Particle Swarm Optimization: Surfing the Waves

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Abstract- A new optimization method has been proposed by Kennedy *et. al.* **in [7, 8], called** *Particle Swarm Optimization (PSO)***. This approach combines social psychology principles in socio-cognition of human (and artificial) agents and evolutionary computation. It has been successfully applied to nonlinear function optimization and neural network training. Preliminary formal analyses for a simple PSO system show that a particle in a simple PSO system follows a path defined by a sinusoidal wave, randomly deciding on both its amplitude and frequency [12]. This paper takes the next step, generalizing to obtain closed form equations for trajectories of particles in a multi-dimensional search space.**

1 Introduction

Evolutionary computation techniques are search methods based on natural systems. For example, Genetic Algorithms (GAs) use principles of genetics and natural selection [4]. "Particle Swarm Optimization" (PSO) [7, 8] is a recently proposed algorithm, motivated by the behavior of organisms such as fish schooling and bird flocking. PSO, as an evolutionary computation technique, utilizes a population-based search procedure, significantly different from the natural selection component of GAs. Comparisons of PSO with genetic algorithms and evolutionary programming (EP) by Eberhart *et al.* [3] and Angeline [1] suggest that the behavior of a PSO system falls in between GAs and EP.

A PSO system conducts search using a population of *particles*, corresponding to individuals. Each particle is a candidate solution to the problem at hand. In a PSO system, particles change their state by "flying" around in a multi-dimensional search space until a relatively unchanging state has been encountered, or until computational limitations are exceeded. In social science context, a PSO system combines a "social-only" model and a "cognition-only" model [6]. The "social-only" component suggests that individuals ignore their own experience and adjust their behavior according to the successful beliefs of individuals in the neighborhood. On the other hand, the "cognition-only" component treats individuals as isolated beings. A particle change its position using these models.

Initial analyses of possible trajectories of a particle in a simple PSO system show that a particle "catches" a "wave" during the search [10], randomly choosing its frequency and amplitude (step size). This paper develops these analyses for a multidimensional and multi particle system.

In Section 2, we explain the PSO algorithm. Section 3 and 4 present the trajectory analysis for a PSO system for different regions of interest. Section 5 presents the analysis of the step size of a particle. Section 6 explains the effect of inertia weight. Section 7 demonstrates trajectories of a *simple* particle. Section 8 contains conclusions.

2 Preliminaries

Each particle in PSO represents a solution to a specific problem. In other words, a particle is a point in a multi-dimensional search space in which we are attempting to find an optimal location with respect to a fitness function f. We will use the following terminology:

- X is a set of positions (states) of m particles in a multi-dimensional space, where

$$X = \{X_1, ..., X_j, ..., X_l, ..., X_m\}$$

- At time step t, position of j^{th} particle $X_j(t) = X_j(X_j(t-1), V_j(t))$ in a $n-dimensional$ space depends on its position at the previous time step and its current velocity, where

$$X_j(t) = (x_{j,1}(t), x_{j,2}(t), ..., x_{j,n}(t))$$

$$V_j(t) = (v_{j,1}(t), v_{j,2}(t), ..., v_{j,n}(t))$$

- $N(X_j)$, the neighborhood of a particle X_j, is defined as all those particles X_k such that X_j and X_k are "near" each other, in the context of a predefined topology.

- Define $X_l^{\#}(t)$, to be the best particle such that X_l is a particle, $X_l \in N(X_j)$ and $f(X_l^{\#}(t)) \geq f(X_k(t))$ for all $t^{\#} \leq t-1$ and $X_k \in N(X_j)$.
- Define $X_l^{*}(t)$ to be the best previous position of X_i, such that $f(X_l^{*}(t)) \geq f(X_j(t))$, for all $t^{*} \leq t-1$.
- Using the above notation, each particle X_j changes its state according to the following equation:

