Soft Computing Journal manuscript No.
(will be inserted by the editor)

A Hybrid Multi-population Framework for Dynamic Environments Combining Online and Offline Learning

Gönül Uludağ · Berna Kiraz · A. Şima Etaner-Uyar · Ender Özcan

Received: date / Accepted: date

Abstract Population based incremental learning algorithms and selection hyper-heuristics are highly adaptive methods which can handle different types of dynamism that may occur while a given problem is being solved. In this study, we present an approach based on a framework hybridizing these approaches to solve dynamic environment problems. A key feature of this hybrid approach is that it also incorporates online learning, which takes place during the search process for a high quality solution to a given instance, mixing it with offline learning which takes place during the training session prior to dealing with the instance. The performance of the approach along with the influence of different heuristic selection methods used within the selection hyper-heuristic is investigated over a range of dynamic environments produced by a well known benchmark generator. The empirical results show that the proposed approach using a particular hyper-heuristic outperforms some of the top approaches in literature for dynamic environment problems.

Keywords Heuristic · Metaheuristic · Hyper-heuristic · Estimation of Distribution Algorithm · Dynamic Environment

1 Introduction

One of the challenges in combinatorial optimization is to develop a solution method for *dynamic environment problems* in which the environment changes over time during the optimization/search process. There is a variety of heuristic search methodologies, such as tabu search and evolutionary algorithms to choose from to solve *static* combinatorial optimization problems (Burke and Kendall, 2005). When performing a search for the best solution in dynamic environments, the dynamism is often ignored and generic search methodologies are utilized. However, the key to success for a search algorithm in dynamic environments is its adaptation ability and speed to react whenever a change occurs. There is a range of approaches in literature proposed for solving dynamic environment problems (Branke, 2002; Cruz et al, 2011; Yang et al, 2007). Often, a given approach performs better than some others for handling a particular type of dynamism in the environment. This implies that the properties of the dynamism need to be known beforehand, if the most appropriate approach is to be chosen. However, even this may be impossible depending on the relevant dynamism associated with the problem. In this study, we use propose a hybrid approach to deal with a variety of dynamic environment problems regardless of the nature of their dynamism.

Most of the approaches for dynamic environments are either *online* or *offline* learning approaches. The on-

A preliminary version of this study was presented in UKCI 2012: 12th Annual Workshop on Computational Intelligence

Gönül Uludağ and Berna Kiraz
Institute of Science and Technology
Istanbul Technical University
Maslak, Istanbul, Turkey 34469
E-mail: uludagg@itu.edu.tr
E-mail: berna.kiraz@marmara.edu.tr

A. Şima Etaner-Uyar
Department of Computer Engineering
Istanbul Technical University
Maslak, Istanbul, Turkey 34469
E-mail: etaner@itu.edu.tr

Ender Özcan
School of Computer Science
University of Nottingham
Nottingham, UK NG8 1BB
E-mail: Ender.Ozcan@nottingham.ac.uk

line learning approaches get feedback/guidance during the search process while a problem instance is being solved. The offline approaches make use of a training session using a set of test instances to learn how to deal with unseen instances. *Statistical Model-based Optimization Algorithms* (SMOAs) are known to be highly adaptive and thus are expected to be able to track the changes, if and when they occur. They are potentially *viable* approaches for solving dynamic environment problems. Consequently, their use has been growing in the recent years. Probabilistic model-based techniques, for example *Estimation of Distribution Algorithms* (EDAs) are among the most common ones used within these approaches (Larrañaga and Lozano, 2002). EDAs are population based search methodologies in which new candidate solutions are produced using the probabilistic distribution model learned from the current best candidate solutions. Univariate marginal distribution algorithm (UMDA) (Ghosh and Muehlenbein, 2004), Bayesian optimization algorithm (BOA) (Kobliha et al, 2006) and population based incremental learning (PBIL) (Yang and Yao, 2005) are among the most commonly used EDAs in literature. There is a growing number of studies which apply *improved* variants of EDAs in dynamic environments (Barlow and Smith, 2009; Fernandes et al, 2008a; Wu et al, 2010b; Yang and Richter, 2009; Peng et al, 2011; Yang and Yao, 2008; Yuan et al, 2008).

Heuristic and many meta-heuristic approaches operate directly on the solution space and utilize problem domain specific information. *Hyper-heuristics* (Burke et al, 2012), on the other hand, are described as more *general* methodologies as compared to such approaches, since they are designed for solving a range of computationally difficult problems without requiring any modification. They conduct search over the space formed by a set of low-level heuristics which perturb or construct a (set of) candidate solution(s) (Cowling et al, 2000; Özcan et al, 2008). Hyper-heuristics operate at a higher level, communicating with the problem domain through a domain barrier as they perform search over the heuristics space. Any type of problem specific information is filtered through the domain barrier. Due to this feature, a hyper-heuristic can be directly employed in various problem domains without requiring any change, of course, through the use of appropriate domain specific low-level heuristics. This gives hyper-heuristics an increased level of generality. There are two main types of hyper-heuristics; methodologies that *generate* and *select* heuristics (Ross, 2005; Burke et al, 2012). This study focuses on the selection hyper-heuristic methodologies. There is strong empirical evidence showing that selection hyper-heuristics are able to quickly adapt without any external intervention in a given dynamic environment providing effective solutions (Kiraz and Topcuoglu, 2010; Kiraz et al, 2011).

In order to exploit the advantages of approaches with learning and those with model-building features in dynamic environments, we proposed a hybridization of EDAs with hyper-heuristics in the form of a two-phase framework, combining offline and online learning mechanisms in Uludağ et al (2012a). A list of probability vectors for generating good solutions is learned in an offline manner in the first phase. In the second phase, *two* sub-populations are maintained. A sub-population is sampled using an EDA, while the other one uses a hyper-heuristic for sampling appropriate probability vectors from the previously learned list in an online manner. In this study, we extend our previous studies and perform exhaustive tests to empirically analyze and explain the behavior of such an EDA and hyper-heuristic hybrid and try to determine a selection method which performs well within the proposed framework. We also try to decrease the computational requirements of the approach while maintaining its high performance through the use of adaptive mechanisms.

The rest of the paper is organized as follows. Section 2 provides an overview of selection hyper-heuristics, components used in the experiments and related studies on dynamic environments. Section 3 describes the proposed multi-phase hybrid approach which combines online and offline learning via a framework hybridizing multi-population EDAs and hyper-heuristics. The empirical analysis of this hybrid approach over a set of dynamic environment benchmark problems and the experimental design are provided in section 4. Finally, section 5 discusses the conclusion and future work.

2 Background and Related Work

2.1 Selection Hyper-heuristics

An iterative selection hyper-heuristic based on a single point search framework, in general, consists of *heuristic selection* and *move acceptance* components (Özcan et al, 2008). Previous studies show that different combinations of these components yield selection hyper-heuristics with differing performances. A selection hyper-heuristic operates at a high level and controls a set of predefined low level heuristics. At each step, a (set of) current solution(s) is modified through the application of a heuristic, which is chosen by the heuristic selection method. Then the new (set of) solution(s) is accepted or rejected using the move acceptance method. This process continues until the termination criteria are satisfied. In this section, we provide an overview of the selection hyper-heuristic components used in this study.

There are many heuristic selection methods proposed in literature. Some of these methods were introduced in Cowling et al (2000) including Simple Random (SR), Random Descent (RD), Random Permutation (RP), Random Permutation Descent (RPD), Greedy (GR) and Choice Function (CF). In Simple Random, a low-level heuristic is randomly selected and applied to the candidate solution once. In Random Descent, a randomly selected heuristic is applied repeatedly to the candidate solution as long as the solution improves. In Random Permutation, a permutation of all low-level heuristics is generated at random and each heuristic is applied successively once. In Random Permutation Descent, a heuristic is selected in the same way as Random Permutation, but it is applied repeatedly to the candidate solution as long as the solution improves. In Greedy, all low-level heuristics are applied to the candidate solution and the one generating the best solution is selected. Choice Function maintains a score for each heuristic, which is based on a weighted average of three measures: the performance of each individual heuristic, the pairwise performance between the heuristic and the previously selected heuristic and the elapsed time since the heuristic was last used. The heuristic with the maximum score is selected at each iteration. The score of each heuristic is updated after the heuristic selection process.

Nareyek (2004) used Reinforcement Learning (RL) to choose from a set of neighborhoods. Reinforcement Learning employs a notion of reward/punishment to maintain the performance of a heuristic which yields an improving/worsening solution after it is chosen and is applied to a solution at hand. In Reinforcement Learning, each heuristic is initialized with the same utility score. After a heuristic is selected and applied to a candidate solution, its score is increased or decreased at a certain rate depending on change (improvement or worsening) in the solution quality. At each iteration, the low level heuristic with the maximum score is selected as in Choice Function.

A heuristic selection method incorporates *online learning*, if the method receives some feedback during the search process and makes its decisions accordingly. In this respect, Random Descent, Random Permutation Descent, Greedy, Choice Function and Reinforcement Learning are all learning heuristic selection methods. Random Descent, Random Permutation Descent and Greedy also receive a feedback during the search process, that is whether or not a given heuristic makes an improvement (or largest improvement). So, they can be considered as learning mechanisms with an extremely short term memory.

