On the Utilisation of Fuzzy Rule-Based Systems for Taxi Time Estimations at Airports

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Abstract

The primary objective of this paper is to introduce Fuzzy Rule-Based Systems (FRBSs) as a relatively new technology into airport transportation research, with a special emphasis on ground movement operations. Hence, a Mamdani FRBS with the capability to learn from data has been adopted for taxi time estimations at Zurich Airport (ZRH). Linear regression is currently the dominating technique for such an estimation task due to its established nature, proven mathematical characteristics and straightforward explanatory ability. In this study, we demonstrate that FRBSs, although having a more complex structure, can offer more accurate estimations due to their proven properties as nonlinear universal approximators. Furthermore, such improvements in accuracy do not come at the cost of the model's interpretability. FRBSs can offer more explanations of the underlying behavior in different regions. Preliminary results on data for ZRH suggest that FRBSs are a valuable alternative to already established linear regression methods. FRBSs have great potential to be further seamlessly integrated into the taxiway routing and scheduling process due to the fact that more information is now available in the explanatory variable space.

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1 Introduction

There has been little study of the problem of predicting taxi times at airports prior to the last decade [1, 3, 9, 11, 19], since accurate taxi times were not often needed in advance. However, the increasing use of automated decision support tools more recently has tremendously increased the value of having accurate taxi time predictions. It has been common practice for airports to use standard mean taxi times for specific source/destination pairs, perhaps further broken down into aircraft sizes but usually with no further discrimination. Any variances from the means were usually considered irrelevant and replaced by the addition of slack time when needed. More recent attempts to work towards a connected system, linking airspace and airports, mean that landing time information is becoming available much sooner and the taxi time can become the main uncertainty in on-stand time predictions. From a departures point of view, increasingly accurate ready time predictions from airlines can

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mean that taxi times become the main uncertainty in the predictions for when aircraft can/will reach the runway. In some cases this can make the difference between being able to predict take-off times (or even perform take-off sequencing) or not before aircraft even leave the stands. Taxi time prediction, of course, becomes even more important when automated decision support is desired for ground movement optimisation, since relatively small deviations from predictions can have increasing knock-on effects.

Due to the number of factors which can influence taxi times, until recently they have been considered to be very unpredictable. Although accurate taxi time prediction for individual aircraft could still be considered to be in its infancy, various researchers have had some success in using linear regression-type approaches to both identify the factors which are more heavily correlated to taxi times and in producing functions which are much more effective at predicting taxi times. The research in this paper moves the taxi time prediction research another step forward by utilising the correlating factors which have been previously identified in [1] and applying a non-linear fuzzy rule-based prediction approach to find functions which make even more accurate taxi time predictions. Obviously, linear regression approaches can cope with non-linear behaviour providing that an appropriate mapping function is utilised in advance, but the ability to cope with non-linearity without an a-priori production of a mapping function is a distinct advantage of the Fuzzy Rule-Based System (FRBS) approach.

The relative sparsity of the research and the concentration of researchers upon linear methods seem to be at odds with the importance of the problem, given the fact that airport ground movement serves as a link between the other airport operations, such as departure sequencing, arrival sequencing and gate/stand allocation [2, 6]. Any analysis requires data, however the previous lack of detailed utilisation of historic data, and the difficulty in capturing certain types of data have meant that it has not always been recorded. As can be seen in [8], even if the correct equipment is available to record the needed information, the installation positions and the way in which different taxiing stages are categorised (such as straight, turn and stop segments) can still induce different options, leading to substantial uncertainties. This explains why the emphasis of the previous work has been upon identifying the significant explanatory variables from the recorded data via a combined statistical approach and linear regression [1, 9, 11, 19], rather than exploring different regression methods. Idris et al. [9] performed a statistical analysis of the taxi-out process for Boston Logan International Airport and concluded that the take-off queue size is the most important factor affecting taxi time. In order to more realistically decide the departure queue lengths, Zhang et al. [19] proposed an iterative algorithm to improve the prediction accuracy of the linear regression models. A sequential forward floating subset selection method was developed in [11] with the aim of selecting the most influential ones from a set of candidate explanatory variables. Finally, Atkin et al. [1] identified in their work that take-off queue size is not the dominating variable for some of the European airports, such as Stockholm-Arlanda Airport (ARN) and Zurich Airport (ZRH). Unlike Boston Logan International Airport, these European hub airports do not have long queues for take-off. Hence, to improve the estimation accuracy for this kind of airport, one has to include information about the surface layout. Their model works for both departure and arrival aircraft and furthermore, can predict taxi time for unimpeded aircraft to be used in routing approaches.