$$\begin{aligned} v_{j,d}(t) &= v_{j,d}(t-1) + \\ & \quad \phi_{j,d}^{(1)}(t)(x_{j,d}(t^{*}) - x_{j,d}(t-1)) + \\ & \quad \phi_{j,d}^{(2)}(t)(x_{l,d}(t^{\#}) - x_{j,d}(t-1)) \quad (1) \\ x_{j,d}(t) &= x_{j,d}(t-1) + v_{j,d}(t) \quad (2) \end{aligned}$$

for $1 \leq d \leq n, 1 \leq i, l \leq m$ and where $\phi^{(k)} = a_k U_k(0,1)$, a_k is a positive constant and $U_k(0,1)$ is a uniformly distributed random number in [0,1] for $k = 1, 2$.

PSO system combines two different models:

- "Cognition-Only" Model:

$$\begin{aligned} v_{j,d}(t) &= v_{j,d}(t-1) + \\ & \quad \phi_{j,d}^{(1)}(t)(x_{j,d}(t^{*}) - x_{j,d}(t-1)) \quad (3) \end{aligned}$$

- "Social-Only" Model:

$$\begin{aligned} v_{j,d}(t) &= v_{j,d}(t-1) + \\ & \quad \phi_{j,d}^{(2)}(t)(x_{l,d}(t^{\#}) - x_{j,d}(t-1)) \quad (4) \end{aligned}$$

Kennedy suggests in [6] that the cognitive part of PSO is represented by Equation 3 and the social part of PSO is represented by Equation 4. The "cognition-only" model represents private thinking, while the "social-only" model represents the social-psychological adaptation of knowledge. In traditional evolution, the first repeats local stress, whereas the latter captures 'regional' history.

Positions of particles are initialized randomly. Particles start moving in the search space with a predefined initial velocity, $v_{j,d}(0)$, generally chosen equal to 2.

3 Trajectory Analyses

Equations 1 and 2 reflect the behavior of a particle in a PSO system in each dimension. Let us assume $\phi_{j,d}^{(1)}, \phi_{j,d}^{(2)}, p_{j,d}, p_{l,d}$ are constants, where
$p_{j,d} = x_{j,d}(t^{*})$ and $p_{l,d} = x_{l,d}(t^{\#})$. Equations 1 and 2 simplify to:

$$\begin{aligned} v_{j,d}(t) &= v_{j,d}(t-1) + \phi_{j,d}^{(1)}(p_{j,d} - x_{j,d}(t-1)) + \\ & \quad \phi_{j,d}^{(2)}(p_{l,d} - x_{j,d}(t-1)) \quad (5) \\ x_{j,d}(t) &= x_{j,d}(t-1) + v_{j,d}(t) \quad (6) \end{aligned}$$

By substitution, Equations 5 and 6 yield the following recursive equation, defining the trajectory of a particle in a PSO system:

$$\begin{aligned} & x_{j,d}(t) - (2 - \phi_{j,d}^{(1)} - \phi_{j,d}^{(2)})x_{j,d}(t-1) + x_{j,d}(t-2) = \\ & \phi_{j,d}^{(1)} p_{j,d} + \phi_{j,d}^{(2)} p_{l,d} \quad (7) \end{aligned}$$

with initial conditions:

- $x_{j,d}(0) = x_{j,d}^{(0)}$
- $v_{j,d}(0) = v_{j,d}^{(0)}$
- $$\begin{aligned} x_{j,d}(1) &= x_{j,d}^{(0)}(1 - \phi_{j,d}^{(1)} - \phi_{j,d}^{(2)}) + \\ & \quad v_{j,d}^{(0)} + \phi_{j,d}^{(1)} p_{j,d} + \phi_{j,d}^{(2)} p_{l,d} \end{aligned}$$