A recent learning heuristic selection method, Ant-based Selection (AbS), was proposed in Kiraz et al (2013b). As in ant colony optimization approaches, Ant-based Selection uses a matrix of pheromone trail values (τ_{h_i,h_j}). A pheromone trail value (τ_{h_i,h_j}) shows the desirability of selecting heuristic h_j after the selection of heuristic h_i. All pheromone trails are initialized with a small value τ_0. In the first step, a low-level heuristic is randomly selected. Then, the most appropriate low-level heuristic is selected based on pheromone trail values. In Ant-based Selection (Kiraz et al, 2013b), there are two successive stages: heuristic selection and pheromone update stages. In the heuristic selection stage, Ant-based Selection chooses the heuristic h_s with the highest pheromone trail ($h_s = \max_{i=1..k} \tau_{h_c,h_j}$) with a probability of q_0 where h_c is the previously invoked heuristic. Otherwise, the authors consider two different methods to decide the next heuristic to invoke. The first method selects the next heuristic based on probabilities proportional to the pheromone trail of each heuristic pair. This method is analogous to the roulette wheel selection of evolutionary computation. In the second method, the next heuristic is selected based on tournament selection. After the selection process, the pheromone matrix is updated. First, all values in the pheromone matrix are decreased by a constant factor (evaporation) ($\tau_{h_i,h_j} = (1-\rho)\tau_{h_i,h_j}$ where $0 < \rho \leq 1$ is the pheromone evaporation rate). Then, only the pheromone trail value between the previously selected heuristic and the last selected heuristic is increased by using Equation 1.

$$\tau_{h_c,h_s} = \tau_{h_c,h_s} + \Delta\tau \qquad (1)$$

where h_c is the previously selected heuristic and h_s is the last selected heuristic. $\Delta\tau$ is the amount of pheromone trail value to be added and is defined as $\Delta\tau = 1/f_c$ where f_c is the fitness value of the new solution generated by applying the last selected heuristic h_s.

Kiraz and Topcuoglu (2010) tested { Simple Random, Random Descent, Random Permutation, Random Permutation Descent, Choice Function } across dynamic generalized assignment problem instances, extending a memory-based evolutionary algorithm with the use of hyper-heuristics. The results showed that Choice Function combined with the method which accepts all moves, outperformed the generic memory-based evolutionary algorithm. Kiraz et al (2011, 2013a) investigated the behavior of hyper-heuristics using a range of heuristic selection methods in combination with various move acceptance schemes on dynamic environment instances generated using the moving peaks benchmark generator. The results indicated the success of Choice Function across a variety of change dynamics, once again.

However, this time, the best move acceptance method, used together with Choice Function, accepted those new solution candidates which were better than or equal to the current solution candidate. A major difference between our approach and previous studies is that our approach uses a population of operators to create solutions, not a population of solutions directly. More on selection hyper-heuristics including their categorization, different components, application areas can be found in Özcan et al (2008); Chakhlevitch and Cowling (2008); Burke et al (2012)

2.2 Dynamic Environments

A dynamic environment problem contains one or more components which may change in time individually or simultaneously. For example, constraints of a given problem instance, objectives, or both may change in time. Branke (2002) identified the following criteria to categorize the change dynamics in an environment:

– *Frequency of change* indicates how often a change occurs,
– *Severity of change* is the magnitude of the change,
– *Predictability of change* is a measure of correlation between the changes,
– *Cycle length/cycle accuracy* is a characteristic defining whether an optimum returns exactly to previous locations or close to them in the search space, periodically.

In order to handle different types of change properties in the environment, a variety of strategies have been utilized which can be grouped under four main categories (Yaochu and Branke, 2005):

– strategies which maintain diversity at all times,
– strategies which increase diversity after a change,
– strategies which use implicit or explicit memory,
– strategies that work with multiple populations.

Most of the existing approaches for solving dynamic environment problems are based on evolutionary algorithms. The use of memory in evolutionary algorithms has been proposed to allow the algorithm to remember solutions which have been successful in previous environments. Commonly memory schemes used in evolutionary algorithms are either implicit, e.g. as in (Lewis et al, 1998; Uyar and Harmanci, 2005), or explicit, e.g. as in (Branke, 1999; Yang, 2007). The main benefit of using memory in an evolutionary algorithm is to enable the algorithm to detect and track changes in a given environment rapidly if the changes are periodic. For similar reasons, some algorithms make use of multiple populations, e.g. as in (Branke et al, 2000; Ursem, 2000; Wineberg and Oppacher, 2000). These approaches explore different regions of the search space by dividing the population into sub-populations. Each sub-population tracks several optima simultaneously in different parts of the search space.

The sentinel-based genetic algorithm (GA) (Morrison, 2004) is another multi-population approach to dynamic environments which makes use of solutions referred to as *sentinels*, uniformly distributed over the search space for maintaining diversity. Sentinels are fixed at the beginning of the search and in general, are not mutated or replaced during the search. Sentinels can be selected for mating and used during crossover. Due to having the sentinels distributed uniformly over the search space, the algorithm can recover quickly when the environment changes and the optimum moves to another location in the search space. Sentinels were reported to be effective in detecting and following the changes in the environment.

There is a growing interest in Statistical Model-based Optimization Algorithms which are adaptive and, thus, have the potential to react quickly to changes in the environment and track them. For example, EDAs, such as, Univariate marginal distribution algorithm (Ghosh and Muehlenbein, 2004), Bayesian optimization algorithm (Kobliha et al, 2006), and PBIL (Yang and Yao, 2005), are among the most common Statistical Model-based Optimization Algorithms used in dynamic environments. There are also some studies based on Statistical Model-based Optimization Algorithms for dynamic environments to estimate both time and direction (pattern) of changes (Simões and Costa, 2008a,b, 2009b,a).

The standard PBIL (PBIL) algorithm was first introduced by Baluja (1994). PBIL builds a probability distribution model based on a *probability vector*, $\overrightarrow{P}$ using a selected set of promising solutions to estimate a new set of candidate solutions. Learning and sampling are the key steps in PBIL. The initial population is sampled from the *central probability vector*, $\overrightarrow{P}_{central}$. During the search process, the probability vector $\overrightarrow{P}(t) = \{p_1, p_2, ..., p_l\}$ (l is the length) is learnt by using the best sample(s) $\overrightarrow{B}(t)$ at each t iteration as $p_i(t+1) := (1-\alpha)p_i(t)+\alpha B_i(t), \quad i = \{1, 2, ..., l\}$, where α is the learning rate. A bitwise mutation is applied to the probability vector for maintaining diversity. Then a set $S(t)$ of n candidate solutions are sampled from the updated probability vector as follows. For each locus i, if a randomly created number $r = rand(0.0, 1.0) < p_i$, it is set to 1; otherwise, it is set to 0. The process is repeated until the termination criteria are met.

In literature, several PBIL variants were proposed for dynamic environments (Yang, 2005b; Yang and Yao, 2005, 2008). One of them is a dual population PBIL

(PBIL2) introduced in Yang and Yao (2005). In PBIL2, the population is divided into two sub-populations. Each sub-population has its own probability vector. Both vectors are maintained in parallel. As in PBIL, the first probability vector $\overrightarrow{P}_1$ is initialized based on the central probability vector, while the second probability vector $\overrightarrow{P}_2$ is initialized randomly. The sizes of the initial sub-populations are equal. Thus, half of the population is initialized using $\overrightarrow{P}_1$, and the other half using $\overrightarrow{P}_2$. After all candidate solutions are evaluated, sub-population sample sizes are slightly adjusted. Then, each probability vector is learnt towards the best solution(s) in the relevant sub-population. Similar to PBIL, a bitwise mutation is applied to both probability vectors before sampling them to obtain the new set of candidate solutions.

Yang (2005b) proposed an explicit associative memory-based PBIL (MPBIL) approach. In memory-based PBIL, the best candidate solution along with the corresponding environmental information, i.e. the probability vector $\overrightarrow{P}(t)$ at a given time, is stored in the memory and it is retrieved when a new environment is encountered. The memory is updated every t generations using a stochastic time pattern based on $t_M = t+rand(5, 10)$, where t_M is the next memory update time. Whenever the memory is full and needs to be updated, first the memory point with its sample $\overrightarrow{B}_M(t)$ closest to the best candidate solution $\overrightarrow{B}(t)$ in terms of Hamming distances, is found. If the best candidate solution has a higher fitness than this memory sample, it is replaced by the candidate solution; otherwise, memory remains unchanged. When the best candidate solution $\overrightarrow{B}(t)$ is stored in the memory, the current working probability vector $\overrightarrow{P}(t)$ is also stored in the memory and is associated with $\overrightarrow{B}(t)$. Likewise, when replacing a memory point, the best candidate solution and the working probability vector replace both the sample and the associated probability vector within the memory point, respectively. The memory is re-evaluated every iteration in order to detect environment changes. When an environment change is detected, the memory probability vector associated with the best re-evaluated memory sample replaces the current working probability vector if the best memory sample is fitter than the best candidate solution created by the current working probability vector. If no environment change is detected, memory-based PBIL progresses just as the standard PBIL does.

Another PBIL variant; dual population memory-based PBIL (MPBIL2) was introduced by Yang and Yao (2008). This scheme employs both memory and multi-population approaches. Similar to PBIL2, $\overrightarrow{P}_1$ is initialized with the central probability vector, and the second probability vector $\overrightarrow{P}_2$ is initialized randomly. The size of each population in dual population memory-based PBIL is adjusted according to its individual performance. When it is time to update the memory, the working probability vector that creates the best overall sample, i.e., the winner of $\overrightarrow{P}_1$ and $\overrightarrow{P}_2$, is stored together with the best sample in the memory, if it is fitter than the closest memory sample. The memory is re-evaluated every iteration. When an environment change is detected, only $\overrightarrow{P}_1$ is replaced by the best memory probability vector, if the associated memory sample is fitter than the best candidate solution generated by $\overrightarrow{P}_1$. This is to avoid having $\overrightarrow{P}_1$ and $\overrightarrow{P}_2$ converge to the same values.