Although previous research efforts have led to a number of promising results, none of them explored the potential in other modeling methods. As pointed out in [1], it is important to be able to accurately estimate taxi times if more realistic ground movement decision support systems are desired. And given the fact that nonlinearity is present in airport data, nonlinear modeling approaches, such as FRBSs with proven ability to approximate any real continuous function on a compact set to an arbitrary accuracy [12, 16], should be very competent for this type of taxi time estimation task.

In this paper, a Mamdani FRBS [5, 13], which can learn from data, has been employed to further improve the estimation accuracy. However, the aim of the introduction of FRBSs into airport transportation research is for more than estimation accuracy improvement. One distinctive characteristic of FRBSs lies in their explanatory ability, distinguishing FRBSs from other nonlinear modeling techniques, such as artificial neural networks [7]. The emphasis of this work is not placed on data pre-processing or analysis, but rather, this work is based on the data which has been prepared and analysed by Atkin et al. [1]. We would like to explore the possibility of including FRBSs as an alternative for consideration by the practitioners in this field, and we consider both the feasibility of using FRBSs and the extra benefits that one could gain from using FRBSs.

Based on such an understanding, the rest of the paper is organised as follows: Section 2 briefly describes the problem of airport taxi time estimation and the data set from ZRH; Section 3 introduces the Mamdani FRBS and its revised version, which is used in this work; experimental results on ZRH and its analysis are presented in Section 4; and finally, conclusions and future directions are given in Section 5. In order to promote the adoption of FRBSs in the field of airport research, we also point out several important issues which could see this approach being accepted by the practitioners and maturing into a systematic approach in this field.

2 Problem description

The problem considered in this paper involves eliciting an aircraft taxi time model using the available historic data from ZRH. In the following two sections, we first emphasise the importance of the airport ground movement problem, and then we briefly describe the data set which was used with an emphasis upon discussing the explanatory variables.

2.1 The airport ground movement problem

Airport ground movement plays a major role in the ever increased annual average delays for flights, as it serves as a link between other airport operations [6, 14]. The problem is basically a routing and scheduling problem. As stated by Atkin et al. [1], "it involves directing aircraft on the surface of an airport to their destinations in a timely manner, with the aim to reduce the overall travelling time, to meet target time windows and/or to absorb the delay at the preferred time." For this reason, it is crucial that one can accurately estimate taxi times for aircraft, since this forms the basis not only for optimal allocation of airport ground resources within a single airport at the tactic level, but also for the optimal flow management across multiple airports at the strategic level [6, 9]. Interested readers in this problem are referred to a recently published survey showing the state-of-the-art in this research area [2]. In this paper, we are interested in building up an approximate model which can not only reproduce taxi times for the past events but also predict them for the future.

2.2 ZRH Airport data

Zurich Airport is the largest airport in Switzerland. The sketch of its layout is shown in Figure 1. As can be seen from the figure, the airport operates with 3 runways [1].