A closed form for Equation 7 can be obtained by using generating functions or any method for solving non-homogeneous linear recurrence equations for the displacement:

$$x_{j,d}(t) = \eta_{j,d}\alpha_{j,d}^{t} + \iota_{j,d}\beta_{j,d}^{t} + \kappa_{j,d} \quad (8)$$

where

$$\begin{aligned} \delta_{j,d} &= \sqrt{(2 - \phi_{j,d}^{(1)} - \phi_{j,d}^{(2)})^2 - 4} \quad (9) \\ \alpha_{j,d} &= (2 - \phi_{j,d}^{(1)} - \phi_{j,d}^{(2)} + \delta_{j,d})/2 \quad (10) \\ \beta_{j,d} &= (2 - \phi_{j,d}^{(1)} - \phi_{j,d}^{(2)} - \delta_{j,d})/2 \quad (11) \\ \eta_{j,d} &= (0.5 - (\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)})/2\delta_{j,d})x_{j,d}^{(0)} + v_{j,d}^{(0)}/\delta_{j,d} + \\ & \quad (\phi_{j,d}^{(1)} p_{j,d} + \phi_{j,d}^{(2)} p_{l,d})/(2\delta_{j,d}) - \\ & \quad (\phi_{j,d}^{(1)} p_{j,d} + \phi_{j,d}^{(2)} p_{l,d})/(2\phi_{j,d}^{(1)} + 2\phi_{j,d}^{(2)})) \quad (12) \\ \iota_{j,d} &= (0.5 + (\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)})/2\delta_{j,d})x_{j,d}^{(0)} - v_{j,d}^{(0)}/\delta_{j,d} - \\ & \quad (\phi_{j,d}^{(1)} p_{j,d} + \phi_{j,d}^{(2)} p_{l,d})/(2\delta_{j,d}) + \\ & \quad (\phi_{j,d}^{(1)} p_{j,d} + \phi_{j,d}^{(2)} p_{l,d})/(2\phi_{j,d}^{(1)} + 2\phi_{j,d}^{(2)})) \quad (13) \\ \kappa_{j,d} &= (\phi_{j,d}^{(1)} p_{j,d} + \phi_{j,d}^{(2)} p_{l,d})/(\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)}) \quad (14) \end{aligned}$$

Note that there are several special cases, such as $\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)} = 2$. Analyses can be divided into two major cases: $\delta_{j,d} \in \Re$ (real) and $\delta_{j,d} \in \Im$ (complex). For real values of $\delta_{j,d}$, possible trajectories of a particle will be governed by Equations 8-14. $\delta_{j,d}$ becomes a complex number for $0 < \phi_{j,d}^{(1)} + \phi_{j,d}^{(2)} < 4$ yielding:

$$\delta_{j,d} = i\sqrt{4 - (2 - \phi_{j,d}^{(1)} - \phi_{j,d}^{(2)})^2}$$

Hence $\alpha_{j,d}$ and $\beta_{j,d}$ become complex numbers. Converting $\alpha_{j,d}$ and $\beta_{j,d}$ into polar form yields:

$$\begin{aligned} \alpha_{j,d} &= (cos(\theta_{j,d}) + isin(\theta_{j,d})) \quad (15) \\ \beta_{j,d} &= (cos(\theta_{j,d}) + isin(-\theta_{j,d})) \quad (16) \end{aligned}$$

where

$$\theta_{j,d} = atan(||\delta_{j,d}||/(|2-\phi^{(1)}_{j,d}-\phi^{(2)}_{j,d}|)) \quad (17)$$

If $(\phi^{(1)}_{j,d}+\phi^{(2)}_{j,d}) > 2$, π is added to $\theta_{j,d}$. Applying *de Moivre's formula*,

$$\alpha^t_{j,d} = (cos(\theta_{j,d}t) + isin(\theta_{j,d}t)) \quad (18)$$
$$\beta^t_{j,d} = (cos(\theta_{j,d}t) - isin(\theta_{j,d}t)) \quad (19)$$