All variants of PBILs using restart and random immigrant schemes were investigated in Yang and Yao (2008). According to the experimental results, the dual population memory-based PBIL approach with restart outperforms other techniques. Cao and Luo (2010) introduced different associative memory updating strategies inspired from memory-based PBIL (Yang and Yao, 2008). The empirical results indicate that the environmental information based updating strategy gives better results only in cyclic dynamic environments. A direct memory scheme and its interaction with random immigrants is examined for Univariate marginal distribution algorithm in Yang (2005a). Yang and Richter (2009) introduced a hyper-learning scheme using restart and hypermutation in PBIL. Moreover, a multi-population scheme is applied successfully to Univariate marginal distribution algorithm by (Wu et al, 2010a,b). Xingguang et al (2011) investigated an environment-triggered population diversity control approach for memory enhanced Univariate marginal distribution algorithm, while Peng et al (2011) examined an environment identification-based memory management scheme for binary coded EDAs.

An EDA based approach in continuous domains has been implemented based on online Gaussian mixture model by Goncalves and Zuben (2011). The proposed online learning approach outperformed mainly in high-frequency changing environments. Yuan et al (2008) implemented continuous Gaussian model EDAs and investigated their potential for solving dynamic optimization problems. Bosman (2005) investigated online time-linkage real valued problems and analyzed how remembering information from the past can help to find new solutions.

EDAs have been applied with good results to some real world problems, such as inventory management problems (Bosman, 2005), the dynamic task allocation problem (Barlow and Smith, 2009) and the dynamic pricing model (Shakya et al, 2007). The main drawback of the EDA-based approaches, such as Univariate marginal distribution algorithm and PBIL, is diversity loss. Some strategies are used to cope with converging to local optima. Fernandes et al (2008b) proposed a new update strategy for the probability model in Univariate marginal distribution algorithm, based on Ant Colony Optimization transition probability equations. The experimental results showed that the proposed strategies increase the adaptation ability of Univariate marginal distribution algorithm in uncertain environments. Li et al (2011) introduced a new Univariate marginal distribution algorithm, referred to as *transfer model* to enhance the diversity of the population. The results show that the proposed algorithm can adapt in dynamic environments, rapidly.

There are different benchmark generators in literature for dynamic environments. The Moving Peaks Benchmark generator (Branke, 2002) is commonly used in continuous domains, while in discrete domains the XOR dynamic problem generator (Yang, 2004, 2005a) is preferred. In this study, we use the XOR dynamic problem generator for creating dynamic environment problems with various degrees of difficulty from any binary-encoded stationary problem using a bitwise exclusive-or (XOR) operator. Given a function $f(\mathbf{x})$ in a stationary environment and $\mathbf{x} \in \{0,1\}^l$, the fitness value of the $\mathbf{x}$ at a given generation g is calculated as $f(\mathbf{x}, g) = f(\mathbf{x} \oplus m_k)$, where m_k is a binary mask for k^{th} stationary environment and $\oplus$ is the XOR operator. Firstly, the mask m is initialized with a zero vector. Then, every τ generations, the mask m_k is changed as $m_k = m_{k-1} \oplus t_k$, where t_k is a binary template.

3 A Hybrid Framework for Dynamic Environments

In this section, we describe our multi-phase hybrid framework, referred to as hyper-heuristic based dual population EDA (HH-EDA2), for solving dynamic environment problems. Our initial investigations in (Uludağ et al, 2012a,b) indicated that this framework has potential for solving dynamic environment problems. Therefore, in this paper, we extend our studies further and provide an analysis of this framework across a variety of dynamic environment problems with different change properties produced by the XOR dynamic problem generator and explore further enhancements and modifications.

Although we chose PBIL2 as the EDA component in our studies, the proposed hybrid framework can combine any multi-population EDA with any selection hyper-heuristic in order to exploit the strengths of both approaches.

HH-EDA2 consists of two main phases: offline learning and online learning. In the offline learning phase, a number of masks to be used in the XOR generator are sampled over the search space. The search space is divided into M sub-spaces and a set of masks is generated randomly in each sub-space, thus making the masks distributed well over the landscape. For the XOR generator, each mask corresponds to a different environment. Then, for each environment (represented by each mask) PBIL is executed. As a result of this, good probability vectors $\overrightarrow{P}list$ corresponding to a set of different environments are learned in an offline manner. These learned probability vectors are stored for later use during the online learning phase of HH-EDA2.

In the online learning phase, the probability vectors $\overrightarrow{P}list$, serve as the low-level heuristics, which a selection hyper-heuristic manages. Figure 1 shows a simple diagram illustrating the structure and execution of HH-EDA2.

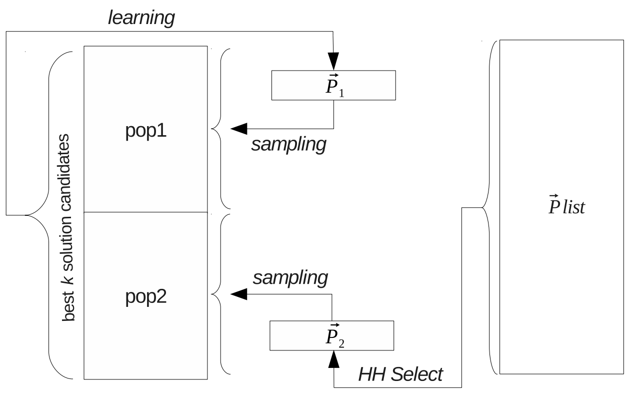


Fig. 1 The framework of HH-EDA2

The online learning phase of the HH-EDA2 framework uses the PBIL2 approach, explained in Section 2. Similar to PBIL2, the population is divided into two sub-populations and two probability vectors, one for each sub-population, are used simultaneously. As seen in Figure 1, *pop*1 represents the first sub-population and $\overrightarrow{P}_1$ is its corresponding probability vector; *pop*2 represents the second sub-population and $\overrightarrow{P}_2$ is its corresponding probability vector. *HH Select* shows selection of $\overrightarrow{P}_2$ in P_{list} using heuristic selection methods. The pseudocode of the proposed HH-EDA2 is shown in Algorithm 1.

In HH-EDA2, the first probability vector $\overrightarrow{P}_1$ is initialized to $\overrightarrow{P}_{central}$, and the second probability vector $\overrightarrow{P}_2$ is initialized to a randomly selected vector from $\overrightarrow{P}list$. Initial sub-populations of equal sizes are sampled independently from their own probability vectors. Af-

Algorithm 1 Pseudocode of the proposed HH-EDA2 approach

```
1: $t := 0$
2: initialize $\overrightarrow{P}_1(0) := \overrightarrow{0.5}$
3: $\overrightarrow{P}_2(0)$ is selected from $\overrightarrow{P}list$
4: $S_1(0) := sample(\overrightarrow{P}_1(0))$ and $S_2(0) := sample(\overrightarrow{P}_2(0))$
5: while (termination criteria not fulfilled) do
6:     evaluate $S_1(t)$ and evaluate $S_2(t)$
7:     adjust next population sizes for $\overrightarrow{P}_1(t)$ and $\overrightarrow{P}_2(t)$ respectively
8:     place $k$ best samples from $S_1(t)$ and $S_2(t)$ into $\overrightarrow{B}(t)$
9:     send best fitness from whole/second population to heuristic selection component
10:    learn $\overrightarrow{P}_1(t)$ toward $\overrightarrow{B}(t)$
11:    mutate $\overrightarrow{P}_1(t)$
12:    $\overrightarrow{P}_2(t)$ is selected using heuristic selection
13:    $S_1(t) := sample(\overrightarrow{P}_1(t))$ and $S_2(t) := sample(\overrightarrow{P}_2(t))$
14:    $t := t + 1$
15: end while
```

ter the fitness evaluation process, sub-population sample sizes are slightly adjusted within the range $[0.3 * n,\ 0.7 * n]$ according to their best fitness values. At each iteration, if the best candidate solution of the first sub-population is better than the best candidate solution of the second sub-population, the sample size of the first sub-population, n_1 is determined by $min(n_1 + 0.05 * n, 0.7 * n)$; otherwise n_1 is defined by $min(n_1 - 0.05 * n, 0.3 * n)$. While, $\overrightarrow{P}_1$ is learned towards the best solution candidate(s) in the whole population and mutation is applied to $\overrightarrow{P}_1$, $\overrightarrow{P}_2$ is selected using the heuristic selection methods from $\overrightarrow{P}list$. No mutation is applied to $\overrightarrow{P}_2$. Then, the two sub-populations are sampled based on their respective probability vectors. The approach repeats this cycle until some termination criteria are met. In the HH-EDA2 framework, different heuristic selection methods can be used for selecting the second probability vector from $\overrightarrow{P}list$.