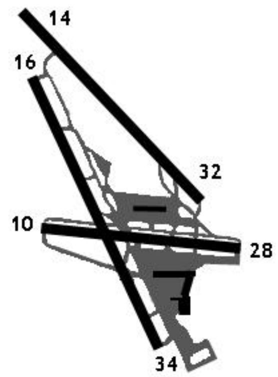

Figure 1 Sketch of airport layout for ZRH (source: [1])

It was confirmed by the field staff that, as long as no heavy winds occur, ZRH operates with three operational modes: a) before 7am, runway 34 is used for arrivals and 32 and 34 for departures; b) during the day, runways 14 and 16 are used for arrivals and 28 and 16 for departures c) after 9pm, only runway 28 is used for arrivals and runways 32 and 34 are used for departures. The mentioned rules only apply on weekdays and outside the holiday times of Baden-Württemberg. In collaboration with Flughafen Zürich AG, we had access to the data for an entire day's operation for the 19th of October 2007. No extraordinary events happened during that day. The data set consists of 679 movements and contains information about each aircraft, the stand and the runway, the start and end time of taxiing, the aircraft type and the information about whether the aircraft was arriving or departing [1]. A rigorous statistical analysis was conducted in [1] and a set of significant explanatory variables were extracted from the original data set. In this work, we are investigating the 14 explanatory variables, listed below, and their relationships with the taxi time.

- **Distance:** This is the approximate distance (in meters) that an aircraft was taxiing. Since only the stand and the runway (source and destination) were available in the supplied data, this factor was calculated by assuming that the shortest route was taken.
- **Log(Distance):** This is the logarithmic transformation of the Distance.
- **Log(Angle):** The angle is calculated as the total angular deviations between adjacent arcs on the shortest path. Taxiing speed is confined by the total amount of turning which an aircraft had to achieve. The larger the total is, the slower the taxiing speed. In the modeling, we take the logarithmic transformation of the turning angle rather than the angle itself.
- **LAN**: This is a binary variable, fixed to 1 for arrival aircraft and 0 for departure aircraft.
- **Q and N values:** These values indicate the amount of other traffic on the airport surface while the aircraft under consideration is taxiing. The N value counts the number of other aircraft which are already taxiing on the airport surface at the time that the particular aircraft starts to taxi. The Q value counts the number of other aircraft which cease taxiing during the time that the particular aircraft is taxiing. In order to be able to account for both arrivals and departures, these values are further differentiated into the combinations of arrivals and departures, which leads to eight integer variables in total.
- **Operational Modes:** There are three operational modes at ZRH, provided that no heavy winds occur, as discussed before. Hence, the two dummy variables $O_{Morning}$ and $O_{Evening}$ are used to represent the operational modes, with the former set to 1 for the

morning period and the latter set to 1 for the evening period. Both of these variables are set to 0 for the day period.

Details regarding how the aforementioned variables were extracted from the available data set can be found in [1].

3 Fuzzy Rule-Based Systems (FRBSs)

In the real world, many systems contain extremely nonlinear, time-varying and uncertain behaviour. This prevents the development of computerised systems for them from being a straightforward algorithmic solution because of the inherent uncertainty which arises as a natural occurrence in these types of applications. In addition, the human operators can often be an adequate controller by being able to construct acceptable models of processes in their own minds. Models which do not include any mathematical equations and more closely match those which humans may mentally develop are, therefore, easier to handle. In other words, the human operator has the ability to interpret linguistic statements about the process and to think in a qualitative rather than in a quantitative fashion. Fuzzy logic theory is inspired by these observations and was first introduced by Zadeh [17]. One strong point of fuzzy inference systems is that they can combine human expertise together with sensory measurements and mathematical models. In this section, a special case of fuzzy inference systems, namely the Mamdani FRBS and its revised version, will be discussed first. The emphasis is then placed on the distinctive features of FRBSs which make them competent candidates for this particular estimation task.

3.1 A Mamdani FRBS and its revised version

Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic [18]. The mapping then provides a basis from which decisions can be made, or patterns discerned. The general process of the fuzzy inference and its schematic diagram is shown in Figure 2.

