively transport particles of ore. In some cases these particles may be of a similar size to the bubbles [14]. Davies and Cox [15] simulated the motion of an elliptical particle, slightly larger than the bubbles, as it sedimented through a foam under gravity, a motion which is opposed by the forces exerted by the network of soap films and the pressures in the bubbles surrounding the particle. They showed that the effect of these forces was to rotate the ellipse until its long axis was parallel to gravity, and so in a foam the equilibrium orientation of the moving ellipse is opposite to that in the Newtonian case.

Figure 4 shows how FoamVis can be used to separate and understand the origins of the torque exerted on an ellipse. Line segments show the direction and magnitude (which is proportional to length) of the foam-induced vector forces, as recorded in the dump file. In addition, the instantaneous position of T1s can be shown.

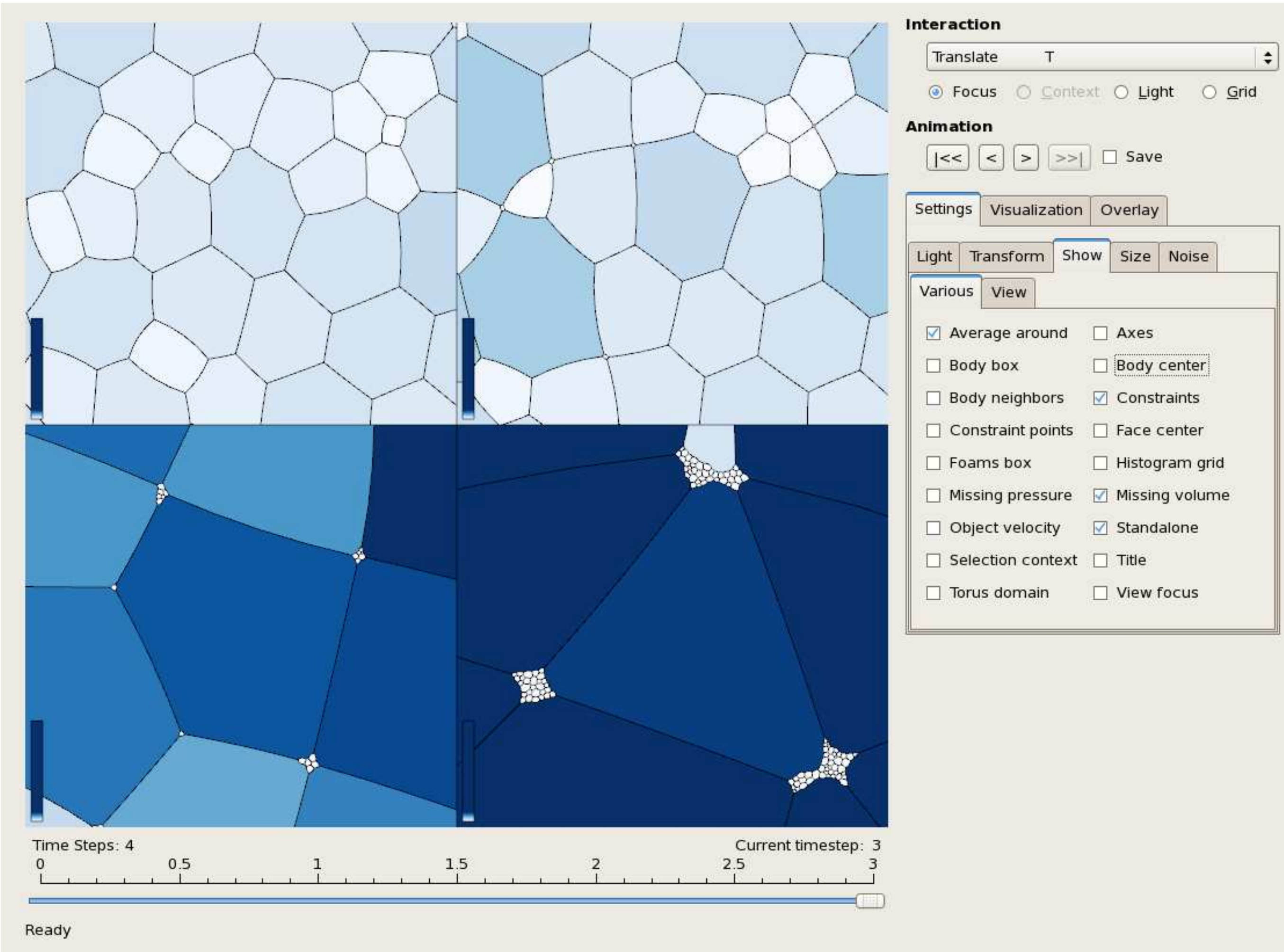
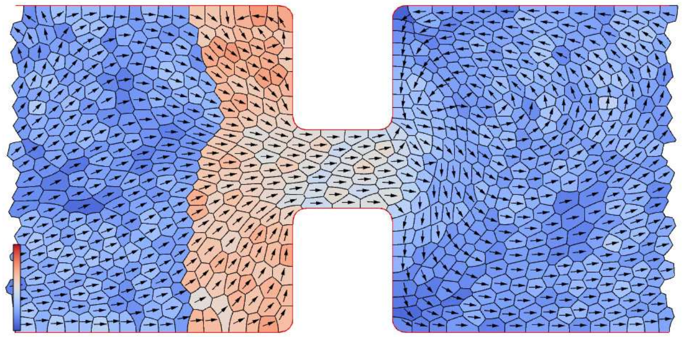
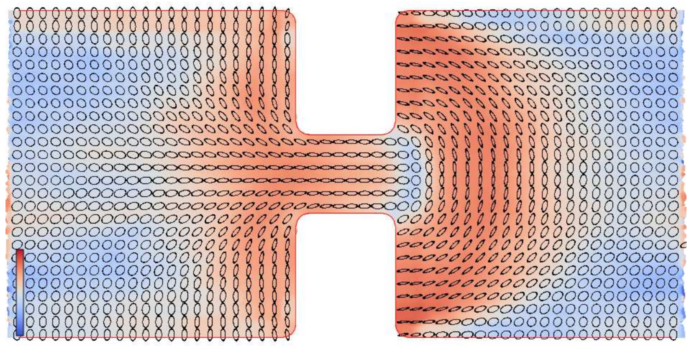