4 Experiments

In this study, we performed four groups of experiments. In the first group, we investigated the influence of different heuristic selection methods on the performance of the proposed framework, to determine the most suitable one for dynamic environment problems. In the second group of experiments, the proposed framework, incorporating the chosen heuristic selection scheme, is compared to similar methods from literature. The third and fourth group of experiments focus on the offline and online learning components of the framework, respectively. We explore the influence of the time spent for offline learning on the performance of the overall approach. For the online learning component, we explore the effects of the learning parameter on the overall performance and then propose and analyze an adaptive version.

4.1 Experimental Design and Settings

In this subsection, we explain the dynamic environment problems used in the experiments and present the general parameter settings for all experiments. Moreover, approaches from literature that are implemented for comparisons and the details of their settings are also provided in this section. Further parameter settings specific to each experiment will be given in the relevant subsections.

4.1.1 Benchmark Problems and Settings of Algorithms

We use three Decomposable Unitation-Based Functions (DUFs) Yang and Yao (2008) within the XOR generator. All Decomposable Unitation-Based Functions are composed of 25 copies of 4-bit building blocks. Each building block is denoted as a unitation-based function $u(x)$ which gives the number of ones in the corresponding building block. Its maximum value is 4. The fitness of a bit string is calculated as the sum of the $u(x)$ values of the building blocks. The optimum fitness value for all Decomposable Unitation-Based Functions is 100. DUF1 is the *OneMax* problem whose objective is to maximize the number of ones in a bit string. DUF2 has a unique optimal solution surrounded by four local optima and a wide plateau with eleven points having a fitness of zero. DUF2 is more difficult than DUF1. DUF3 is fully deceptive. The mathematical formulations of the Decomposable Unitation-Based Functions, as given in Yang and Yao (2008), can be seen below.

$$f_{DUF1} = u(x) \tag{2}$$

$$f_{DUF2} = \begin{cases} 4 \text{ , if } u(x) = 4 \\ 2 \text{ , if } u(x) = 3 \\ 0 \text{ , if } u(x) < 3 \end{cases} \tag{3}$$

$$f_{DUF3} = \begin{cases} 4 & \text{, if } u(x) = 4 \\ 3 - u(x) & \text{, if } u(x) < 4 \end{cases} \tag{4}$$

In the offline learning phase, first a set of M XOR masks are generated. In order to have the XOR masks distributed uniformly on the search space, an approach similar to stratified sampling is used. Then, for each mask, PBIL is executed for 100 independent runs where each run consists of G generations. During offline learning, each environment is stationary and 3 best candidate solutions are used to learn probability vectors. The population size is set to 100. At the end of the offline learning stage, the probability vector producing the best solution found so far over all runs for each environment, is stored in $\overrightarrow{P}list$. The parameter settings for PBIL used in this stage is given in Table 1

Table 1 Parameter settings for PBILs

Parameter	Setting	Parameter	Setting
Solution length	100	Mutation rate P_m	0.02
Population size	100	Mutation shift δ_m	0.05
Number of runs	100	Learning rate α	0.25

After the offline learning stage, we experiment with four main types of dynamic environments: randomly changing environments (Random), environments with cyclic changes of type 1 (Cyclic1), environments with cyclic changes of type 1 with noise (Cyclic1-with-Noise) and environments with cyclic changes of type 2 (Cyclic2). In the Cyclic1 type environments, the masks representing the environments, which repeat in a cycle, are selected from among the sampled M masks used in the offline learning phase of HH-EDA2. To construct Cyclic1-with-Noise type environments, we added a random bitwise noise to the masks used in the Cyclic1 type environments. In Cyclic2 type environments, the masks representing the environments, which repeat in a cycle, are generated randomly.

To generate dynamic environments showing different dynamism properties, we consider different change frequencies τ, change severities ρ and cycle lengths CL. We determined the change periods which correspond to low frequency (LF), medium frequency (MF) and high frequency (HF) changes as a result of some preliminary experiments where we executed PBIL on stationary versions of all the Decomposable Unitation-Based Functions. The corresponding convergence plots for the Decomposable Unitation-Based Functions are given in Figure 4. As can be seen in the plots, the selected settings for low frequency, medium frequency and high frequency for each Decomposable Unitation-Based Function correspond respectively to stages where the PBIL algorithm has been converged for some time, where it has not yet fully converged and where it is very early on in the search. Table 2 shows the determined change periods for each Decomposable Unitation-Based Function.

Table 2 The value of the change periods

Functions	LF	MF	HF
DUF1	50	25	5
DUF2	50	25	5
DUF3	100	35	10

In the Random type environments, the severity of changes are determined based on the definition of the XOR generator and are chosen as 0.1 for low severity (LS), 0.2 for medium severity (MS), 0.5 for high severity (HS), and 0.75 for very high severity (VHS) changes. For all types of cyclic environments, the cycle lengths CL are selected as 2, 4 and 8. Except for Cyclic1-with-Noise type of environments, the environments return to their exact previous locations.

In our previous study Uludağ et al (2012b), we explored the effects of restart schemes for HH-EDA2. Our experiments showed that a restart scheme significantly improves the performance of HH-EDA2. In the best performing restart scheme for HH-EDA2, only the first probability vector $\overrightarrow{P}_1$ is reset to the to $\overrightarrow{P}_{central}$, whenever an environment change is detected.

Since HH-EDA2 is a multi-population approach, which also uses a kind of memory, for our comparison experiments, we focused on memory based approaches as well as multi-population ones which were shown in literature to be successful in dynamic environments. Therefore, we used different variants of PBILs with restart schemes and a sentinel-based genetic algorithm.

In Yang and Yao (2008), experiments show that a restart scheme combined with the multi-population PBIL, significantly outperforms dual population memory-based PBIL on most Decomposable Unitation-Based Functions in different kinds of dynamic environments. In the version of PBIL that utilizes a restart scheme (PBILr), the probability vector $\overrightarrow{P}$ is reset to $\overrightarrow{P}_{central}$ when an environment change is detected. In the version of PBIL2 that utilizes a restart scheme (PBIL2r), whenever an environment change is detected, only the first probability vector $\overrightarrow{P}_1$ is reset to $\overrightarrow{P}_{central}$. In the restart variant for memory-based PBIL (MPBILr), the probability vector $\overrightarrow{P}$ is reset to $\overrightarrow{P}_{central}$ when change is detected. The parameter settings of memory-based PBIL with restart are the same as the PBIL used in the offline learning phase 1. In the dual population memory-based PBIL approach with a restart scheme (MPBIL2r), whenever an environment change is detected, the second probability vector $\overrightarrow{P}_2$ is reset to $\overrightarrow{P}_{central}$. The population size n is set to 100 and the memory size is fixed to $0.1{*}n = 10$. Initial sub-populations are $0.45 * n = 45$ and sub-population sample sizes are slightly adjusted within the range of $[30, 60]$. The memory is updated using a stochastic time pattern. After each memory update, the next memory updating time is set as $t_M = t + rand(5, 10)$.

For the sentinel-based genetic algorithm, we used tournament selection where the tournament size is 2, uniform crossover with a probability of 1.0, mutation with a mutation rate of $1/l$ where l is the chromosome length. The population size is set to 100. We tested two different values for the number of sentinels: 8 and 16. These values are chosen for two reasons. First of all, (Morrison, 2004) suggests working with 10% of the population as sentinels. Secondly, in our previous study, we experimented with storing $M = 8$ and $M = 16$ probability vectors in $\overrightarrow{P}list$ for HH-EDA2 and found $M = 8$ to be better. At the beginning of the search, sentinels are initialized to locations of the masks representing different parts of the search space. For HH-EDA2, the masks used in the offline learning stage were chosen in such a way as to ensure that they are distributed uniformly on the search space. Therefore $M = 8$ or $M = 16$ masks are used as the sentinels.

Both in PBIL2 and HH-EDA2, each sub-population size is initialized as 50 and adjusted within the range of [30, 70].

In Reinforcement Learning, score of each heuristic is initialized to 15 and is allowed to vary between 0 and 30. If the selected heuristic yields a solution with an improved fitness, its score is increased by 1, otherwise it is decreased by 1. The Reinforcement Learning settings are taken as recommended in Özcan et al (2010).

In (Kiraz et al, 2013b), the results show that Ant-based Selection with roulette wheel selection is better than the version with tournament selection. Therefore, we work Ant-based Selection with roulette wheel selection in this paper. In (Kiraz et al, 2013b), $\Delta\tau$ is calculated as $\Delta\tau = 0.1 * (1/f_c)$ so that pheromone values increase gradually. For Ant-based Selection, q_0 and ρ are set to 0.5 and 0.1, respectively. These are the settings recommended in (Kiraz et al, 2013b).

For each run of the algorithms, 128 changes occur after the initial environment. Therefore, the total number of generations in a run is calculated as $maxGenerations = changeFrequency * changeCount$.

4.1.2 Performance Evaluation Criteria

In order to compare the performance of the algorithms, the results are reported in terms of offline error Branke (2002), which is calculated as the cumulative average of the differences between the best values found so far and the optimum value at each time step, as given below.

$$\frac{1}{T}\sum_{t=1}^{T} \mid opt_t - e_t* \mid \tag{5}$$

$e*_t = max(e_\tau, e_{\tau+1}, ..., e_t)$ where T is the total number of evaluations and τ is the last time step ($\tau < t$) when change occurred.

In the result tables, each entry shows the average offline error values averaged over 100 independent runs. In the rows of the tables, we can see the performance of each approach under a variety of change frequency-severity pair settings in randomly changing environments and under different cycle length and change frequency settings in cyclic environments for three Decomposable Unitation-Based Functions. Each column shows the performance of all the approaches for the corresponding change frequency-severity pair settings in randomly changing environments and for the cycle length-change frequency pair settings in cyclic environments. In addition, in all the tables, the best performing approach(es) in each row is marked in bold.