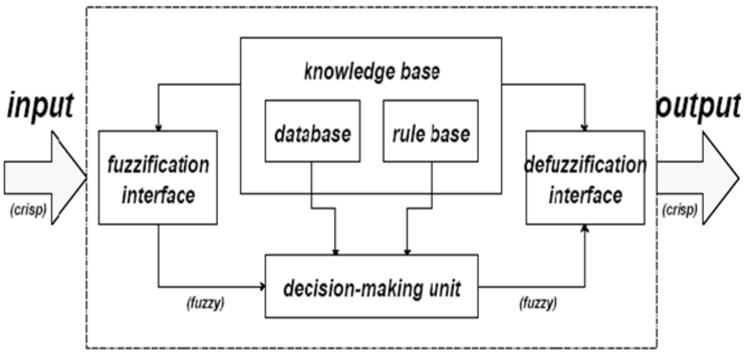

Figure 2 Fuzzy Inference Systems (source: [10])

The 'rule base' contains a number of fuzzy if-then rules in the following form:

R_i : If x_1 is A_i^1 and x_2 is A_i^2, ... , and x_j is A_i^j, ... , and x_n is A_i^n Then $y_i = Z_i$,

where R_i represents the ith rule in the rule base, x_j is the value of the jth explanatory variable ($j = 1, \ldots, n$) and is defined over the universe of discourse $\mho_j$, y_i is the output of the ith rule, A_i^j is the linguistic value (fuzzy set) for the jth linguistic (explanatory) variable x_j of this ith rule, defined over the universe of discourse $\mho_j$, and Z_i is the consequence, or output, of the rule and is discussed below.

For each A_i^j, there is a membership function $\mu_{A_i^j}(x_j)$ associated with it which maps $\mho_j$ to $[0, 1]$. In this work, Gaussian membership functions are used for all of the explanatory variables as described in (1).

$$\mu_{A_i^j}(x_j) = \exp\left[-\frac{1}{2} \cdot \frac{(x_j - c_i^j)^2}{(\sigma_i^j)^2}\right], \tag{1}$$

where c_i^j denotes the centre of the bell-shape curve and σ_i^j denotes the standard deviation. For illustration, Figure 3 shows an example of a Gaussian membership function with its centre at 0.5 and its standard deviation being 0.2.

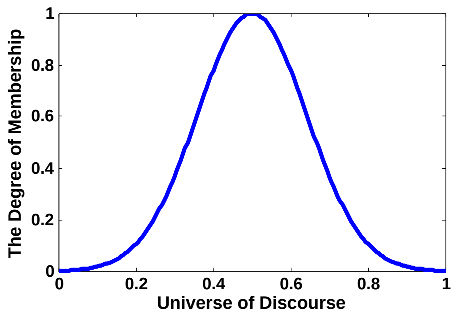

Figure 3 The shape of a Gaussian membership function for the explanatory variables

With the link between A_i^j and $\mu_{A_i^j}(x_j)$, each rule can be expressed to the end user using linguistic terms without showing the mathematical details, so, for example, R_i, could also be rewritten as follows:

$$R_i: \text{ If } x_1 \text{ is big and } x_2 \text{ is small}, \ldots, \text{ and } x_j \text{ is big}, \ldots, \text{ and } x_n \text{ is medium Then } y_i = Z_i,$$

where 'big', 'small' and 'medium' are linguistic values defined by $\mu_{A_i^j}(x_j)$.

The 'database' in Figure 2 contains all such membership functions for the fuzzy sets used in the fuzzy rules. Usually, the rule base and the database are jointly referred to as the 'knowledge base'. The 'decision-making unit' performs the inference operations on the rules and two interfaces perform fuzzification and defuzzification respectively. Defuzzification is an important module since it converts a set of output values or output membership functions from different fuzzy rules into a single crisp output value.