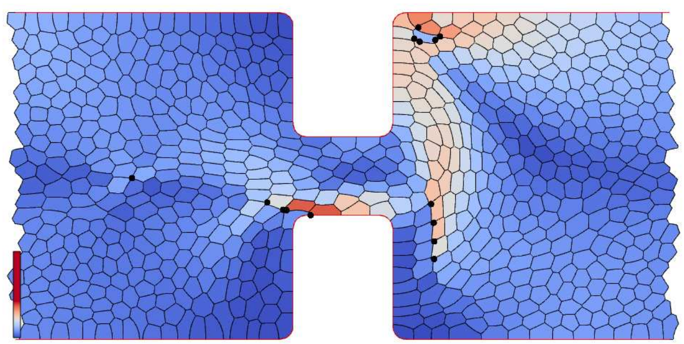

Figure 1: A screenshot of FoamVis for a simulation in which a foam containing a mixture of two gases undergoes diffusion-driven coarsening. The foam is shown at timesteps $10^2$, $10^3$, $10^4$ and $10^5$. Bubbles are coloured by area, and the same bubble is kept at the centre of the field of view. The foam gradually segregates into a few large bubbles with equal pressure, with many smaller bubbles of roughly equal size clustered around their vertices.



(a)



(b)



(c)

Figure 2: Flow of a foam through a 4:1:4 constriction from left to right. (a) Bubbles coloured by instantaneous pressure, with their direction of motion (velocity vectors with equal length) indicated. Note the region of high pressure just upstream of the constriction, and the apparent vortex in one of the upstream corners. (b) A rolling average of the texture tensor (ellipses), with pixels coloured by the average of its largest eigenvalue, indicating the local strain. The foam is isotropic far from the constriction, but the bubbles are elongated parallel and perpendicular to the direction of flow up- and downstream respectively. (c) Bubbles with large instantaneous velocities appear to correlate with the position of T1s (black dots); it is also apparent that T1s often occur in avalanches, and that their effect on bubble motion is extensive.