We also perform One-way ANOVA and Tukey HSD tests at a confidence level of 95% for testing whether the differences between the approaches are statistically significant or not. To provide a summary of the statistical comparison results, we count the number of times an approach obtains a significance state over the others on the three Decomposable Unitation-Based Functions for different change severity and frequency settings in randomly changing environments and for different cycle length and change frequency settings in cyclic environments. In the tables providing the summary of statistical comparisons, $s+$ shows the total number of times the corresponding approach performs statistically better than the others and $s-$ shows the vice versa; $\geq$ shows the total number of times the corresponding approach performs slightly better than the others, however, the performance difference is not statistically significant and $\leq$ shows the vice versa.

To compare the performance of approaches over different dynamic environments, the approaches are scored in the same way as in the CHeSC competition [1]. The scoring system in CHeSC is based on the Formula 1 scoring system used before 2010. For each approach, median, best and average values over 100 runs are calculated. Then, the results of the approaches are sorted with respect to these values. The top 8 approaches eventually get the following points for each problem instance: 10, 8, 6, 5, 4, 3, 2 and 1, respectively. The sum of scores over all problem instances is the final score of an algorithm. Considering random and cyclic environments, there are 117 problem instances, therefore, 1170 is the maximum overall score that an algorithm can get in this scoring system.

4.2 Results

In this subsection, we provide and discuss the results of each group of experiments separately.

[1] http://www.asap.cs.nott.ac.uk/external/chesc2011/

4.2.1 Comparison of heuristic selection methods

In this set of experiments, we test different heuristic selection methods within the proposed framework. The tested heuristic selection methods are Simple Random (SR), Random Descent (RD), Random Permutation (RP), Random Permutation Descent (RPD), Reinforcement Learning (RL) and Ant-based Selection (AbS). We use all change frequency and severity settings for the Random dynamic environments; we also use all change frequency and cycle length settings for the Cyclic1, Cylic1-with-Noise and Cyclic2 type dynamic environments. Tests are performed on all Decomposable Unitation-Based Functions, i.e. DUF1, DUF2 and DUF3. The results are summarized in Table 3 and 4. Table 3 provides the statistical comparison summary, whereas Table 4 shows the ranking results obtained based on median, best and average offline error values.

Table 3 Overall ($s+$, $s-$, $\geq$ and $\leq$) counts for the different heuristic selection schemes.

Heuristic Selection	$s+$	$s-$	$\geq$	$\leq$
RP	247	58	192	88
RPD	196	68	122	199
SR	139	123	197	126
AbS	129	173	148	135
RD	91	181	144	169
RL	52	251	98	184

Table 4 The overall score according to the Formula 1 ranking based on median, best and average offline error values for the different heuristic selection schemes.

Heuristic Selection	Median	Best	Average
RP	930	908	927
SR	738	706	733
RPD	737	767	731
AbS	668	626	688
RD	602	626	605
RL	537	579	528

As seen in Table 3, Random Permutation generates the best average performance across all dynamic environment problems, performing significantly/slightly better than the rest for 247/192 instances. The second best approach is Random Permutation Descent on average. Random Permutation is still the best approach if the median and best performances are considered as well (Table 4) based on the Formula 1 ranking. It can be seen from the table that Random Permutation scores 930 and 908, respectively. Learning via the PBIL process helps, but using an additional learning mechanism on top of that turns out to be misleading for the search process. For example, the use of reinforcement learning in the selection hyper-heuristic (RL) yields the worst average performance. Random Permutation as a non-learning heuristic selection combines the learnt probability vectors effectively yielding an improved performance which outperforms Simple Random.

For randomly changing environments, all heuristic selection schemes performed well and there were no statistically significant differences between the results. The results show that for DUF1 and DUF2, in the tested cyclic environments, Random Permutation performs the best as a heuristic selection method in the HH-EDA2 framework. For DUF3, Random Permutation Descent seems to produce better results than Random Permutation, however this performance difference is not statistically significant and actual offline error values from Random Permutation are close to the ones produced by Random Permutation Descent. Due to space limitations, these results are omitted here.

4.2.2 Comparisons to selected approaches from literature

In this set of experiments, we compare our approach to some well known and successful previously proposed approaches from literature as described in Section 4.1.1. As a result of the experiments in Subsection 4.2.1, we fixed the heuristic selection component as Random Permutation during these experiments and used the same problems, change settings and dynamic environment types as in Subsection 4.2.1.

In a randomly changing environment, HH-EDA2 outperforms the rest of the previously proposed approaches on DUF1 and DUF2 regardless of the frequency or severity of the changes as illustrated in Tables 5 and 6, based on average offline error values, respectively. The same phenomenon is observed for DUF3, except for the low frequency cases (see Table 7). HH-EDA2 performs the best only when the changes occur at a low frequency and a very high severity on DUF3. On the other hand, sentinel-based genetic algorithm with 16 sentinels performs the best for the low and high severity change cases, while memory-based PBIL approaches with restart perform the best for the changes with medium severity on DUF3.

In a cyclically changing environment of type 1 with and without noise, HH-EDA2 again outperforms the rest of the previously proposed approaches on DUF1 and DUF2 regardless of the cycle length or frequency of change as illustrated in Tables 8 and 9 based on average offline error values, respectively. For the Cyclic2 case, HH-EDA2 still performs the best on DUF1, except when the changes occur at a high frequency and

Table 5 Offline errors generated by different approaches averaged over 100 runs, on the DUF1 for different change severity and frequency settings in randomly changing environments.

Algorithm	LF				MF				HF			
	LS	MS	HS	VHS	LS	MS	HS	VHS	LS	MS	HS	VHS
Random												
HH-EDA2	**0.06**	**0.06**	**0.08**	**0.09**	**0.17**	**0.25**	**0.86**	**0.99**	**21.94**	**23.60**	**26.79**	**28.26**
PBILr	4.13	7.84	16.71	21.75	9.51	16.24	26.68	30.44	27.91	33.91	38.02	38.76
PBIL2r	3.47	7.20	16.16	20.76	9.04	15.80	25.95	29.14	27.56	33.32	37.23	38.11
MPBILr	0.56	0.67	0.91	0.09	1.84	2.21	2.78	1.29	26.88	28.66	30.33	30.41
MPBIL2r	0.67	0.81	1.02	0.11	4.85	4.30	4.32	2.83	26.98	29.70	31.52	31.60
Sentinel8	20.11	20.20	4.40	0.78	22.84	23.00	11.83	9.21	28.29	29.43	32.12	33.91
Sentinel16	7.26	9.10	2.21	1.19	12.35	15.10	12.42	12.36	24.36	27.56	31.91	33.57

Table 6 Offline errors generated by different approaches averaged over 100 runs, on the DUF2 for different change severity and frequency settings in randomly changing environments.

Algorithm	LF				MF				HF			
	LS	MS	HS	VHS	LS	MS	HS	VHS	LS	MS	HS	VHS
Random												
HH-EDA2	**0.12**	**0.16**	**0.49**	**0.53**	**0.43**	**0.85**	**4.13**	**4.54**	**42.92**	**45.74**	**50.86**	**52.95**
PBILr	8.96	18.45	38.93	45.80	20.58	34.65	51.43	54.83	52.69	60.41	65.11	65.70
PBIL2r	7.66	17.29	37.03	42.99	19.51	33.58	49.67	52.67	51.55	59.38	64.05	64.46
MPBILr	0.98	1.23	1.81	1.92	4.81	5.48	6.78	7.07	51.11	53.81	55.77	56.25
MPBIL2r	1.51	1.74	2.14	1.97	12.28	10.50	10.40	10.61	51.50	55.02	57.46	57.94
Sentinel8	39.11	38.87	13.82	3.33	43.52	42.72	27.69	23.73	51.33	52.93	57.52	59.64
Sentinel16	16.00	20.25	8.75	5.08	25.71	30.43	28.11	28.38	45.78	50.67	57.41	59.49

Table 7 Offline errors generated by different approaches averaged over 100 runs, on the DUF3 for different change severity and frequency settings in randomly changing environments.

Algorithm	LF				MF				HF			
	LS	MS	HS	VHS	LS	MS	HS	VHS	LS	MS	HS	VHS
Random												
HH-EDA2	19.44	18.46	16.04	**14.18**	**19.75**	**18.99**	**17.26**	**15.49**	**38.44**	**39.99**	**41.29**	**40.75**
PBILr	25.44	25.85	23.96	19.49	30.06	33.11	35.27	31.56	40.11	44.48	47.18	45.85
PBIL2r	24.98	25.23	23.15	18.55	29.38	32.42	34.37	30.73	39.55	43.66	46.41	45.07
MPBILr	17.10	**17.01**	17.02	16.91	19.20	19.26	19.34	19.04	44.67	45.78	46.28	46.08
MPBIL2r	18.18	18.04	17.48	17.24	24.07	23.18	21.60	21.87	41.18	44.93	46.49	45.82
Sentinel8	36.98	33.85	17.59	22.54	40.04	36.56	26.88	28.13	43.07	41.79	43.88	41.74
Sentinel16	**14.66**	20.51	**14.05**	17.26	31.04	31.22	30.94	27.82	38.94	42.76	46.20	45.59

the cycle length is low (2 and 4). For those problem instances, PBIL with restart approaches perform better. For DUF2 of type Cyclic2, HH-EDA2 is the best approach when the frequency of change is medium.

