Recent Developments on Combinatorial Optimisation Problems

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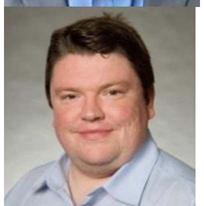
Recent Algorithms on Combinatorial Optimisation Problems

Campuses, The University of Nottingham

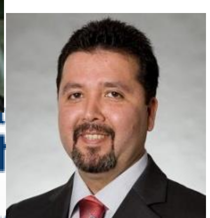
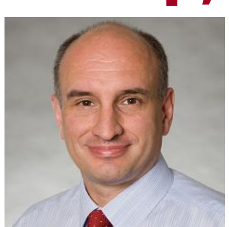
Recent Algorithms on Combinatorial Optimisation Problems






ASAP Group, The University of Nottingham

ASAP
research group

Main Algorithms & Applications

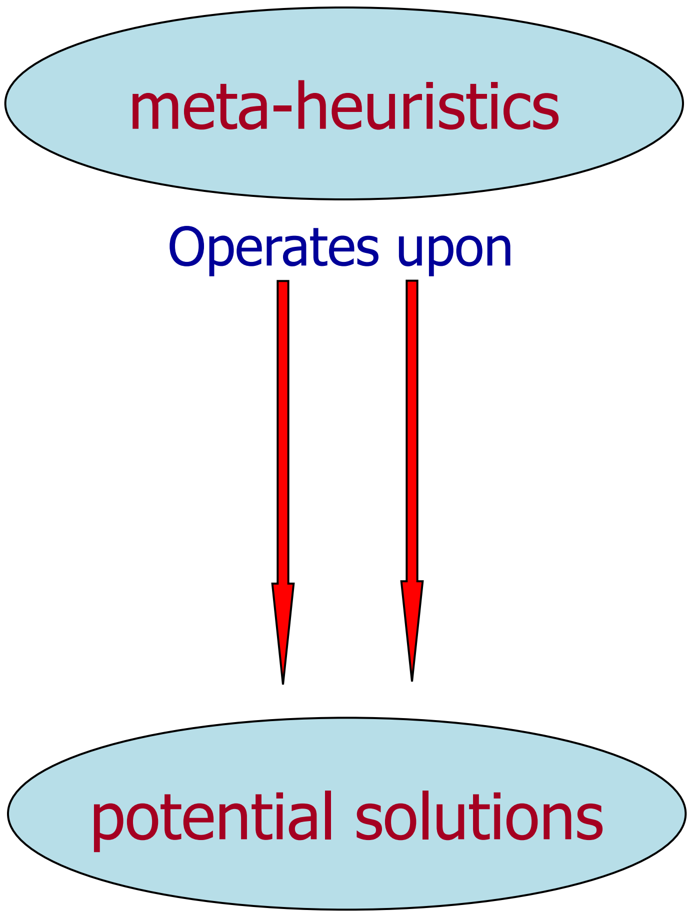
- Methodologies
  - Meta-heuristics
    - Evolutionary algorithms, Local search
    - **Hyper-heuristics**
    - Hybrids
  - Exact approaches
    - Constraint programming
    - Integer / linear programming
  - Hybridisations

Main Algorithms & Applications

- Applications
  - Personnel/workforce scheduling
  - Portfolio optimisation
  - Telecommunication network routing
  - Vehicle routing in logistics
  - **Timetabling**
  - …