When we substitute $\alpha_{j,d}$ and $\beta_{j,d}$ into Equation 8, after the cancellations we obtain:

$$x_{j,d}(t) = \Lambda_{j,d}sin(\theta_{j,d}t) + \Upsilon_{j,d}cos(\theta_{j,d}t) + \kappa_{j,d} \quad (20)$$

where

$$\begin{aligned}\Lambda_{j,d} &= (2v^{(0)}_{j,d} - (\phi^{(1)}_{j,d}+\phi^{(2)}_{j,d})x^{(0)}_{j,d} + \\ &\quad \phi^{(1)}_{j,d}p_{j,d} + \phi^{(2)}_{j,d}p_{l,d})/||\delta_{j,d}|| \quad (21)\\ \Upsilon_{j,d} &= x^{(0)}_{j,d} - \kappa_{j,d} \quad (22)\end{aligned}$$

4 Regions of interest

Let $\phi_{j,d} = \phi^{(1)}_{j,d} + \phi^{(2)}_{j,d}$. There are several interesting regions for the analyses based on various $\phi_{j,d}$s. Special cases are $\phi_{j,d}$ =0, 2, 4.

In regions $\phi_{j,d} \in (0, 2-\sqrt{3}]$ and $\phi_{j,d} \in (2+\sqrt{3}, 4]$, $||\delta_{j,d}|| \leq 1$. As $\phi_{j,d}$ decreases, amplitude $\Lambda_{j,d}$ will grow, meanwhile frequency will be diminish. The frequency is inversely proportional to the period. This means larger step sizes. The opposite effect results for the intervals $\phi_{j,d} \in (2-\sqrt{3}, 2)$ and $\phi_{j,d} \in (2, 2+\sqrt{3})$, where $||\delta_{j,d}|| > 1$. A particle then moves around a point which is the random weighted average of points of the personal best and neighborhood best using step sizes obtained from a sinusoidal wave randomly. The amplitude and the frequency of the sine wave are determined by the random variables used in taking the weighted average.

For $\phi_{j,d} > 4$, the trajectory equation implies exponentially growing amplitudes, causing the danger of a particle moving out of a search space, without returning. This possibility can be eliminated by choosing $a_1 + a_2 < 4$ in Equation 1.

4.1 Special Cases

For $\phi_{j,d}$ = 0, 2, 4, recursive trajectory equations change yielding different closed forms.

4.1.1 $\phi_{j,d} = 0$

Recall that $\phi^{(1)}_{j,d}, \phi^{(2)}_{j,d} \geq 0$, so $\phi_{j,d} = 0$, if and only if $\phi^{(1)}_{j,d} = 0$ and $\phi^{(2)}_{j,d} = 0$. This produces the following recursive trajectory equation:

$$x_{j,d}(t) - 2x_{j,d}(t-1) + x_{j,d}(t-2) = 0 \quad (23)$$

Solving this equation yields

$$x_{j,d}(t) = A_{j,d} + B_{j,d}t \quad (24)$$

where

$$A_{j,d} = x^{(0)}_{j,d} \quad (25)$$
$$B_{j,d} = v^{(0)}_{j,d} \quad (26)$$

4.1.2 $\phi_{j,d} = 2$

Since the term $x_{j,d}(t-1)$ drops out in Equation 7 for $\phi_{j,d} = 2$, the trajectory equation becomes:

$$\begin{aligned}x_{j,d}(t) &= v^{(0)}_{j,d}cos(t\pi/2) + (v^{(0)}_{j,d} + 4x^{(0)}_{j,d} + 2\phi^{(1)}_{j,d}p_{j,d} + \\ &\quad 2\phi^{(2)}_{j,d}p_{l,d})sin(t\pi/2) + \\ &\quad (\phi^{(1)}_{j,d}p_{j,d} + \phi^{(2)}_{j,d}p_{l,d})/2 \quad (27)\end{aligned}$$