HH-EDA2 delivers a poor performance on DUF3 for all cases, except for the high frequency cases for Cyclic1 with and without noise (see Table 10). The advantage of combining offline learning and online learning mechanisms disappear when the problem being solved is deceptive. The use of sentinels produces a better performance on DUF3 for a change type of Cyclic2.

It should be noted that only for the sentinel-based genetic algorithm schemes and HH-EDA2, cyclic environments of type 1 and type 2 are different. In cyclic environments of type 1, the environment cycles between environments represented by the masks used in the offline learning stage. These masks are used as sentinels in the sentinel-based genetic algorithm schemes and the probability vectors obtained as a result of training on these masks are used as low level heuristics in HH-EDA2.

An overall comparison of all approaches are provided in Tables 11 and 12. HH-EDA2 generates the best average performance across all dynamic environment problems (Table 11) performing significantly/slightly better than the rest for 609/18 instances. The second best approach is memory-based PBIL using a single population and restart. Moreover, HH-EDA2 is the top approach if the median and best performances are considered as well (see Table 12) based on Formula 1 rankings, scoring 1035 and 998, respectively. The closest competitor accumulates a score of 711 and 639 for its

Table 8 Offline errors generated by different approaches averaged over 100 runs, on the DUF1 for different cycle length and change frequency settings in different cyclic dynamic environments.

Algorithm	LF			MF			HF		
	CL=2	CL=4	CL=8	CL=2	CL=4	CL=8	CL=2	CL=4	CL=8
Cyclic1									
HH-EDA2	**0.03**	**0.02**	**0.02**	**0.05**	**0.04**	**0.05**	**14.20**	**13.82**	**14.59**
PBILr	11.96	14.69	17.09	15.47	19.11	25.64	18.73	20.59	28.65
PBIL2r	10.65	13.78	16.53	12.10	17.76	24.54	19.02	20.44	28.43
MPBILr	0.08	0.08	1.76	1.30	1.29	4.59	30.42	30.39	30.30
MPBIL2r	0.12	0.11	1.98	3.98	3.36	6.18	19.85	21.61	29.53
Sentinel8	2.54	6.02	4.94	19.02	14.05	10.19	23.30	24.21	31.88
Sentinel16	9.04	1.33	3.55	14.81	11.96	13.19	19.37	33.08	32.96
Cyclic1-with-Noise									
HH-EDA2	**0.02**	**0.02**	**0.02**	**0.05**	**0.04**	**0.05**	**14.48**	**13.86**	**14.66**
PBILr	11.93	14.59	17.06	15.50	19.09	25.70	18.69	20.57	28.54
PBIL2r	10.64	13.78	16.48	12.03	17.75	24.52	19.12	20.51	28.41
MPBILr	0.08	0.08	1.76	1.29	1.29	4.59	30.40	30.40	30.29
MPBIL2r	0.12	0.11	1.98	3.97	3.36	6.16	19.99	21.50	29.47
Sentinel8	2.49	5.97	4.93	19.11	13.97	10.18	23.23	24.11	31.93
Sentinel16	9.05	1.35	3.59	14.80	11.92	13.24	19.37	33.05	32.93
Cyclic2									
HH-EDA2	**0.08**	**0.08**	**0.08**	**0.85**	**0.86**	**0.89**	25.83	26.80	**26.98**
PBILr	11.67	15.75	17.18	15.16	21.39	25.60	**18.39**	24.77	30.15
PBIL2r	10.44	14.94	16.63	11.91	20.03	24.36	18.82	24.63	29.85
MPBILr	**0.08**	**0.08**	**0.08**	1.30	1.29	1.30	30.39	30.40	30.43
MPBIL2r	0.12	0.11	0.11	4.03	3.03	2.83	19.54	**25.88**	30.55
Sentinel8	0.50	4.79	5.11	19.89	17.65	11.81	25.38	28.02	33.17
Sentinel16	1.17	1.31	1.73	14.70	12.08	12.36	22.11	33.36	33.34

Table 9 Offline errors generated by different approaches averaged over 100 runs, on the DUF2 for different cycle length and change frequency settings in different cyclic dynamic environments.

Algorithm	LF			MF			HF		
	CL=2	CL=4	CL=8	CL=2	CL=4	CL=8	CL=2	CL=4	CL=8
Cyclic1									
HH-EDA2	**0.04**	**0.04**	**0.04**	**0.09**	**0.08**	**0.08**	**27.33**	**27.38**	**26.53**
PBILr	23.57	32.85	38.10	25.14	38.31	47.93	29.92	38.36	50.70
PBIL2r	19.97	30.46	35.90	21.67	35.70	45.52	30.76	38.50	50.45
MPBILr	0.23	14.39	6.76	4.39	20.65	11.89	55.93	55.31	55.35
MPBIL2r	0.51	11.68	6.76	12.31	22.22	14.95	32.37	39.73	51.32
Sentinel8	33.16	15.29	10.92	35.72	33.72	23.56	39.09	46.03	57.70
Sentinel16	21.11	6.21	11.01	28.68	27.47	28.85	35.43	58.80	59.14
Cyclic1-with-Noise									
HH-EDA2	**0.04**	**0.04**	**0.05**	**0.08**	**0.09**	**0.09**	**26.96**	**26.37**	**27.34**
PBILr	23.56	32.79	38.04	24.86	38.12	47.94	29.63	38.33	50.66
PBIL2r	19.89	30.42	35.85	21.77	35.66	45.64	30.79	38.54	50.35
MPBILr	0.24	14.38	6.77	4.39	20.65	11.96	55.97	55.33	55.36
MPBIL2r	0.53	11.74	6.78	12.30	22.25	14.88	32.57	39.81	51.39
Sentinel8	33.12	15.61	10.93	35.64	33.87	23.58	39.27	45.97	57.65
Sentinel16	21.16	6.24	11.08	28.73	27.54	28.78	35.36	58.79	59.11
Cyclic2									
HH-EDA2	0.45	0.46	**0.51**	**3.93**	**4.06**	**4.25**	49.34	**50.82**	**51.20**
PBILr	24.00	34.29	39.54	26.38	41.27	49.84	**30.39**	43.54	53.57
PBIL2r	20.20	32.08	37.46	22.54	38.68	47.46	31.04	43.69	53.02
MPBILr	**0.25**	**0.24**	2.06	4.38	4.38	7.36	55.95	55.92	55.62
MPBIL2r	0.51	0.39	2.36	12.23	8.97	10.87	32.74	45.69	53.97
Sentinel8	4.79	12.58	13.68	37.78	36.54	27.94	41.23	51.41	58.90
Sentinel16	5.04	5.81	7.25	29.84	27.87	28.38	39.86	59.35	59.36

Table 10 Offline errors generated by different approaches averaged over 100 runs, on the DUF3 for different cycle length and change frequency settings in different cyclic dynamic environments.

Algorithm	LF			MF			HF		
	CL=2	CL=4	CL=8	CL=2	CL=4	CL=8	CL=2	CL=4	CL=8
Cyclic1									
HH-EDA2	10.09	11.36	11.33	10.35	11.60	11.58	**22.23**	**22.42**	**22.76**
PBILr	24.11	24.34	23.10	29.51	34.91	33.94	26.72	37.40	41.63
PBIL2r	23.37	23.60	22.22	28.41	33.86	33.02	26.43	36.52	40.57
MPBILr	16.76	16.79	16.79	18.87	18.85	18.88	46.64	46.63	46.59
MPBIL2r	17.54	17.25	17.35	24.14	21.60	21.45	27.67	38.32	42.24
Sentinel8	**2.51**	**2.19**	**3.27**	**7.32**	**6.22**	**9.05**	24.22	24.87	24.91
Sentinel16	3.21	3.38	3.41	9.72	10.90	11.04	24.78	24.95	24.93
Cyclic1-with-Noise									
HH-EDA2	10.09	11.35	11.34	10.35	11.59	11.59	**22.21**	**23.20**	**23.20**
PBILr	24.05	24.38	23.05	29.65	35.02	33.91	26.71	37.61	41.54
PBIL2r	23.40	23.62	22.24	28.38	33.84	33.00	26.35	36.38	40.53
MPBILr	16.81	16.78	16.82	18.86	18.84	18.86	46.62	46.70	46.60
MPBIL2r	17.50	17.23	17.33	24.08	21.54	21.45	28.13	38.21	42.23
Sentinel8	**2.49**	**2.20**	**3.28**	**7.35**	**6.26**	**9.11**	24.30	24.89	24.90
Sentinel16	3.17	3.39	3.38	9.82	10.99	10.94	24.74	24.95	24.93
Cyclic2									
HH-EDA2	16.27	16.60	16.02	17.47	17.73	17.24	40.67	41.19	41.34
PBILr	24.86	24.61	23.68	30.67	35.05	35.22	27.22	37.87	42.89
PBIL2r	24.10	23.86	22.84	29.41	33.85	34.34	27.03	37.23	41.72
MPBILr	16.80	16.80	17.17	18.87	18.86	19.85	46.69	46.67	45.78
MPBIL2r	17.32	17.27	17.64	23.80	21.54	21.96	28.31	38.60	43.25
Sentinel8	**2.27**	**2.24**	**2.71**	**6.57**	**6.46**	**7.80**	**24.80**	**24.90**	**24.90**
Sentinel16	3.36	3.58	3.44	10.71	12.00	11.10	24.89	24.96	24.95

median and best performances, respectively. These results also indicate that the use of a dual population and the selection hyper-heuristic both improves the performance of the overall algorithm.