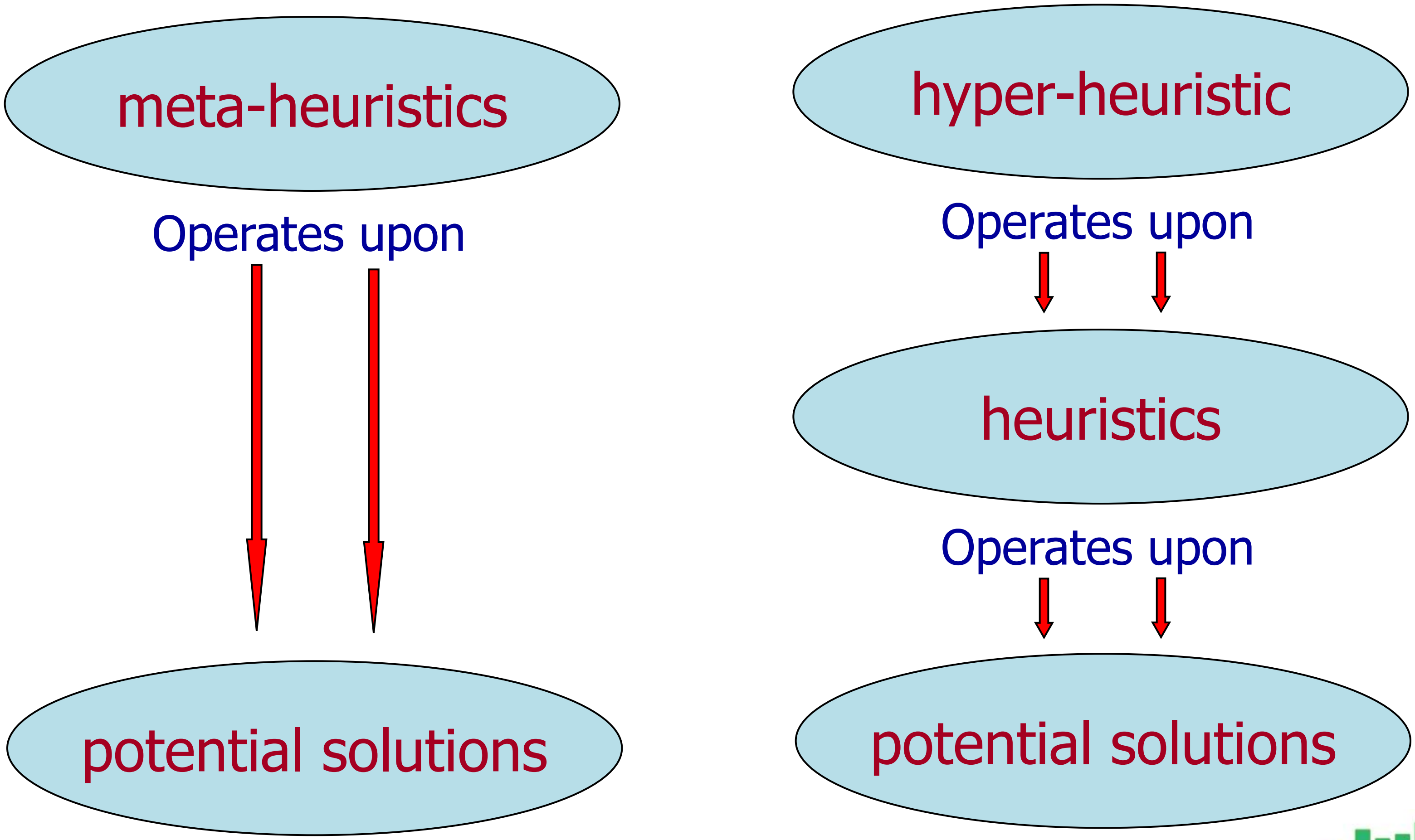
HYPER-HEURISTICS

Background



- Search space
  - All possible solutions
- Design of algorithms
  - Problem specific information hard coded
  - Parameters find tuned for different problems (or instances)

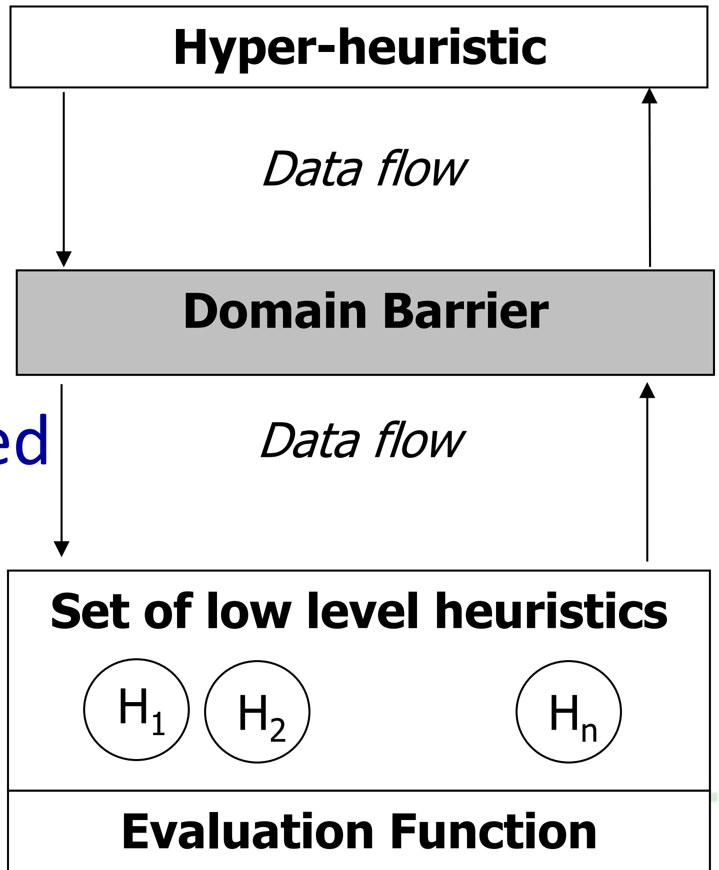
Background


meta-heuristics
Operates upon
potential solutions
hyper-heuristic
Operates upon
heuristics
Operates upon
potential solutions

Background

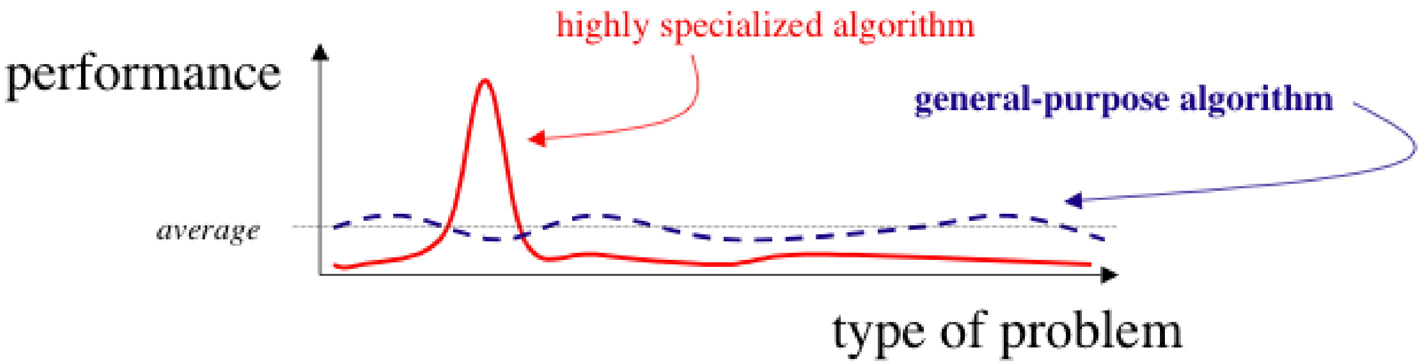
All the term *hyper-heuristic* says is: "***Operate on a search space of heuristics***" or "***Heuristics that choose heuristics***"

