G51APS, Algorithmic Problem Solving
Coursework 3, 2011/2012
Logic Puzzles

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September 28, 2012

Abstract

Model solutions.

a) $S = A$
b) $R = (Q = A)$. (Answer $R \equiv Q \equiv A$, or any permutation of the letters is also correct. If equality symbol is used, expression must be parenthesised.)
c) The response is always "no". From the first native's statement, we are given that $A = A \wedge B$. The response to the question is $B \equiv \neg A \wedge \neg B$. But

$$
\begin{array}{ll}
 & B \equiv \neg A \wedge \neg B \\
= & \quad \{ \quad A = A \wedge B \quad \} \\
 & B \equiv \neg(A \wedge B) \wedge \neg B \\
= & \quad \{ \quad \text{De Morgan's rule} \quad \} \\
 & B \equiv (\neg A \vee \neg B) \wedge \neg B \\
= & \quad \{ \quad \text{absorption} \quad \} \\
 & B \equiv \neg B \\
= & \quad \{ \quad \text{definition of negation} \quad \} \\
 & \mathsf{false} \;\; .
\end{array}
$$

d) Statement is $A \Leftarrow \neg A \wedge \neg B$. This is simplified as follows:

$$
\begin{array}{ll}
 & A \Leftarrow \neg A \wedge \neg B \\
= & \quad \{ \quad \text{definition of } \Leftarrow \quad \} \\
 & A \wedge \neg A \wedge \neg B \equiv \neg A \wedge \neg B
\end{array}
$$

$$
\begin{array}{ll}
= & \{ \quad \text{law of contradiction} \quad \} \\
& \mathsf{false} \equiv \neg A \wedge \neg B \\
= & \{ \quad \text{De Morgan's law} \quad \} \\
& A \vee B \quad .
\end{array}
$$

Thus the native's statement is one of us is a knight.

For the second part, the solution is independent of the statement made by the first native. Let it be S. Then, from the first native's statement, we are given that $A = S$. The response to the question is $B = S$. Substituting equals for equals, the response is thus $B = A$. That is, the response says whether or not the two natives are the same type (both knights or both knaves).