Table 11 Overall ($s+$, $s-$, $\geq$ and $\leq$) counts for the algorithms used

Algorithm	$s+$	$s-$	$\geq$	$\leq$
HH-EDA2	609	72	18	3
MPBILr	390	278	20	14
MPBIL2r	367	297	7	31
Sentinel16	335	346	5	16
Sentinel8	298	384	15	5
PBIL2r	236	442	14	10
PBILr	132	548	11	11

Table 12 The overall score according to the Formula 1 ranking based on median, best and average offline error values for the algorithms used

Algorithm	Median	Best	Average
HH-EDA2	1035	998	1035
MPBILr	711	639	709
MPBIL2r	608	812	611
Sentinel16	606	581	606
Sentinel8	598	583	598
PBIL2r	498	468	497
PBILr	390	365	390

4.2.3 Duration of offline learning

In this set of experiments, we look into the effect of the offline learning phase. In normal operation, for each problem (DUF1, DUF2 and DUF3 in this paper), before running the algorithm we execute an offline learning phase. In the XORing generator, each different environment is represented with an XOR mask which is applied to the solution candidate during fitness evaluations. We sample the space of the XOR masks, generating M of them which are distributed uniformly over the landscape. Then for each environment, represented by each mask, we train an PBIL algorithm for G iterations to learn a good probability vector for that environment. In this set of experiments, we explore the effect of the number of iterations G performed during the offline learning stage. In the experiments we choose $M = 8$ masks to represent 8 environments. We train PBIL for various the number of iteration G settings as: $G = \{0, 1, 5, 10, 20, 25, 30, 40, 50, 75, 100, 150, 200, 250, 500, 1000, 10000\}$. Then, we execute HH-EDA2 incorporating the Random Permutation heuristic selection, using the set of probability vectors created by each the number of iteration G setting and record the final offline errors. For these experiments, we use all change frequency and severity settings for the Random type dynamic environments; we also use all change frequency and cycle length settings for the Cyclic1 and Cyclic1-with-Noise type dynamic environments. The tests are performed using DUF1, DUF2 and DUF3.

If the number of iteration G is 0, then this indicates that there is no offline learning. Although offline learning improves the performance of the overall algorithm slightly for any given problem, the value of the number of iteration G does not matter much if the environment changes randomly. Figure 2 illustrates this phenomenon on the Decomposable Unitation-Based Functions for medium frequency and medium severity changes (MF-MS). We can observe that small G values is sufficient to handle any type of dynamic environment. Figure 3 illustrates that the number of iteration G should be set larger than or equal to 20 for an improved performance to solve DUF1 and DUF2 when the frequency of change is medium, the type is Cyclic1 and the cycle length is 4, while the choice of values greater than 50 for the number of iteration G is sufficient on DUF3. Due to lack of space, the plots for other dynamic environment instances are not provided here, however, similar observations were made for those cases too. Figure 4 illustrates the convergence behavior of PBIL on the stationary versions of the Decomposable Unitation-Based Functions. The frequency levels corresponding to low frequency, medium frequency and high frequency were determined using these plots. It is interesting to note that the values of the number of iteration G which are seen to be sufficient for good performance, approximately coincide with our medium frequency settings for the different Decomposable Unitation-Based Functions. This shows that, since for random type changes, the value of the number of iteration G does not make a difference, to achieve a good level of performance, offline learning should be done until PBIL partially converges. This will provide a heuristic way to determine a good the number of iteration G value for other types of problems encountered in the future.

4.2.4 Adaptive online learning and mutation rates

During the tests in Uludağ et al (2012b), we experimented with different learning rates α and mutation rates P_m. The experiments showed that the selection of these rates are important for algorithm performance. According to our experiments, a good value for learning rate α is 0.1 and for P_m is 0.35 for the tested dynamic environment problems. To be able to decrease the number of parameters needing to be tuned, thus making our approach more general, here we propose adaptive versions for the mutation rate parameter and the learning rate parameter. We use the same adaptive approach for both parameters as given in Equation 6 and Equation 7.

$$\alpha^t = \begin{cases} \beta\alpha^{t-1} & \text{, if } \quad \Delta E < 0 \\ \alpha^{t-1} & \text{, if } \quad \Delta E = 0 \\ \frac{1}{\beta}\alpha^{t-1} & \text{, if } \quad \Delta E > 0 \end{cases} \tag{6}$$

where, E^t is the error value for the generation t. $\Delta E = E^t - E^{t-1}$ is the difference between the current and the former error value. β is the learning factor and γ is the mutation factor. The lower and upper bounds of the interval for learning rate α is chosen as $0.25 \leq \alpha \leq 0.75$ and for mutation rate P_m as $0.05 \leq P_m \leq 0.3$.

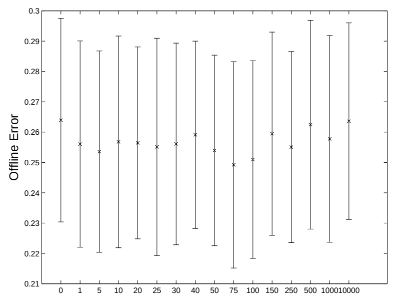
$$P_m{}^t = \begin{cases} \gamma P_m{}^{t-1} & \text{, if } \quad \Delta E < 0 \\ P_m{}^{t-1} & \text{, if } \quad \Delta E = 0 \\ \frac{1}{\gamma}P_m{}^{t-1} & \text{, if } \quad \Delta E > 0 \end{cases} \tag{7}$$

The initial values of these parameters ($t = 0$) are chosen as $\alpha^0 = 0.75$ and $P_m^0 = 0.3$. Throughout the generations, if the values become less than the lower bound or greater than the upper bound, learning rate α and mutation rate P_m are reset to their tuned values ($\alpha = 0.35$, $P_m = 0.1$), found in our previous study Uludağ et al (2012b).

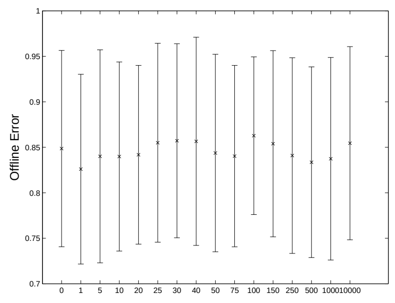
To see the effects of adaptive learning rate α and adaptive mutation rate P_m separately, we perform the same set of experiments three times: with only adaptive α and fixed mutation rate P_m; with only adaptive mutation rate P_m and fixed learning rate α; with both adaptive learning rate α and mutation rate P_m. For the first set, we fixed the mutation rate value to $P_m = 0.1$. For the second set, we fixed the learning rate value to $\alpha = 0.35$. For the third set, both parameters are allowed to vary between their predetermined lower and upper bounds.

For the learning factor β and the mutation factor γ, we experimented with various setting combinations between 0.8 and 0.99 and we chose an acceptable one as being $\beta = 0.99$ and $\gamma = 0.99$. We did not perform extensive experiments to set these parameters, since we did not want to fine tune too much, as this would contradict our initial aim of trying to decrease the amount of fine tuning required. Besides, our results showed that the settings for these parameters are not very sensitive. For this experiment also, we used the same problems, change settings and dynamic environment types as those used in Subsection 4.2.3. The results of these experiments are provided in Tables 13 and 14.

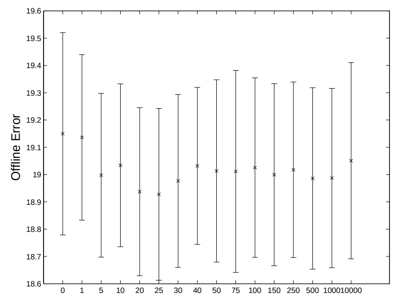
The results in Table 13 and Table 14 show that having only one of the parameters as adaptive decreases solution quality. However, the cases where both parameters are adaptive, produces results which are equivalent to those obtained when the parameters are fixed



(a) DUF1

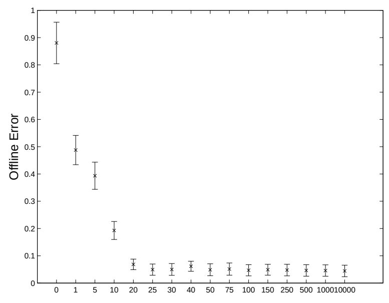


(b) DUF2

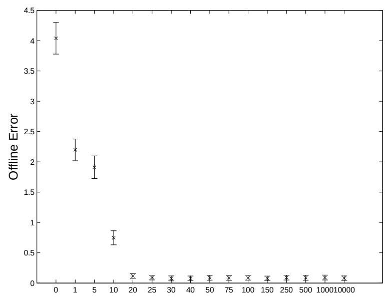


(c) DUF3

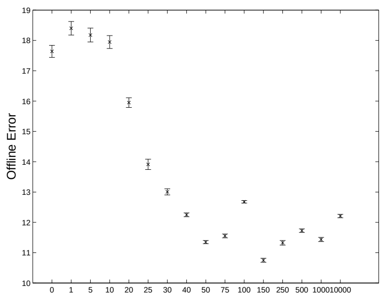
Fig. 2 Error bar of different the number of iteration G settings for all Decomposable Unitation-Based Functions in random environment