- Most meta-heuristics operate directly on problems
- Hyper-heuristics operate on heuristics, which are then applied on the actual problems
  - *automatically* work well on *different* problems



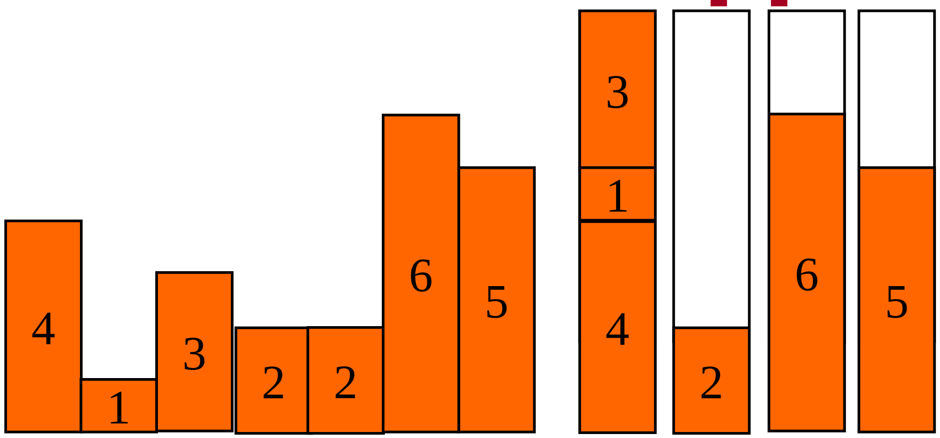
Background

- Research challenges
  - Automate heuristic design
    - Now made by human experts
    - Not cheap!
  - How general we could make hyper-heuristics
    - No free lunch theorem[WOL97]



[WOL97] WOLPERT, D.H. and W.G. MACREADY, 1997. "No free lunch theorems for search". *IEEE Transactions on Evolutionary Computation*

Applications



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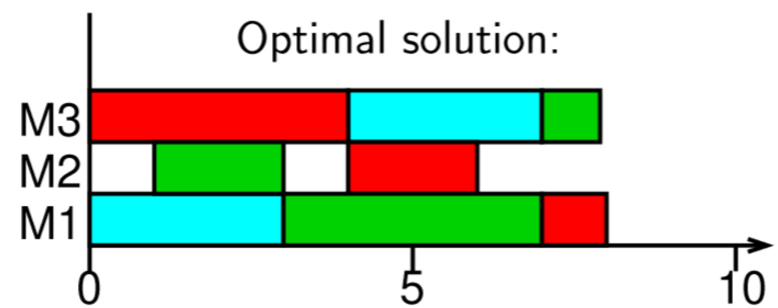
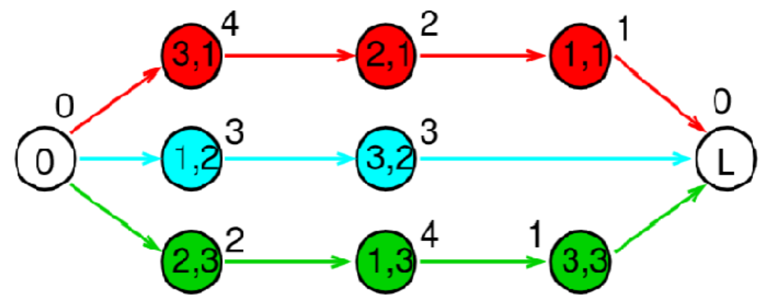
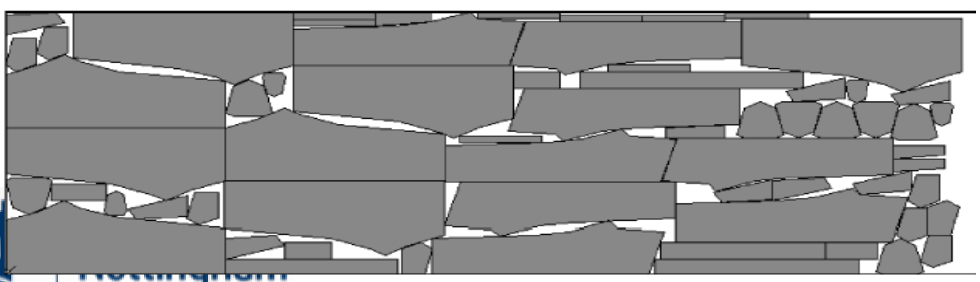
1A	D	E	E	E	L			E	E	E	E		D	D	D	N	N	N				L	L	L	L				51	11
A	DH	DH	DH	DH	DH			DH	DH	DH		DH	DH	DH	DH		DH	DH				DH	DH	DH	DH	DH			20	3
B	N	N	N	N				D	D	L	L	L				L	L	L				E	E	E	D	D			0	0
C	D	D	D	D	D				N	N	N		L	L	L				L	L	L		E	E	E	L			25	4
D				L	N	N	N	N			DH	D				E	E	E	DH	E	E		N	N			E	E	13	7
E					D	DH	DH	D					E	E		DH	E	E	E	DH	DH		D	D	E	E	DH	DH	21	10
F	L	L	L			L	L	L	L			N	N	N	N			D	D			D				D	D	D	10	3
G				E	E	E	E			D	D	D			E	E			D	D	D	D			N	N	N	N	10	2
H	E	E	E			D	D		E	E	E	E			D	D	D		N	N	N	N					L	L	26	6

Total Penalty 176

Unassigned Shifts 0







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ASAP research group

Algorithms

- Low level heuristics
  - Constructive: Construct solutions step by step
    - Graph colouring heuristics, etc.
  - Improvement: Initial solutions improved iteratively
    - Different improvement strategies
- High level heuristics
  - Genetic Algorithms, Tabu Search, Simulated Annealing, Genetic Programming, etc.
  - Case Based Reasoning, Multi-objective techniques, Fuzzy Techniques, choice function, etc.