There are many types of fuzzy inference systems. The two most popular types of fuzzy inference systems are the Mamdani-type [13] and Sugeno-type [15] FRBSs. These two types vary in the form of the consequence part Z_i. The consequence part of the Mamdani-type FRBS is a fuzzy set while the consequence part of the Sugeno-type FRBS is a set of functions with the arguments that are the explanatory variables of the antecedent (input) part. In this work, we concentrate on a Mamdani FRBS due to its ability to approximate nonlinear systems and its unique property to interpret the underlying system via linguistic terms, in both the outputs and inputs rather than only the inputs.

The output membership function (fuzzy set) is also a bell-shaped function, and is defined as follows:

$$\mu_{B_i}(y) = \frac{1}{1 + \left(\frac{y - c_i^y}{\sigma_i^y}\right)^2}. \tag{2}$$

For a Mamdani FRBS with r rules, the defuzzification method used in this work is fully defined in (3).

$$y^{crisp} = \frac{\sum_{i=1}^{r} c_i^y \cdot \mu_i(X) \cdot \int_y \mu_{B_i}(y)dy}{\sum_{i=1}^{r} \mu_i(X) \cdot \int_y \mu_{B_i}(y)dy}, \tag{3}$$

where $\mu_i(X)$ is defined as $\mu_{A_i^1}(x_1) \times \mu_{A_i^2}(x_2) \times \ldots \times \mu_{A_i^n}(x_n)$ and represents the degree of certainty for a data sample associated with the ith rule.

For a Mamdani FRBS, the predominant approach in the traditional design is highly dependent upon human experts who will decide upon the values of c_i^j, σ_i^j, c_i^y and σ_i^y according to their domain experience. Hence, learning components are not necessarily required in the traditional Mamdani FRBS, making it unsuitable for a data-driven modeling task such as the one studied in this work. In [5], the authors utilised a combined k-means clustering algorithm and genetic algorithms to automatically identify the initial values of c_i^j, σ_i^j, c_i^y and σ_i^y from the historic data set and then fine-tune these values through a back-error propagation algorithm to further improve the estimation accuracy of the Mamdani FRBS. With the help of automatic knowledge induction and learning, the revised Mamdani FRBS becomes very appealing for the data-driven estimation task that we are investigating in this work.

3.2 Distinctive features of FRBSs

From the above description, we conclude that the following points are the distinctive features of a Mamdani FRBS, and argue that FRBSs deserve more attention in airport operations research:

- **Ability to approximate complex nonlinear systems.** When several rules concurrently describe a system under investigation, a FRBS decomposes the system into several sub regions, modeling these via different combinations of rules in the rule base. For this reason, and because of the nonlinearity embedded within membership functions, a FRBS is suitable for modelling complex nonlinear systems. Linear regression methods tend to need manual intervention to tune, for example by applying transformations to explanatory variables. For illustration, [1] found that it was necessary to use log(Distance) rather than Distance in order to get linear correlations. If logarithmic transformations were not used, then the resulting system would have had a poorer estimation performance. In contrast, this kind of learning is already a part of the FRBS approach. Box-Cox [4] started to look at automating the determination of transformation functions of polynomial form, but this type of automation is not yet standard practice in linear regression.
- **Ability for rules to differ in different regions.** FRBS will utilise different rules for different parts of the explanatory variable space, which makes it easier to understand how the effects of different explanatory variables change according to the values of other explanatory variables. For instance, how the effects of the distance or turning angle differ depending upon the runway which is in use.

- **Ability to integrate human expertise.** The main advantage of using a Mamdani FRBS as a regression tool for airport ground movements over other regression methods lies in its additional ability for integrating human expertise in the form of vague or imprecise statements rather than crisp mathematical representations. This is useful since the knowledge of many real-world systems can only be described by experts using natural language rather than mathematics. The rule-based structure makes it possible to update a FRBS model by adding new rules elicited from experts or extra data without the need to rebuild the whole model. In particular, experts can specify initial rules which apply over specific regions (value ranges for explanatory variables), which can then be refined by the system if data is available in that region. This is a very promising property if one requires a model to have online adaptive ability or the ability to synergise different types of models.
- **Ability to interpret the underlying system.** Because of the linguistic terms involved in each rule, it is possible to interpret the meaning of the rules.