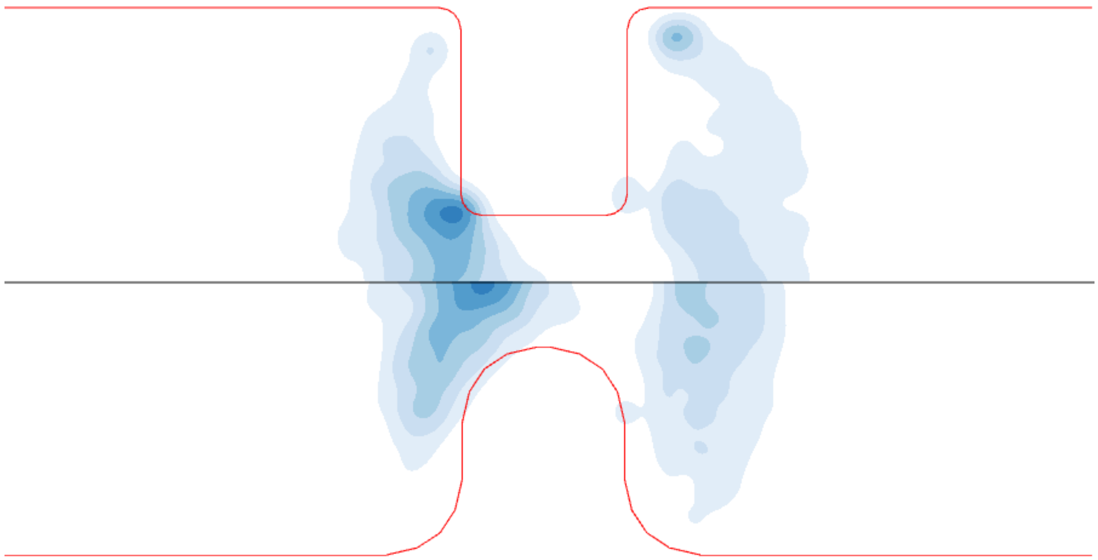


Figure 3: Flow of a foam through a 4:1:4 constriction, showing a smoothed profile of T1 density. The cumulative distribution of T1s changes when the corners of the constriction are either sharp (top) or rounded (bottom). It is apparent that there are more T1s upstream than downstream, and that for the case of a sharp corner most T1s occur close to the corners and the maximum in T1 intensity is shifted upstream.

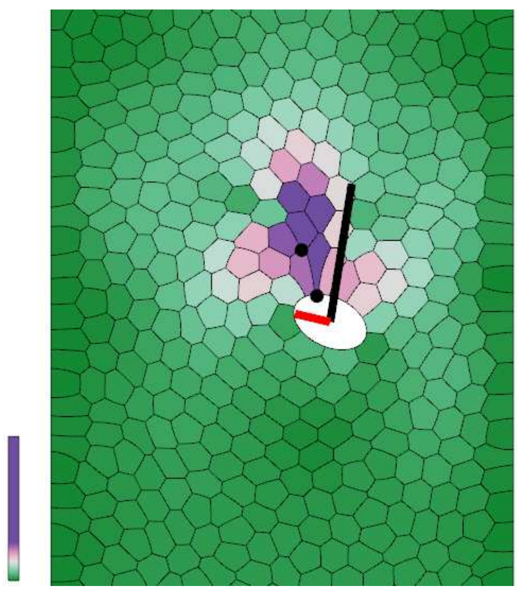


Figure 4: The instantaneous network (black line) and pressure (red line) forces on an ellipse sedimenting under its own weight. The torque due to these forces is not shown. The bubbles are coloured by the magnitude of the velocity of their centres, and the position of T1s is shown as black dots.