HYPER-HEURISTICS
- A GRAPH BASED HYPER-HEURISTIC

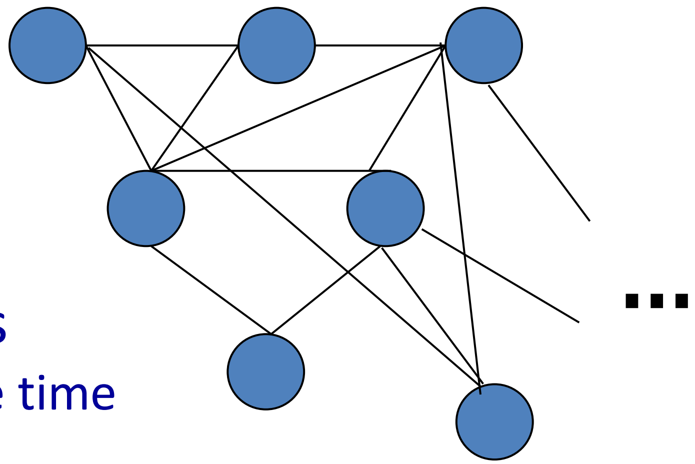
Background

- Constructive heuristics in scheduling
  - Job shop scheduling: dispatching rules
  - Timetabling: graph heuristics
  - Bin packing: 2D/3D packing heuristics
  - Simple and fast

  In complex scheduling problems, using only the basic constructive heuristics often produce unacceptable solutions
- Automated hybridisation / combination of simple heuristics

An example – Timetabling Problems

- Important activities in all universities
- A general timetabling problem
  - A set of events
  - A set of timeslots
  - A set of rooms
  - Schedule the events to timeslots
    - No events for students at the same time
    - Spread students' events
    - …



An example – Timetabling Problems

- Timetabling problems
  - Assign a set of events into a number of time slots, minimising violations of soft constraints
    - Hard constraints
      - Conflicted events in different time slots
      - Room capacity to hold the events, etc.
    - Soft constraints
      - Spread out events over time slots / at least n events or no event on a day
      - No event scheduled on specific time slots, etc.

