G51APS, Algorithmic Problem Solving
Coursework 5, 2012-2013
Games

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Abstract

Model solutions.

(a) See fig. 1.

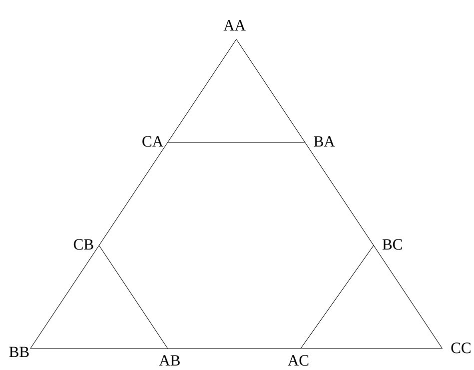

Figure 1: Tower of Hanoi State-Transition Diagram for Two Disks

(b) $T.0{=}0$, $T.1{=}3$, $T.2{=}12$.

$$T.(n{+}1){=}3{\times}T.n{+}3$$

Basis: $0{=}\frac{3}{2}{\times}(3^0{-}1)$.

Induction step:

$$
\begin{array}{ll}
 & \frac{3}{2}{\times}(3^{n+1}{-}1) \\
= & \{ \quad 3^{n+1}=3{\times}3^n \quad \} \\
 & \frac{3}{2}{\times}(3{\times}3^n-1) \\
= & \{ \quad \text{preparing for use of distributivity} \quad \} \\
 & \frac{3}{2}{\times}(3{\times}3^n-(3{\times}1)+2) \\
= & \{ \quad \text{distributivity} \quad \} \\
 & 3{\times}(\frac{3}{2}{\times}(3^n{-}1))+\frac{3}{2}{\times}2 \\
= & \{ \quad \text{arithmetic} \quad \} \\
 & 3{\times}(\frac{3}{2}{\times}(3^n{-}1))+3
\end{array}
$$

(c) See fig. 2.

(d) The function L solves the Tower of Hanoi problem in the longest possible way without repeating a state (i.e. constructs a longest simple path in the state-transition graph for the problem).

$$
\begin{array}{rcl}
L(0,d) & = & () \\
L(n+1,d) & = & L(n,d);[(n+1,\neg d)];L(n,\neg d);[(n+1,\neg d)];L(n,d)
\end{array}
$$

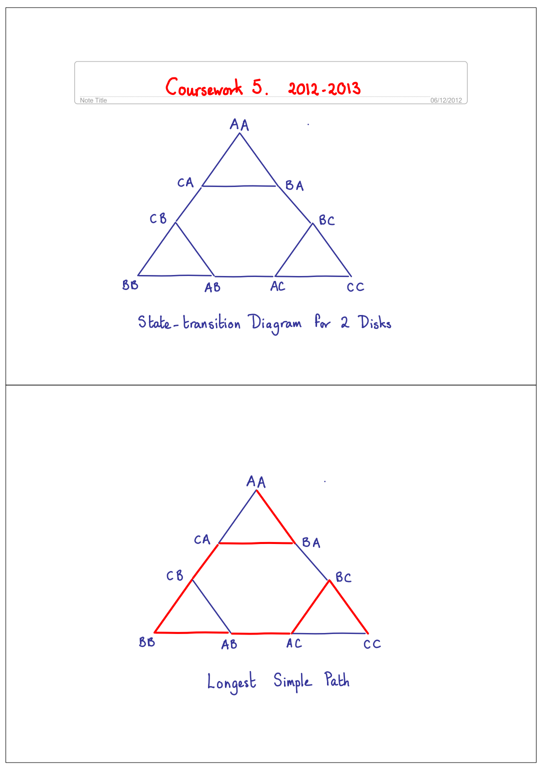

Figure 2: Tower of Hanoi State-Transition Diagram for Two Disks and Longest Simple Path