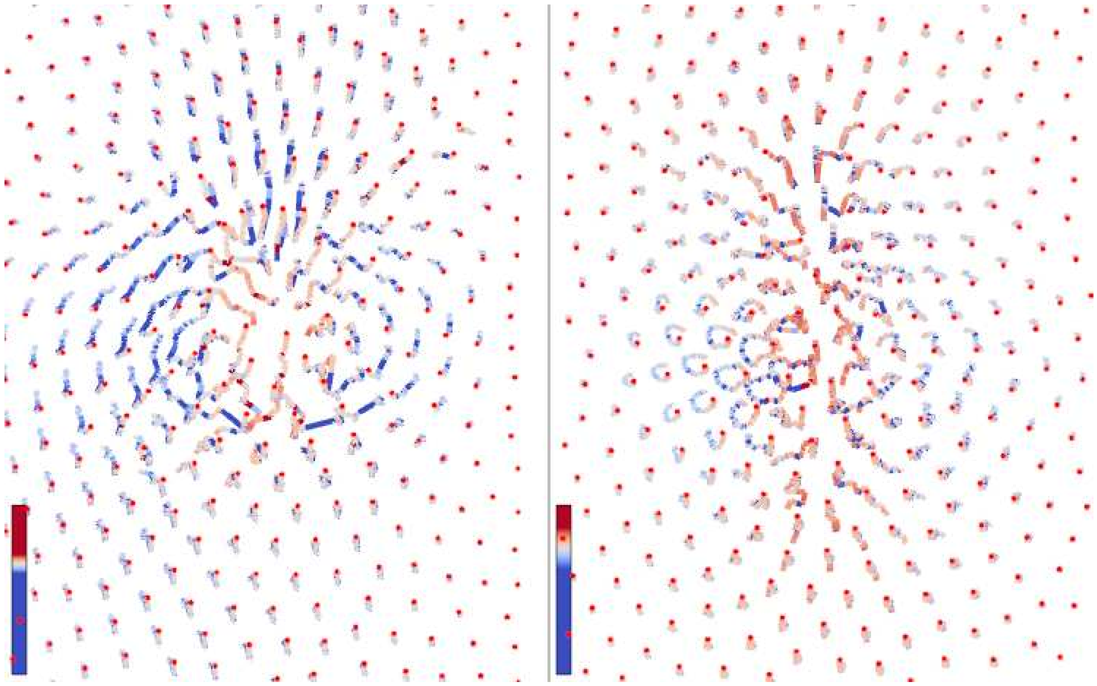


Figure 5: Bubble paths, coloured by vertical velocity, near a sedimenting ellipse which has its long axis horizontal (left) or vertical (right). The colour scale is the same in each case, symmetric about zero velocity, although the range of velocities is larger in the horizontal case. The initial position of each bubble is shown with a red dot. Only in the vertical case, when the ellipse moves more quickly, do the bubbles describe loops.

As the ellipse descends, each bubble that it passes is displaced, and some return to their initial position, describing a loop. Tracking the bubble paths, defined as the motion of the centre of each bubble, in FoamVis allows us to compare the effect of the ellipse having its long axis horizontal or vertical (i.e. perpendicular or parallel to gravity). Figure 5 shows 200 iterations from a simulation in each case, compared side-by-side, indicating that in the latter case, when the ellipse has higher velocity and the torque is small, the displacement of the bubbles is smaller and the loops appear more frequent. We believe that this is because the ellipse remains in the same orientation.

## 4. Conclusions

FoamVis [16] is a visualization tool: the hope is that by *visualising* the results of Surface Evolver simulations of foams, the user "sees" information that was not apparent in either the Evolver's graphics window, the ASCII data, or a simple plot. We have demonstrated how FoamVis offers insight into, and helps intuition in predicting, foam behaviour. In particular, FoamVis offers the possibility (i) to view different time-steps of the same simulation simultaneously, (ii) to view together up to four simulations with slightly different parameters, (iii) to view the same simulation step-by-step in up to four different ways simultaneously, (iv) to highlight those bubbles satisfying a certain characteristic, for example high velocity, and (v) to compare different fields measured from a simulation either instantaneously or averaged in time. All of the visualizations shown have been extracted directly from the usual Surface Evolver output, but presented in a new, and we believe more useful, way.

