G51APS, Algorithmic Problem Solving
Coursework 2, 2012/2013
River Crossing

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1 River Crossing

1.

(a) (5 marks) We first find out how many valid states for the general problem with N couples (the pair of president-bodyguard). Let lB be the number of bodyguards on the left bank and lP be the number of presidents on the left bank.

Obviously, valid states are those that have no bodyguard on the left bank or all bodyguards are on the left bank or bodyguards and presidents must be in pairs, i.e.,

(1) $lB = 0 \vee lB = N \vee lB = lP.$

We can find how many valid states by enumerating all president-bodyguard configurations (lB, lP) that satisfy $lB = 0 \vee lP = 0 \vee lB = lP$. The set S of all such configurations is given by:

$\{lB, lP : 0 \leq lB \leq N \wedge 0 \leq lB \leq N \wedge (lB = 0 \vee lP = 0 \vee lB = lP) : (lB, lP)\}.$

The number of all valid states is just $|S|$.

We can rewrite Equation 1 to $lB = 0 \vee lB = N \vee 0 < lB = lP < N$ and subsequently split the set S into three disjoint sets with states satisfying $lB = 0$, $lB = N$, and $0 < lB = lP < N$.

Assuming $N \neq 0$, there are $N+1$ states satisfying $lB = 0$, $N+1$ states satisfying $lB = N$, and $N-1$ states satisfying $0 < lB = lP < N$. Hence, $S = N + 1N + 1 + N - 1 = 3N + 1$. Combining this with the two boat positions, there are $2 \times (3N + 1) = 6N + 2$

states in total. For the given problem of two couples ($N = 2$), there are $6(2) + 2 = 14$ valid states in total.

Listing out all these states (with $||$ indicating the river, and $-$ indicating the position of the boat): $-2C||$, $2C||-$, $-||2C$, $||2C-$, $-2P||2B$, $2P||2B-$, $-2B||2P$, $2B||2P-$, $-1C1B||1P$, $1C1B||1P-$, $-1P||1C1B$, $1P||1C1B-$, $-1C||1C$, and $1C||1C-$.

(b) (10 marks) See figure 1.

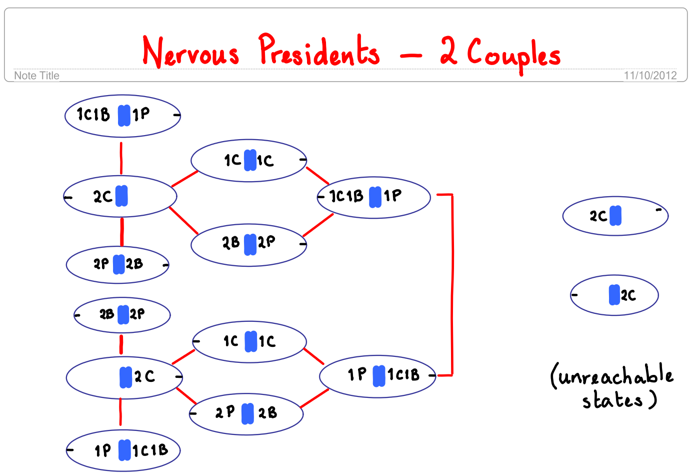

Figure 1: State Transition Diagram

(c) (10 marks) Suppose we allow individual naming. Let the couples be numbered from 1 thru N and the presidents and bodyguards are named $P1, P2, ..., PN$ and $B1, B2, ..., BN$, respectively. Let the set $\{1..N\}$ denote the set of all numbers from 1 thru N. We note that the number of subsets of a set of size N is the size of its power set 2^N.

Using our earlier analysis in (a), we have valid states satisfying the property that $lB = \emptyset \vee lP = \{1..N\} \vee lB = lP$. This can be rewritten as:

$lB = \emptyset \vee lP = \{1..N\} \vee \emptyset \subset lB = lP \subset \{1..N\}$.

Combining with the two boat positions, we have $2 \times (2^N + 2^N + (2^N - 2)) = 3 \times 2^{N+1} - 4$ valid states. For the given problem of $N = 2$, we have $3 \times 2^{2+1} - 4 = 20$ valid states.