Invariants

Note Title 22/09/2008

Invariant = not changing

Fundamental concept in algorithm design.

Knockout Tournament

In a knockout tournament there are 1234 players.
How many games are played ?

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1. Understand the problem.

2. Determine the goal. Determine what is given.

3. Model the problem.

4. Solve the model problem.

5. Interpret the solution.

Invariants of Assignments

xs	list of variables (e.g. m,n,p)
es	list of expressions (e.g. m+1, n-m, 2×p)
xs := es	simultaneous assignment (length of xs must equal length of es e.g. m,n,p := m+1, n-m, 2×p)
E	expression (e.g. 2×m + n - p)
E[xs := es]	expression obtained by replacing each occurrence of a variable in xs by the corresponding expression in es.

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Example

$$(p + g)[p, g := p-1, g+1]$$

$$(m \bmod 3)[m := m+3]$$

(Non-) Examples

$$m = m[m := m+1]$$

$$n = n[n := 2 \times n]$$

$(m \bmod 3)[m := m+4]$

$(m+n \bmod 2)[m,n := n, m+n]$

Empty Boxes

Eleven large empty boxes are placed on a table.

An unknown number of the boxes is selected and, into each, eight medium boxes are placed.

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At the end of this process there are 102 empty boxes.

How many boxes are there in total?

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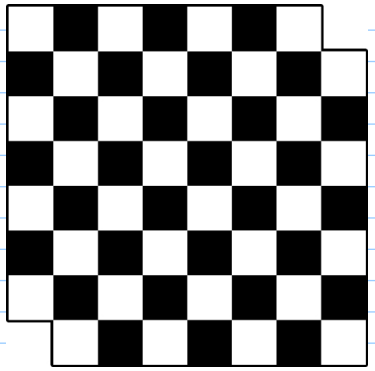
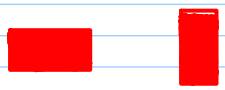
3. Model the problem.

Mutilated Chess Board

chessboard



dominoes



Is it possible to tile the chessboard with (non-overlapping) dominoes ?

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Invariant :