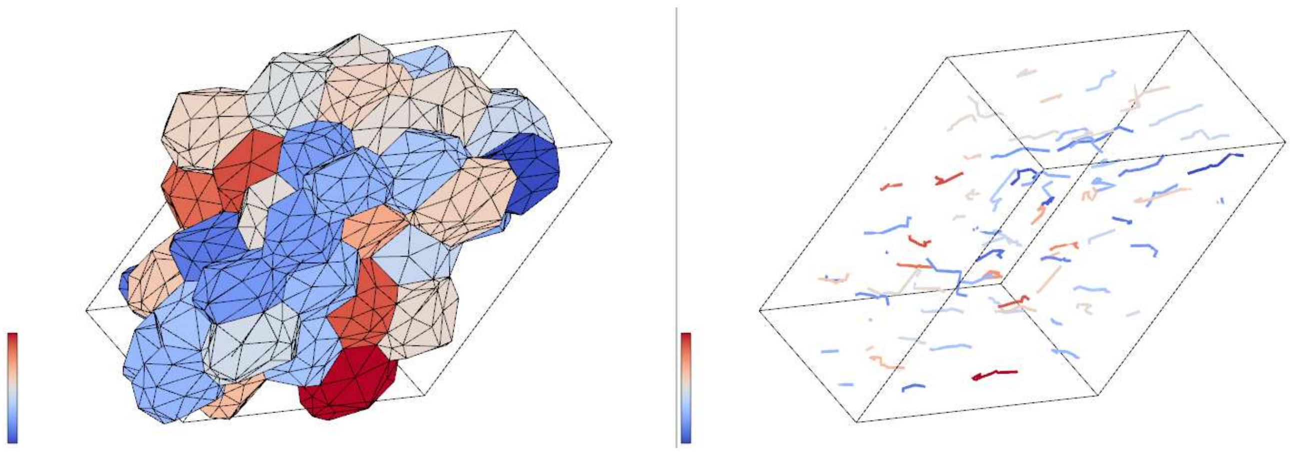


Figure 6: Quasistatic simple shear of a polydisperse dry three-dimensional foam of 65 bubbles, with periodic boundary conditions (shown as a parallelepiped, cf. [17]). Using the multiple views feature, FoamVis animates the foam on the left and shows the corresponding bubble paths on the right. Bubbles and paths are coloured by bubble volume. Sudden changes in path direction are a result of T1s.

Any Surface Evolver file can be visualized, and FoamVis is currently being developed in 3D (see figure 6). Since the format of a dump file is clear, it is possible to render many forms of data in FoamVis. For example, each frame of a video of a foam experiment can be analyzed to extract the positions of vertices and edges, the faces reconstructed by seeking closed loops of edges, and then written out in the dump file format. An example is shown in figure 7 for flow through a constriction. Note that to show velocity requires that the label of a given bubble is the same in successive frames.

In fact, any discrete simulation that can be described as lists of vertices, edges and faces can be visualized in this way, for example the vertex model [18].

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### References

[1] D. Weaire and S. Hutzler. *The Physics of Foams*. Clarendon Press, Oxford, 1999.

[2] I. Cantat, S. Cohen-Addad, F. Elias, F. Graner, R. Höhler, O. Pitois, F. Rouyer, and A. Saint-Jalmes. *Les mousses - structure et dynamique*. Belin, Paris, 2010.

[3] K. Brakke. The Surface Evolver. *Exp. Math.*, **1**:141–165, 1992.

[4] D. Weaire and N. Rivier. Soap, cells and statistics–random patterns in two dimensions. *Contemp. Phys.*, **25**: 59–99, 1984.

[5] D.R. Lipsa, R.S. Laramee, S.J. Cox, and I.T. Davies. FoamVis: Visualization of 2D Foam Simulation Data. *IEEE Transactions on Visualization and Computer Graphics*, **17**:2096–2105, 2011.