4.1.3 $\phi_{j,d} = 4$

The following is the recursive trajectory equation for $\phi_{j,d} = 4$:

$$x_{j,d}(t) + 2x_{j,d}(t-1) + x_{j,d}(t-2) = \phi^{(1)}_{j,d}p_{j,d} + \phi^{(2)}_{j,d}p_{l,d} \quad (28)$$

The closed form of Equation 28 is:

$$\begin{aligned}x_{j,d}(t) &= (C_{j,d} + D_{j,d}t)(-1)^t + \\ &\quad (\phi^{(1)}_{j,d}p_{j,d} + \phi^{(2)}_{j,d}p_{l,d})/4 \quad (29)\end{aligned}$$

where

$$C_{j,d} = x^{(0)}_{j,d} - (\phi^{(1)}_{j,d}p_{j,d} + \phi^{(2)}_{j,d}p_{l,d})/4 \quad (30)$$
$$D_{j,d} = 2x^{(0)}_{j,d} - (\phi^{(1)}_{j,d}p_{j,d} + \phi^{(2)}_{j,d}p_{l,d})/2 - v^{(0)}_{j,d} \quad (31)$$

5 Step Size Analyses

The amplitude of the velocity determines the step size of a particle for its next move. Using substitution, Equations 5 and 6 yield:

$$v_{j,d}(t) - (2 - \phi^{(1)}_{j,d} - \phi^{(2)}_{j,d})v_{j,d}(t-1) + v_{j,d}(t-2) = 0 \quad (32)$$

with initial conditions:

- $x_{j,d}(0) = x^{(0)}_{j,d}$
- $v_{j,d}(0) = v^{(0)}_{j,d}$
- $v_{j,d}(1) = v^{(0)}_{j,d} - (\phi^{(1)}_{j,d}+\phi^{(2)}_{j,d})x^{(0)}_{j,d} + \phi^{(1)}_{j,d}p_{j,d} + \phi^{(2)}_{j,d}p_{l,d}$

Equation 32 has terms similar to the homogeneous part of Equation 7. A closed form can be obtained in a similar manner to that of the trajectory analyses:

$$v_{j,d}(t) = \rho_{j,d}\alpha^t_{j,d} + \tau_{j,d}\beta^t_{j,d} \quad (33)$$

where

$$\begin{aligned}
\rho_{j,d} &= v_{j,d}^{(0)}/2 + ((\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)})/2\delta_{j,d})v_{j,d}^{(0)} + \\
&\quad ((\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)})/\delta_{j,d})x_{j,d}^{(0)} + \\
&\quad (\phi_{j,d}^{(1)}p_{j,d} + \phi_{j,d}^{(2)}p_{l,d})/\delta_{j,d} \qquad (34) \\
\tau_{j,d} &= v_{j,d}^{(0)}/2 - ((\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)})/2\delta_{j,d})v_{j,d}^{(0)} - \\
&\quad ((\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)})/\delta_{j,d})x_{j,d}^{(0)} - \\
&\quad (\phi_{j,d}^{(1)}p_{j,d} + \phi_{j,d}^{(2)}p_{l,d})/\delta_{j,d} \qquad (35)
\end{aligned}$$

Similar observations are valid for the regions of interest for step sizes. Equations 33- 35 will define the possible step size for a particle, where $\delta_{j,d}$ is a real number. For values $0 < \phi_{j,d}^{(1)} + \phi_{j,d}^{(2)} < 4$ velocity equation can be reduced to:

$$v_{j,d}(t) = \Psi_{j,d}sin(\theta_{j,d}t) + \Omega_{j,d}cos(\theta_{j,d}t) \qquad (36)$$

where

$$\begin{aligned}
\Psi_{j,d} &= ((\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)})2x_{j,d}^{(0)} + 2(\phi_{j,d}^{(1)}p_{j,d} + \phi_{j,d}^{(2)}p_{l,d}) + \\
&\quad \phi_{j,d}^{(1)} + \phi_{j,d}^{(2)})/||\delta_{j,d}|| \qquad (37) \\
\Omega_{j,d} &= v_{j,d}^{(0)} \qquad (38) \\
\theta_{j,d} &= atan(||\delta_{j,d}||/(|2 - \phi_{j,d}^{(1)} - \phi_{j,d}^{(2)}|)) \qquad (39)
\end{aligned}$$

If $(\phi_{j,d}^{(1)} + \phi_{j,d}^{(2)}) > 2$, π is added to $\theta_{j,d}$.