We will revisit some of these features in Sections 4 and 5 after presenting the preliminary experimental results.

3.3 Automatic induction of the rules from the data

As mentioned in Section 3.1, a genetic algorithm based k-means clustering algorithm is used to first categorise the data set into different clusters. Each of these clusters is then represented by a rule in the rule base. The projections of the centres and disperses of these clusters on each explanatory variable dimension and the output dimension provide the initial values of c_i^j, σ_i^j, c_i^y and σ_i^y in (1) and (2). In this way, an initial FRBS can be automatically determined from the historical data. A back-error propagation learning algorithm is then used to adjust the values of c_i^j, σ_i^j, c_i^y and σ_i^y in order to further improve the estimation accuracy of an FRBS. Interested readers are referred to [5] for more details.

4 Results

In this section, the introduced Mamdani FRBS was applied to the problem of estimating taxi times for the data from ZRH. The following two measures were utilised:

- R^2**:** The R-square value is used to evaluate how well the model fits the data and is defined by:

$$R^2 = 1 - \frac{\sum_{k=1}^{m} (y_k - \hat{y}_k)^2}{\sum_{k=1}^{m} (y_k - \bar{y})^2}, \tag{4}$$

  where y_k is the observed output of the kth data sample; $\hat{y}_k$ is the estimated output for the kth data sample; $\bar{y}$ is the mean of the observed outputs; and m is the total number of the data samples.
- **Prediction accuracy:** The accuracy of the predictions is measured as the percentage of predictions which are within a specified number of minutes of the actual taxi times. 3 and 5 minute accuracy [3] are the most common measures for taxi times and the two values $\pm$ 3 minutes and $\pm$ 5 minutes measure the ability of the model to predict taxi times to this accuracy.

4.1 Comparison to linear regression

In this experiment, the prediction accuracy of a Mamdani FRBS is compared against a linear regression approach for the same data set from ZRH. The same 14 explanatory variables, as introduced in Section 2.2, are used to estimate taxi times for both methods. A Mamdani FRBS with twelve rules has been used throughout the experiments. Table 1 summarises the average results of 20 runs of a Mamdani FRBS. The comparative results of linear regression were obtained from [1].

Table 1 Comparison of prediction accuracy of the Mamdani FRBS and linear regression

	± 3 minutes	± 5 minutes
Linear regression	95.6%	99.4%
Mamdani FRBS	98.8%	100%

As can be seen from Table 1, taxi time estimations from the Mamdani FRBS were more accurate than the results which were obtained by linear regression. Such improvement in the estimation accuracy is largely attributed to the fact that a FRBS approach decomposes the system into sub regions and tackles each region with a set of cooperative rules. Hence, it provides an extra degree of freedom to fine-tune a fuzzy model in order to fit more accurately into the data. Also, the learning capability of the Mamdani FRBS provides an extra power to learn the hidden transformation functions which may not be included within an a priori transformation. Figure 4 shows the fit of the taxi time estimation of the Mamdani FRBS.

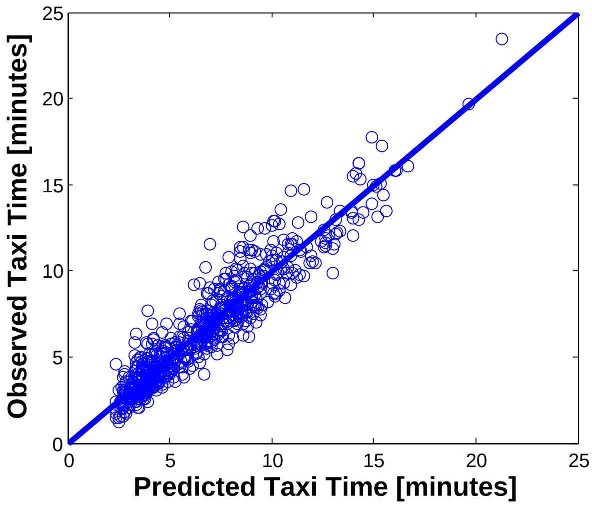

Figure 4 The scatterplot showing the fit of a Mamdani FRBS for taxi times for Zurich Airport

