

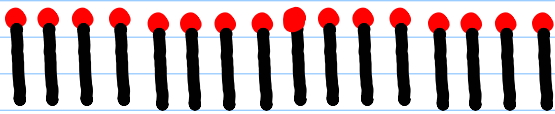
# GAMES

Note Title

22/10/2004

Study impartial, 2-person combinatorial games with complete information.

## Matchstick Game

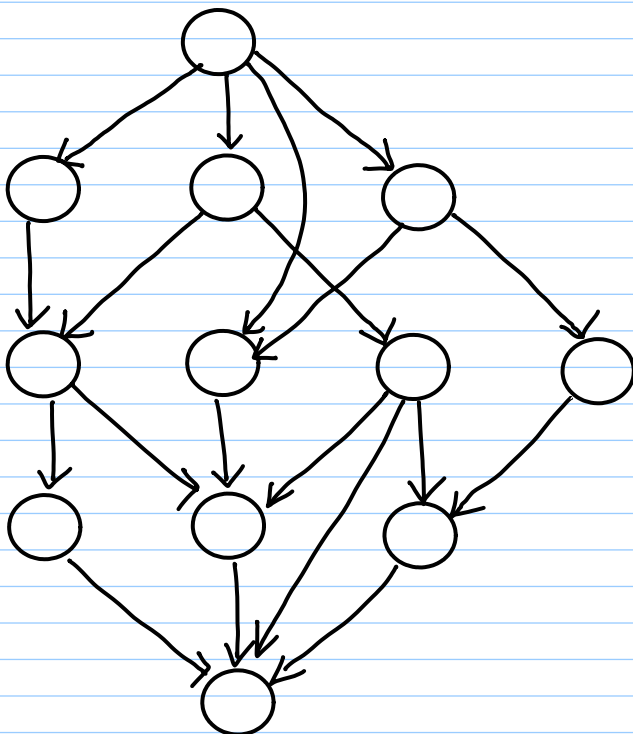


Move: remove 1 or 2 matches.

# Game Graph

Move: remove 1 or 2 matches.

## A Game



Positions are the nodes of the graph.  
Moves are directed edges.

## Summary. Single-pile matchstick games.

| Moves              | Losing Position     | Strategy                 |
|--------------------|---------------------|--------------------------|
| 1 or 2 matches     | $m \bmod 3 = 0$     | take $m \bmod 3$ matches |
| 1, 2 or 3 matches  | $m \bmod 4 = 0$     | -- $m \bmod 4$ --        |
| 1 thru $M$ matches | $m \bmod (M+1) = 0$ | -- $m \bmod (M+1)$ --    |

Key:

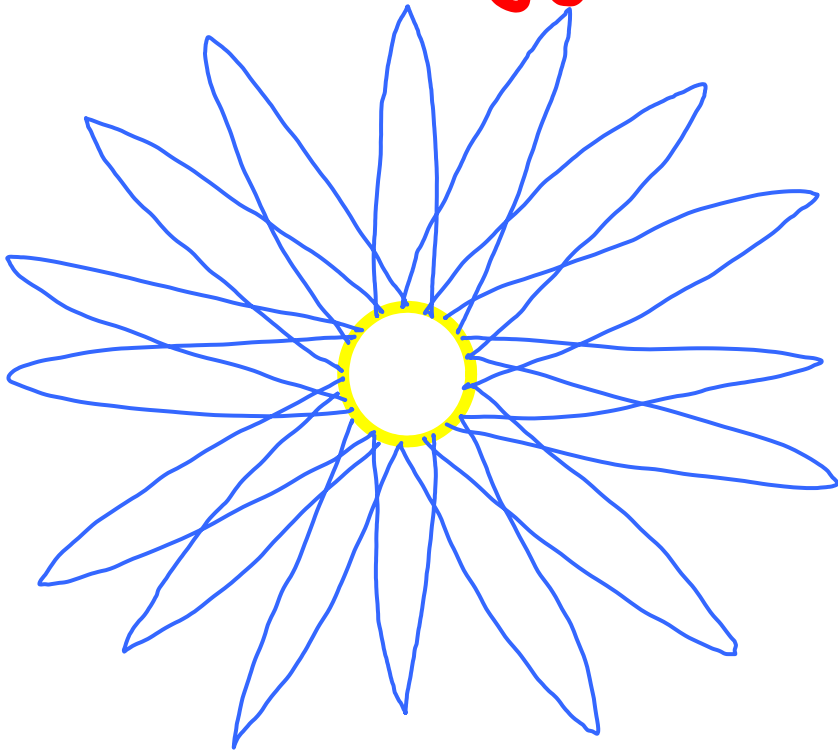
$m$  no. of matches  
 $\bmod$  remainder  
after dividing

## How to win.

Identify a property of positions  
(the *strategy invariant*) such that

- all end positions satisfy the property
- *every* move from a position satisfying the property *falsifies* the property.
- for every position that does not satisfy the property *there is* a move that *truthifies* the property.

# Strategy Invariant



## Move

Remove one or two adjacent petals

## Strategy

Maintain symmetry