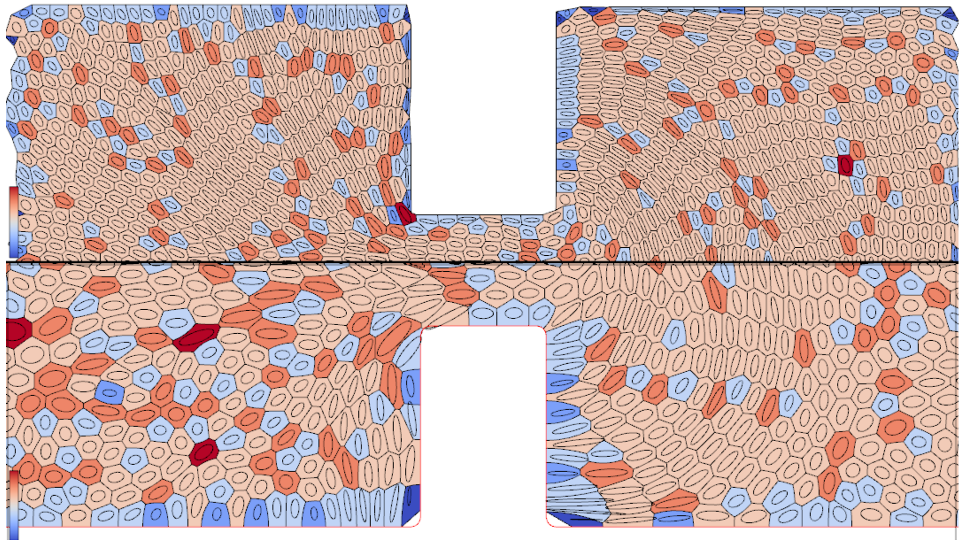

Figure 7: A photograph of an experiment in which a foam flows through a constriction can be reconstructed in Surface Evolver [12], and then visualized in FoamVis. Here we compare such an experiment (top) with a simulation (bottom), as described in §3.2. Each bubble is coloured by its number of sides, and contains an ellipse (a texture tensor) indicating the direction and magnitude of the local strain. Note that there are many more bubbles in the experiment, and that the strain is lower, presumably because of the finite liquid fraction.


[6] Y. Chen, B.M. Schaeffer, M.M. Weislogel, and G.A. Zimmerli. Introducing SE-FIT: Surface Evolver – Fluid Interface Tool for Studying Capillary Surfaces. *Proc. 49th AIAA Aerospace Sciences Meeting*, 2011. DOI: 10.2514/6.2011-1319. http://www.se-fit.com/

[7] F. Graner, B. Dollet, C. Raufaste, and P. Marmottant. Discrete rearranging disordered patterns, part I: Robust statistical tools in two or three dimensions. *Eur. Phys. J. E*, **25**:349–369, 2008.

[8] D. Weaire and V. Pageron. Frustrated froth: evolution of foam inhibited by an insoluble gaseous component. *Phil. Mag. Lett.*, **62**:417–421, 1990.

[9] A.J. Webster and M.E. Cates. Osmotic Stabilization of Concentrated Emulsions and Foams. *Langmuir*, **17**: 595–608, 2001.

[10] Y. Yip Cheung Sang, E. Lorenceau, S. Wahl, M. Stoffel, D. Angelescu and R. Höhler. A microfluidic technique for generating monodisperse submicron-sized drops. *RSC Adv.*, 2013, In press.

[11] S.A. Jones, B. Dollet, N. Slosse, Y. Jiang, S.J. Cox, and F. Graner. Two-dimensional constriction flows of foams. *Coll. Surf. A*, **382**:18–23, 2011.

[12] S.A. Jones and S.J. Cox. On the effectiveness of a quasi-static bubble-scale simulation in predicting the constriction flow of a two-dimensional foam. *J. Rheol.*, **56**:457–471, 2012.

[13] O. Daae Lampe and H. Hauser. Interactive Visualization of Streaming Data with Kernel Density Estimation. In *Proc. 4th IEEE Pacific Visualization Symposium*, pages 171–178, 2011.

[14] P.M. Ireland and G.J. Jameson. Drag force on a spherical particle moving through a foam: The role of wettability. *Int. J. Miner. Process.*, **102–103**:78–88, 2012.

[15] I.T. Davies and S.J. Cox. Sedimentation of an elliptical object in a two-dimensional foam. *J. Non-Newt. Fl. Mech.*, **165**:793–799, 2010.

[16] `http://csgalati.swansea.ac.uk/foam/html/`. Accessed 11th October 2012.

[17] A.M. Kraynik, D.A. Reinelt, and F. van Swol. Foam microrheology: from Kelvin cells to random froth. In D.M. Binding, N.E. Hudson, J. Mewis, J.M. Piau, C.J.S. Petrie, P. Townsend, and M. Wagner, editors, *Proc. XIIIth Int. Congr. on Rheology, Cambridge, UK*, volume **1**, pages 43–48, 2002.

[18] T. Okuzono, K. Kawasaki, and T. Nagai. Rheology of Random Foams. *J. Rheol.*, **37**:571–586, 1993.