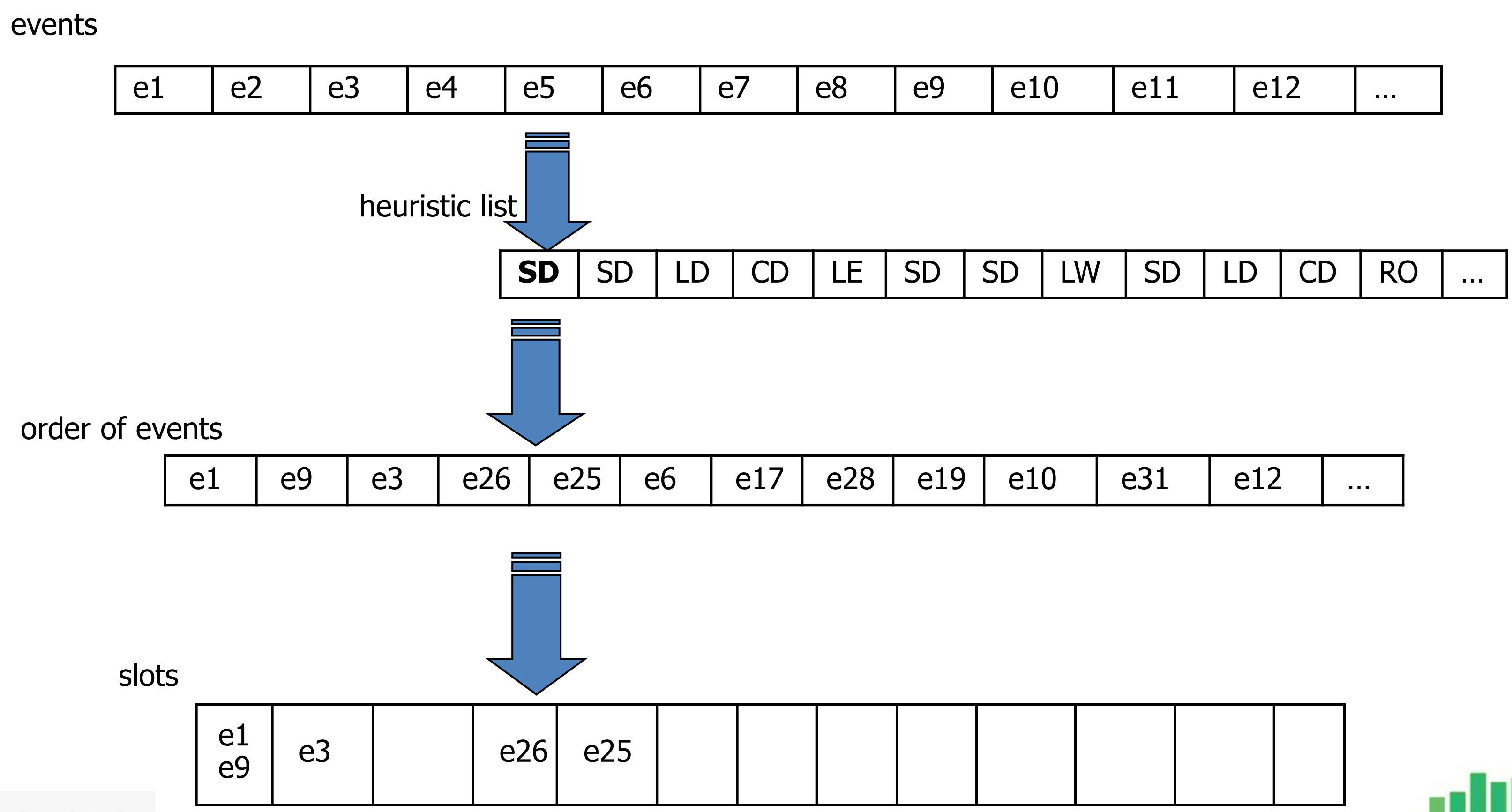
An example – Timetabling Problems

- Timetabling problems
  - Exact methods
    - IP/MILP
  - Constructive heuristics
    - Graph heuristics
    - Constraint satisfaction, etc
  - Meta-heuristics
    - Local search based algorithms
    - Population based algorithms
    - Hybridisations, etc

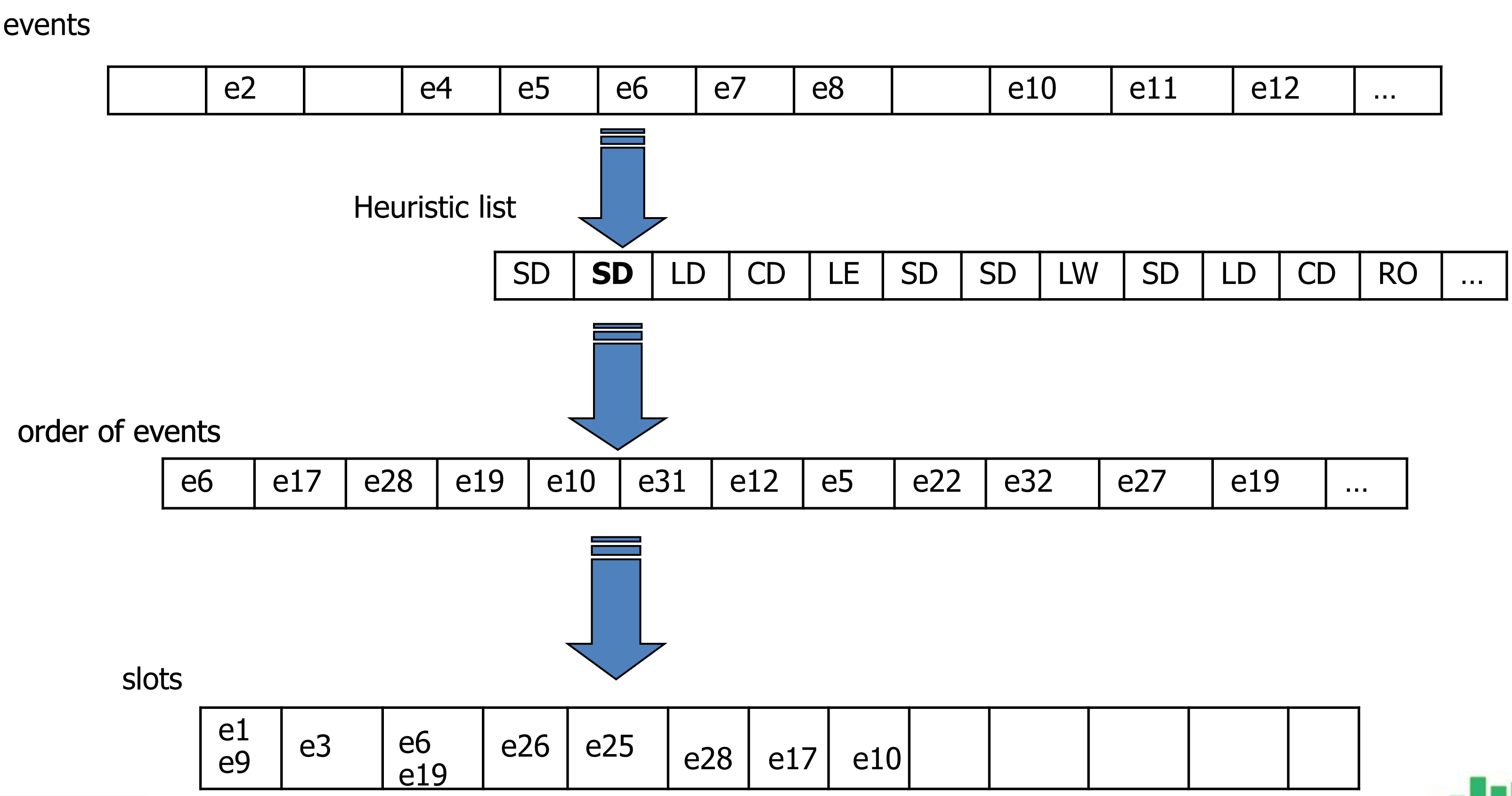
Framework

- The high level framework
  - Any meta-heuristics or learning/search methodology
- The low level graph heuristics: order events by how *difficult* to schedule them
  - Saturation Degree: least available slots
  - Colour Degree: most conflicted with those scheduled
  - Largest Degree: most conflicted with the others
  - Largest Weighted Degree: LD + students involved
  - Largest Enrolment: students enrolled
- Hyper-heuristics: Heuristics to choose heuristics