4.2 Validity of approach without explicit transformations

As mentioned before, one distinctive benefit of FRBSs lies in their ability to approximate complex nonlinear systems without the need to explicitly identify transformation functions for explanatory variables and the output. To test the nonlinear approximation power of the Mamdani FRBS, logarithmic transformations for the explanatory variables (such as the ones for the distance and turning angles) were not included in this experiment. Similarly, Angle was used rather than log(Angle). All other explanatory variables were kept the same as those mentioned in Section 2.2. Again, a Mamdani FRBS with twelve rules was utilised for taxi time estimations based upon the same data set for Zurich Airport. The results presented in Table 2 are the average results from 20 independent runs.

Table 2 The results from the Mamdani FRBS with and without explicit input transformations

	± 2 minutes	± 3 minutes	± 5 minutes	R^2
With log transformations	93.4%	98.8%	100%	0.894
Without log transformations	92.9%	98.7%	100%	0.890

As shown in Table 2, the results are similar for both configurations, which suggests that the Mamdani FRBS can cope with nonlinearity even without explicit transformations. In fact, the Mamdani FRBS can automatically learn such hidden transformation functions from historic data. This is useful for practitioners who are not familiar with statistical analysis and do not know how to choose appropriate transformation techniques.

4.3 Explanatory ability via linguistic terms

Another distinctive feature of FRBS lies in its explanatory ability via linguistic terms, which can facilitate their comprehension by airport staff and allow analysis of the airport ground movement in a qualitative way. Figure 5 illustrates how linguistic terms can be associated with membership functions. Three rules out of twelve are presented due to the space limitation and the membership functions for Q and N values are also omitted.

The linguistic terms attached to the membership functions are obtained by investigating their positions in the corresponding variable intervals. Hence, Rule 1 in Figure 5 can be interpreted as follows: if the aircraft is taxiing during the 'day period' and the total turning angle is 'medium small', and the distance is 'medium long', then taxi time is going to take 'long'. In a similar way, one can interpret the other rules. By investigating the whole rule base one can actually gain an understanding of the general principles of the ground movement, e.g.:

- LAN is an important correlating factor with the taxi time; generally arrivals tend to taxi quicker than departing aircraft, due to the departure queue time.
- Distance and Angle are also two important correlating factors, with a positive impact on the taxi time.
- Evening Mode tends to be more efficient in terms of taxing.

We examined the generated rules and found that the knowledge presented by the fuzzy rules for the ZRH data is consistent with the conclusions which were made in [1], but in a more qualitative way. We believe this unique property will be appealing to some airport operators who do not need detailed quantitative interpretation of the taxiing process.

5 Conclusions and future research directions

To our knowledge, this paper represents the first attempt to introduce the Mamdani FRBS into the airport research field, especially in the area of the ground movement. Preliminary results for taxi time estimations for Zurich Airport are very promising and show that FRBSs can produce more accurate estimations given similar explanatory variables compared to linear regression for this particular data set. Furthermore, FRBSs do not necessarily need to include explicit transformation functions since those mappings can be learnt automatically