6 Inertia weight

Each particle in a PSO system randomly chooses a path on a sinusoidal wave by manipulating its frequency and amplitude, even if it means taking only one step using the wave. Trajectory equations obtained have several regions of interest based on the frequency and amplitude of the related sine wave which depend on initial (previous) velocity and position and random manipulation of step size in each dimension. Shi *et al.* suggest the use of an *inertia weight* (w) in [12], modifying the velocity component in Equation 5:

$$\begin{aligned}
v_{j,d}(t) &= wv_{j,d}(t-1) + \\
&\quad \phi_{j,d}^{(1)}(t)(x_{j,d}(t^*) - x_{j,d}(t-1)) + \\
&\quad \phi_{j,d}^{(2)}(t)(x_{l,d}(t^\#) - x_{j,d}(t-1)) \qquad (40)
\end{aligned}$$

Our analyses show that the use of a constant inertia weight modifies the boundaries between those regions of interest. Furthermore, the choice of inertia weight might create a region where a particle moves out of the search space even for lower values of $\phi_{j,d}$. It does not seem to help the PSO algorithm much. This might be the reason why inertia weights of magnitude approximately 1 were favored. Shi *et al.* experimented using Schaffer' s f6 function [2] in [11] with various weights. The experimental results are in favor of inertia weights within the range of [0.9,1.2], supporting our conclusion. In this range PSO takes the least average number of iterations to find the global optimum for *w*. In their paper, Shi *et al.* also suggests a time varying inertia weight yielding better results. In order to make conclusions, extensive experiments are required.

7 Trajectory Analysis of a Simple Particle

We can further simplify the PSO system by using following assumptions:

- There is only one particle moving along one dimension (*simple particle*).
- $p_{j,d} = p_{l,d} = p$.
- $\phi = \phi_{j,d}^{(1)} + \phi_{j,d}^{(2)}$.
- $x(0) = x_0$.
- $v(0) = v_0$.

Then the behavior of a simple particle is governed by the following equations:

$$\begin{aligned}
v(t) &= v(t-1) - \phi x(t-1) + \phi p \qquad (41) \\
x(t) &= x(t-1) - v(t) \qquad (42)
\end{aligned}$$

Following trajectory equations are obtained from our analysis:

Case $\phi = 0$:

$$x(t) = (x_0 - p) + v_0 t \qquad (43)$$

Case $\phi \in (0, 2)$ and $\phi \in (2, 4)$:

$$x(t) = A_S sin(\theta t) + A_C cos(\theta t) \qquad (44)$$

where

$$\begin{aligned}
A_S &= (2v_0 - \phi x_0 - \phi p)/\delta \qquad (45) \\
A_C &= x_0 - p \qquad (46) \\
\theta &= atan(\delta/|2 - \phi|) \qquad (47) \\
\delta &= \sqrt{|\phi^2 - 4\phi|} \qquad (48) \\
& \qquad (49)
\end{aligned}$$

If $\phi > 2$, π is added to θ.