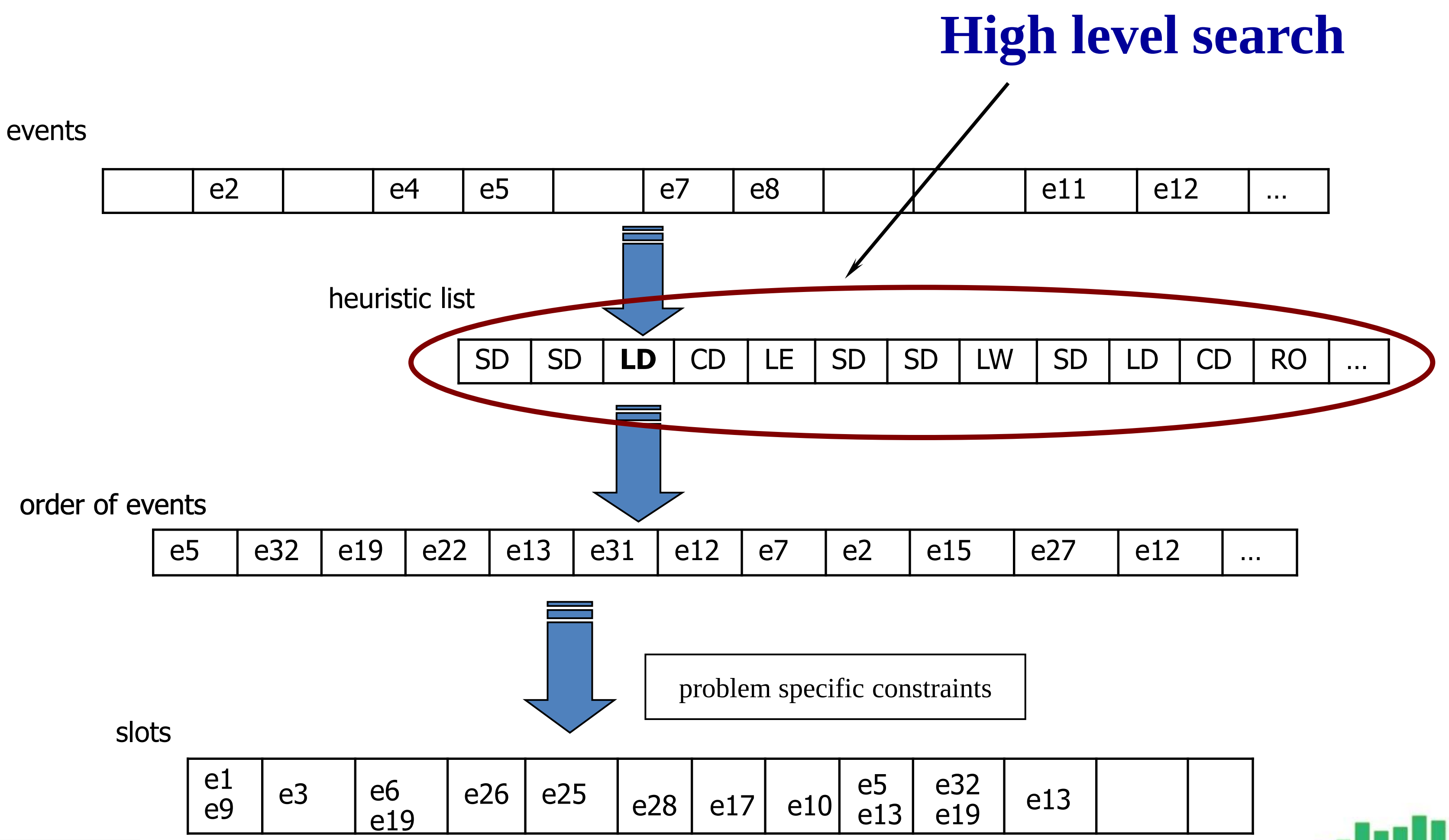
Framework


events
e1 e2 e3 e4 e5 e6 e7 e8 e9 e10 e11 e12 ...
heuristic list
SD SD LD CD LE SD SD LW SD LD CD RO ...
order of events
e1 e9 e3 e26 e25 e6 e17 e28 e19 e10 e31 e12 ...
slots
e1
e9
e3
e26
e25

Framework


events
e2
e4
e5
e6
e7
e8
e10
e11
e12
...
Heuristic list
SD SD LD CD LE SD SD LW SD LD CD RO ...
order of events
e6 e17 e28 e19 e10 e31 e12 e5 e22 e32 e27 e19 ...
slots
e1 e9
e3
e6 e19
e26
e25
e28
e17
e10

Framework


High level search
events
e2
e4
e5
e7
e8
e11
e12
...
heuristic list
SD
SD
LD
CD
LE
SD
SD
LW
SD
LD
CD
RO
...
order of events
e5
e32
e19
e22
e13
e31
e12
e7
e2
e15
e27
e12
...
problem specific constraints
slots
e1
e9
e3
e6
e19
e26
e25
e28
e17
e10
e5
e13
e32
e19
e13

Research issues

- Which high/low level search heuristics?
- Search in two search spaces
- Heuristic hybridisations
- Landscape analysis on heuristic spaces
- Extensions on the framework and other problems

