Logic Puzzles

Note Title 21/11/2005

Portia's Caskets.

Two caskets: silver and gold.
Exactly one has a picture of Portia in it.
Each casket has an inscription which may be true or false.

Silver: exactly one of these inscriptions is true.
Gold: the picture is not in this casket.

Substitution of Equals for Equals

A student buys two beers, one for themself and one for a friend.
The beer for the friend costs three times as much as the one for themself.
The student pays with a £5 note and receives £1 in change.

£ f price of beer for friend
£ t price of beer for themself
£ c change

Exactly one has a picture of Portia in it.

Silver: exactly one of these inscriptions is true.

Gold: the picture is not in this casket.

Island of Knights and Knaves

Knights always tell the truth.

Knaves always lie.

Inhabitant says:

There is gold on this island is the same as I am a knight.

What can you deduce?

Rule: If a native makes a statement, then

$$A = S$$

where A is the truth value of "the native is a knight",
and S is the truth value of the statement.

Rule: If a native is asked a question, their response is the truth value of $A = Q$ where A is the truth value of "the native is a knight", and Q is the answer to the question.

Equality

- reflexive
- symmetric
- transitive
- substitutive

of booleans

- associative

In order to exploit both transitivity and associativity of boolean equality, sometimes write

$$p = q$$

and sometimes write

$$p \equiv q .$$

A *continued* equality

$$p = q = r = \ldots$$

is read *conjunctionally*.

A continued equivales

$$p \equiv q \equiv r \equiv \ldots$$

is read *associatively*.

Examples

$$[\, p \equiv p \equiv \text{true} \,]$$

$$[\, \neg p \equiv p \equiv \text{false} \,]$$

Golden Rule

$$[\, p \vee q \equiv p \equiv q \equiv p \wedge q \,]$$

Inhabitant says:

I am a knave if there is gold on the island.

There is gold on the island if I am a knight.