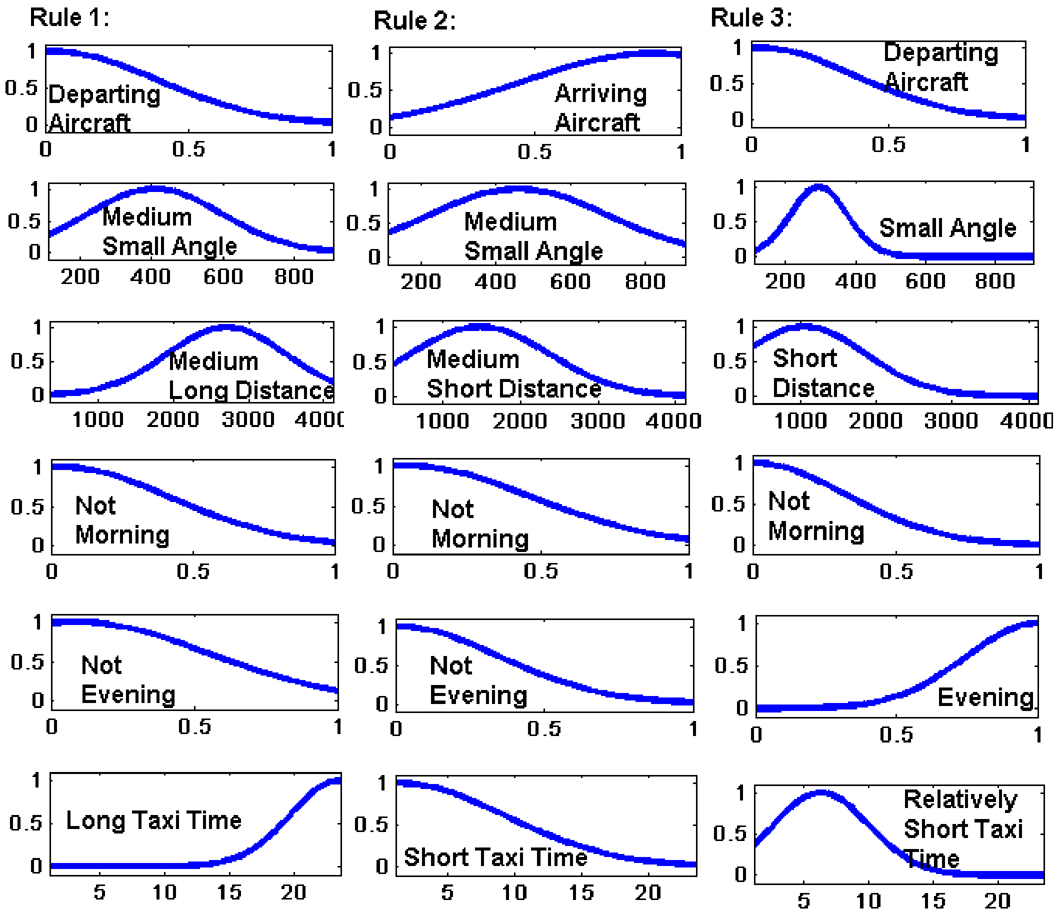

Figure 5 Three fuzzy rules extracted from the data for Zurich Airport

from historic data. Unlike other black-box nonlinear regression techniques, FRBSs can also interpret the underlying systems via linguistic terms which can be understood by humans. We believe that, with all of these distinctive features, the FRBS approach could very well be an alternative for the practitioners in this field.

Building upon this work, we believe that the Sugeno FRBS also deserves more attention in future research for taxi time estimation problems. As discussed in Section 3.1, the consequence part of a Sugeno FRBS is a set of functions of explanatory variables. Hence, if one takes a linear combination of the explanatory variables as the function for the consequence part, the Sugeno FRBS could in some ways be viewed as an extension of multiple linear regression. In such a case, each rule in the rule base resembles a multiple linear regression model for a decomposed explanatory variable region. It is worth highlighting that all these rules are not independent. They work cooperatively to produce estimations. While in the case of multiple linear regression, even if one can build different multiple linear regression models for different regions, they are isolated and cannot deal with the transition behaviour between different sub regions. It is this cooperativeness in FRBSs that may also bring more accurate estimations. Although one could lose certain linguistic meanings in the output compared to the Mamdani FRBS, a function form should be able to approximate the sub region far more accurately than a fuzzy set.

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