Research issues

- High level search methods
  - Iterated Local Search
  - Tabu Search
  - Steepest Descent
  - Variable Neighbourhood Search

  - Objective function
    - heuristic lists → penalties (costs of timetables constructed)
  - "Walks" are allowed

Research issues

- High level search methods
  - Similar performance within the same framework (same total no. of evaluations, same initials, etc)
  - ILS and VNS are slightly better
  - Results are comparable to best approaches on both course and exam benchmark problems

	car91	car92	ear83	hec92	kfu93	lse91	sta83	tre92	ute92	uta92	yor83
SDM avg	6.18	5.3	***36.8***	12.74	15.63	13.51	163.7	9.37	32.6	4.5	43.6
ILS avg	***6.01***	5.18	39.58	13.01	15.35	13.1	***161.6***	8.92	***31.3***	***4.01***	***43.15***
TS avg	6.3	5.34	45.56	14.6	19.55	14.29	169.1	9.67	37.02	4.38	47.97
VNS avg	6.1	***5.1***	38.63	***12.72***	***15.24***	***13.06***	163.3	***8.88***	31.7	4.05	43.93

	s1	s2	s3	s4	s5	m1	M2	m3	m4	m5	l
SDM avg	10.8	15.6	***5***	11.8	12.2	382.5	100%	383	***374.5***	194.5	100%
ILS avg	***8.8***	***13.2***	5.4	***7.6***	12	375	480.5	377.5	380.5	179.7	***1144 60%***
TS avg	12.2	16.4	9.2	12.2	18.2	511.5	533 80%	468	539	236	1164 80%
VNS avg	10	14.8	5.2	8	***10.6***	***365***	***443 40%***	***369.5***	377.5	***165.5***	1148 80%

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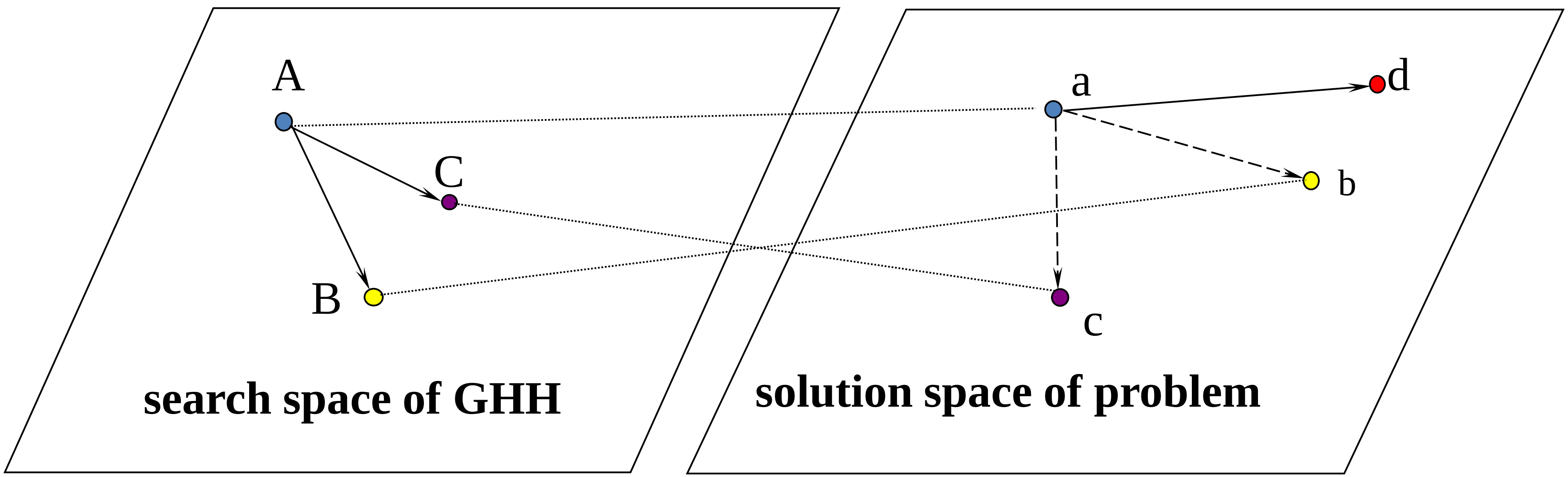
Research issues

- Low level heuristics
  - Different subsets of graph heuristics (SD+LD, SD+LWD, SD+LE, SD+LWD+CD, etc)
  - With a limited computational time
    - SD + LWD performed the best
  - With more graph heuristics
    - Longer time given, the better the results
    - h^l (l: length of the sequence, h: number of graph heuristics
  - Random ordering also contributes the performance

Research issues

Heuristic space **Solution space**



GHH: search is upon heuristics, not solutions

– not all the solutions in solution space are reachable?