Case $\phi = 2$:

$$x(t) = v_0 sin(t\pi/2) \qquad (50)$$

Case $\phi = 4$:

$$\begin{aligned}
x(t) &= ((x_0 - p) + \\
&\quad (2(x_0 - p) - v_0)t)(-1)^t + p \qquad (51)
\end{aligned}$$

Case $\phi > 4$:

$$\begin{aligned} x(t) &= (v_0/\delta)(((2-\phi+\delta)/2)^t - \\ &\quad ((2-\phi-\delta)/2)^t) \end{aligned} \tag{52}$$

where

$$\delta = \sqrt{|\phi^2 - 4\phi|} \tag{53}$$

$$x_0 = p = 0 \tag{54}$$

Details of this analysis can be found in [10]. Trajectories of a particle in a simplified PSO system for $v_0 = 0, x_0 = p = 0$ and various values of ϕ are shown in Figures 1, 2, 3 and 4 [1].

8 Conclusions

This paper formally analyzes the trajectory of a particle in a PSO system, as well as the step sizes used during search. Based on our results, intelligent parameter adaptation methods can be incorporated into the original equations of PSO.

Analytical Results: In [10], we have presented the formal analyses of a simple particle system. More details can be found in [9]. Our analytical results are supported by the simulations of Kennedy in [5]. In this paper, we have generalized our analyses to a multi-dimensional search space where particles attempt to find an optimal solution.

Comparing Regions: Search is conducted in a different manner in different regions. A particle may step back or forward depending on the past experience of itself as well its neighbors' along each dimension. This step is sampled from a random sine wave. In some regions this step size is larger than the others.

Our analysis leads to a potential new approach in applying PSO techniques. Particles may be equipped with a sensor that detects the type of search the particle performing; if it continues to remain in the same region, parameter values may be modified to force the particle to explore a different, more favorable search region. Adaptive inertia weight might provide such a utility.

'Surfing' the waves: We have shown that in the general case, a particle does not "fly" in the search space, but rather "surfs" it on sine waves. An optimal location seeking particle attempts to "catch" another wave randomly, by manipulating its frequency and amplitude. A wave with a high frequency or amplitude is not necessarily a good wave for a particle. Limiting the velocity (V_{max}) seems to help the particle to "jump" onto another wave. Velocity cut-off occurs only in regions where the amplitude of the sine wave is large.

[1] Gnuplot Linux version 3.5 was used for plotting the graphs in the figures, some plotting errors occur due to roundoffs.

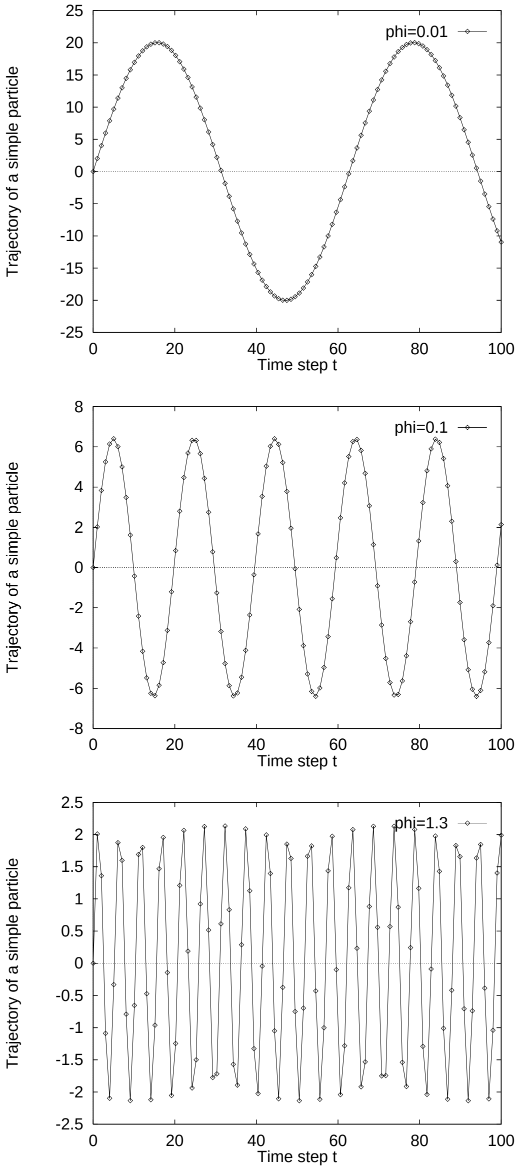

Figure 1: Trajectories of a particle in a simplified PSO system for $\phi \in (0, 2)$ values