(a) DUF1

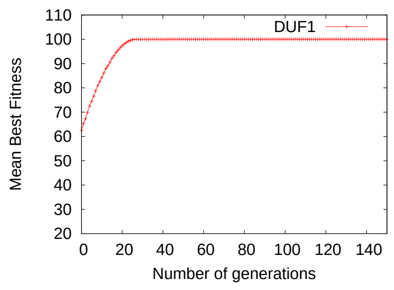


(b) DUF2

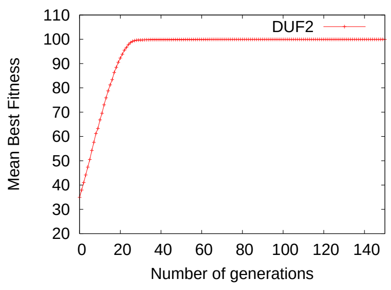


(c) DUF3

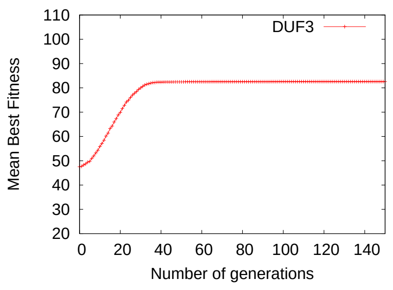
Fig. 3 Error bar of different the number of iteration G settings for all Decomposable Unitation-Based Functions in cyclic environment



(a) DUF1



(b) DUF2



(c) DUF3

Fig. 4 Convergence of mean (100 runs) of best fitness in each generation for all Decomposable Unitation-Based Functions

as a result of initial fine tuning experiments. This observation fails for high frequency change cases both for random and cyclic type of environments. The mutation rate P_m value is set initially to its upper bound value. Since the stationary periods between the changes is very short in the high frequency cases, with a decrease rate of $\gamma = 0.99$, the mutation rate P_m value does not decrease much before the environment changes and the solution quality drops, causing an increase in the mutation rate P_m value. A higher mutation rate seems to help in high frequency change cases. This needs to be further explored. However, overall, the results shows that an adaptive learning rate α and an adaptive mutation rate P_m approach can be used within HH-EDA2 without performance loss.

5 Conclusion and Future Work

In this study, we investigated the performance of a framework which enables hybridization of EDAs and selection hyper-heuristics based on online and offline learning mechanisms for solving dynamic environment problems (Uludağ et al, 2012a). A dual population approach is implemented, referred to as HH-EDA2 which uses PBIL as the EDA. The performance of the overall algorithm is tested using different heuristic selection methods to determine the best one for HH-EDA2. The

Table 13 Offline errors generated by HH-EDA2 using the tuned $\alpha = 0.35$, $P_m = 0.1$ settings and HH-PBIL2 with the various adaptive schemes for α and P_m, averaged over 100 runs, on the three Decomposable Unitation-Based Functions for different change severity and frequency settings in randomly changing environments.

Algorithm	LF				MF				HF			
	LS	MS	HS	VHS	LS	MS	HS	VHS	LS	MS	HS	VHS
DUF1												
Tuned	**0.06**	**0.06**	**0.08**	**0.09**	**0.17**	**0.25**	0.86	0.99	21.94	23.60	26.79	28.26
Adp. α & P_m	**0.06**	**0.06**	**0.08**	**0.09**	**0.17**	0.26	**0.85**	**0.98**	21.92	23.64	26.77	28.24
Adp. α	0.91	0.95	1.03	1.05	2.85	3.19	4.20	4.42	23.67	25.04	27.45	28.70
Adp. P_m	0.07	0.12	0.38	0.41	0.52	1.29	4.02	4.27	**7.68**	**13.96**	**23.85**	**25.99**
DUF2												
Tuned	**0.12**	**0.16**	**0.49**	**0.53**	**0.43**	0.85	4.13	4.54	42.92	45.74	50.86	52.95
Adp. α & P_m	0.13	**0.16**	**0.49**	**0.53**	**0.43**	**0.84**	**4.12**	**4.53**	42.87	45.74	50.83	52.88
Adp. α	1.89	1.99	2.30	2.35	6.30	7.22	10.43	10.99	45.70	47.95	51.87	53.64
Adp. P_m	0.28	1.04	7.15	7.29	1.36	4.27	15.54	16.20	**16.33**	**29.14**	**45.20**	**48.23**
DUF3												
Tuned	**19.44**	18.46	**16.04**	14.18	**19.75**	18.99	**17.26**	**15.49**	38.44	39.99	41.29	40.75
Adp. α & P_m	19.46	**18.39**	16.10	14.17	19.77	**18.98**	17.29	15.50	38.44	39.98	41.33	40.70
Adp. α	19.77	19.30	17.90	16.56	22.86	23.04	23.04	22.11	42.49	43.43	44.13	43.85
Adp. P_m	22.12	20.93	16.81	**14.09**	22.44	21.46	18.30	15.62	**24.47**	**26.53**	**27.97**	**25.50**

Table 14 Offline errors generated by HH-EDA2 using the tuned $\alpha = 0.35$, $P_m = 0.1$ settings and HH-EDA2 with the various adaptive schemes for α and P_m, averaged over 100 runs, on the three Decomposable Unitation-Based Functions for different change severity and frequency settings in cyclic environments of type 1 (Cyclic1).

Algorithm	LF			MF			HF		
	CL=2	CL=4	CL=8	CL=2	CL=4	CL=8	CL=2	CL=4	CL=8
DUF1									
Tuned	0.03	**0.02**	**0.02**	**0.05**	**0.04**	0.05	14.20	13.82	14.59
Adp. α & P_m	0.03	**0.02**	**0.02**	**0.05**	0.05	0.05	14.20	13.47	14.01
Adp. α	0.43	0.26	0.33	0.95	0.61	0.77	14.58	14.48	15.60
Adp. P_m	**0.02**	**0.02**	**0.02**	**0.05**	**0.04**	**0.04**	**8.07**	**7.76**	**7.72**
DUF2									
Tuned	**0.04**	**0.04**	**0.04**	**0.09**	**0.08**	0.08	27.33	27.38	26.53
Adp. α & P_m	**0.04**	**0.04**	**0.04**	**0.09**	**0.08**	0.09	26.63	28.48	27.13
Adp. α	0.54	0.51	0.62	1.33	1.27	1.50	27.92	28.44	28.70
Adp. P_m	**0.04**	**0.04**	**0.04**	**0.08**	**0.08**	**0.07**	**14.37**	**14.13**	**14.66**
DUF3									
Tuned	10.09	11.36	11.33	10.35	11.60	11.58	22.23	22.42	22.76
Adp. α & P_m	10.10	11.36	11.33	10.35	11.61	11.57	22.34	22.49	23.01
Adp. α	11.11	11.99	12.07	12.25	12.72	12.96	24.26	23.97	24.03
Adp. P_m	**10.06**	**11.26**	**11.26**	**10.23**	**11.38**	**11.37**	**12.39**	**13.07**	**13.21**

results revealed that, contrary to our initial intuition, the heuristic selection mechanism with learning isn't the most successful one for the HH-EDA2 framework. The selection scheme that relies on a fixed permutation of the underlying low-level heuristics (Random Permutation) is the most successful one. For the cases when the change period is long enough to allow all the vectors in the permutation to be applied at least once, the Random Permutation heuristic selection mechanism becomes equivalent to *Greedy Selection*. In HH-EDA2, the move acceptance stage of a hyper-heuristic is not used. This is the same as using the *Accept All Moves* strategy. This move acceptance scheme is known to perform the best with the *Greedy Selection* method Kiraz et al (2011).

HH-EDA2 is in general capable of adapting itself to the changes rapidly whether the change is random or cyclic. Even though the hybrid method provides good performance in the overall, it generates an outstanding performance particularly in cyclic environments. This is somewhat expected, since the hybridization technique based on a dual population acts similar to a memory scheme, which is already known to be successful in cyclic dynamic environments Yang and Yao (2008). The use of offline learning and then the us of online learning mechanism works and provides sufficient amount of diversification needed during the search process even under different change dynamics. The overall performance of the algorithm worsens if the offline learning phase is ignored. HH-EDA2 outperforms well know approaches from the literature for almost all cases, except for some deceptive problems.

As future work, we will experiment with the other types of more complex EDA based methods, in particular Bayesian optimization algorithm within the proposed framework. We will also verify our findings in a real-world problem domain, for example the dynamic vehicle routing problem.

A Appendix

Table 15 summarizes the list of abbreviations used mostly in the paper.

Table 15 List of Abbreviations

EDA	Estimation of Distribution Algorithms
PBIL	Population Based Incremental Learning
PBIL2	Dual Population PBIL
HH-EDA2	Hyper-heuristic Based Dual Population EDA
PBILr	PBIL with restart
PBIL2r	PBIL2 with restart
MPBILr	Memory-based PBIL with restart
MPBIL2r	Dual Population Memory-based PBIL with restart
Sentinel8	Sentinel-based Genetic Algorithm with 8 sentinels
Sentinel16	Sentinel-based Genetic Algorithm with 16 sentinels
LF	Low Frequency
MF	Medium Frequency
HF	High Frequency
LS	Low Severity
MS	Medium Severity
HS	High Severity
VHS	Very High Severity
CL	Cycle Length

Acknowledgements This work is supported in part by the EPSRC, grant EP/F033214/1 (The LANCS Initiative Postdoctoral Training Scheme) and Berna Kiraz is supported by the TUBITAK 2211-National Scholarship Programme for PhD students.

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