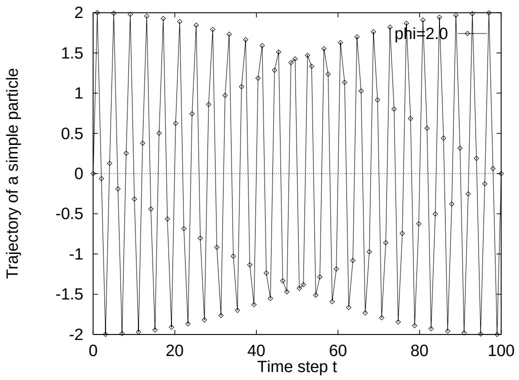

Figure 2: Trajectory of a particle in a simplified PSO system for $\phi = 2$

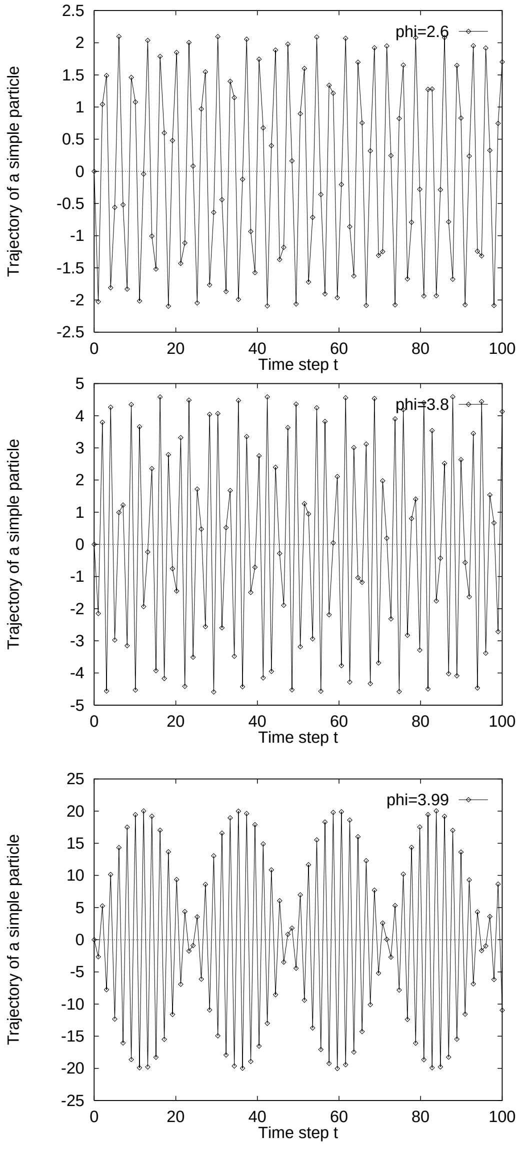

Figure 3: Trajectories of a particle in a simplified PSO system for different $\phi \in (2, 4)$.

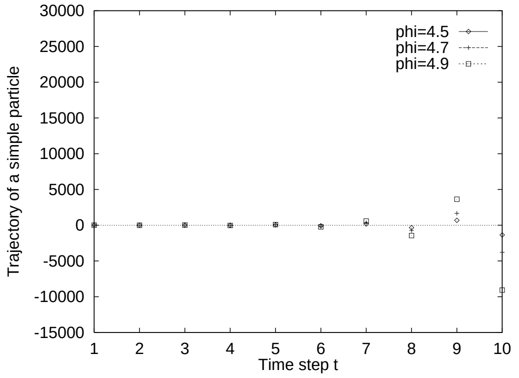

Figure 4: Trajectories of a particle in a simplified PSO system for $\phi > 4$.

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