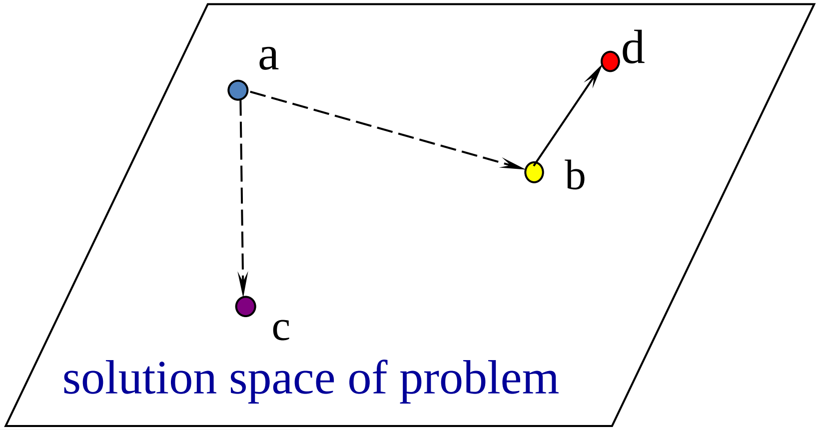
Research issues

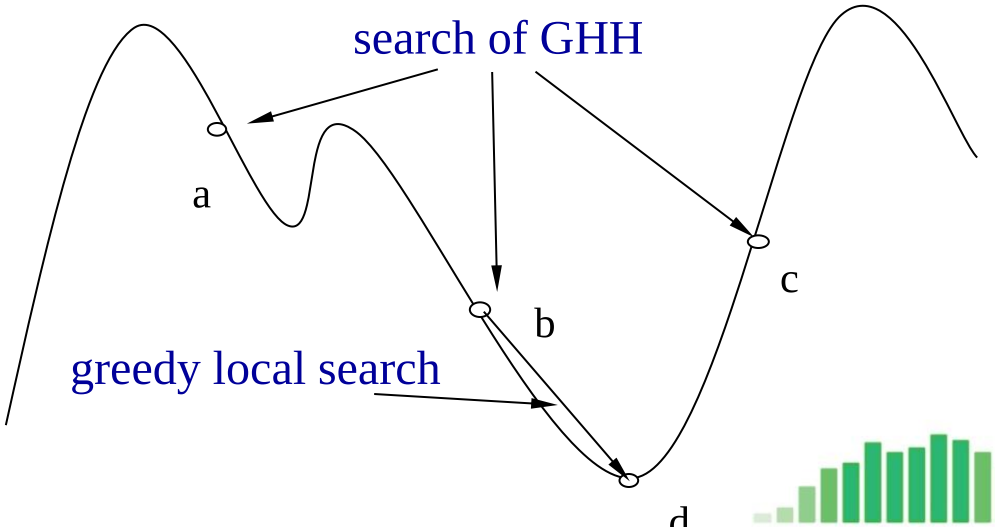
Heuristic space **Solution space**

Representa-tion
Size (Upper Bound)
Neighbor-hood Operator
Objective Function

Research issues

- Hybridisation in the framework with simple greedy search
  - High level search in heuristic space: a, b, c, …
  - Greedy search in solution space: b -> d, …
    - Coverage of the solution space





Research issues

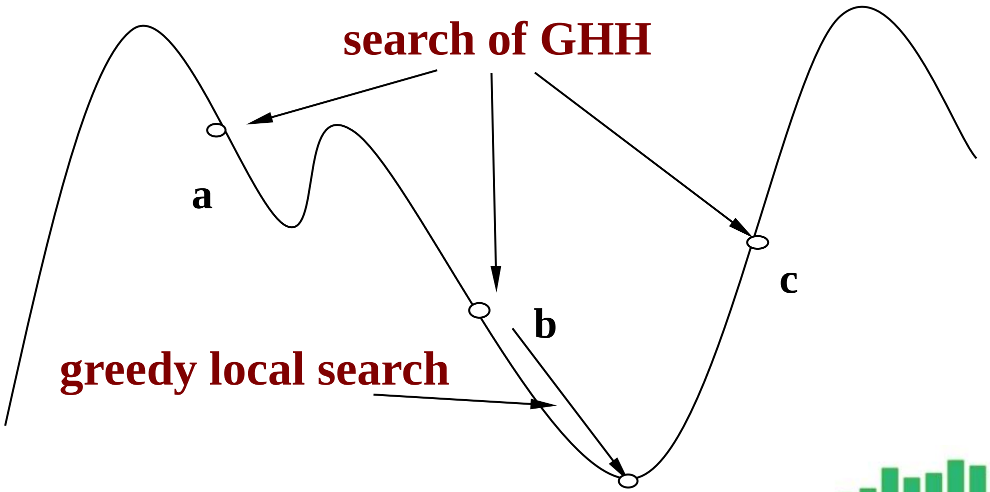
- Hybridisation in the framework with simple greedy search
  - Results greatly improved!

	car91	car92	ear83	hec92	kfu93	lse91	sta83	tre92	ute92	uta92	yor83
GHH2 best	5.16	4.16	35.86	11.94	14.79	11.15	159	8.6	28.3	3.59	41.81
GHH2 avg	**5.21**	**4.20**	**36.2**	**12.1**	**15.01**	**11.24**	**160.81**	**8.65**	**28.64**	**3.62**	**41.96**
GHH2 time	26001	11666	740	105	3417	2015	128	2293	131	10045	641
GHH1 best	5.3	4.77	38.39	12.01	15.09	12.72	159.2	8.74	30.32	3.42	40.24
GHH1 avg	6.01	5.18	39.58	12.33	15.35	13.1	161.6	9.0	31.3	4.01	43.15
GHH1 time	13684	6553	462	70	1887	1125	72	1433	101	5429	340

	s1	s2	s3	s4	s5	m1	m2	m3	m4	m5	l
GHH2 best	0	0	0	0	0	257	259	192	235	112	0.8/1132
GHH2 avg	***0.2***	***0.6***	***0***	***0.4***	***0.1***	***261***	***273***	***214.5***	***242***	***116***	***1135***
GHH2 time	50	54	48	45	65	19411	15750	18512	18782	9725	20328
GHH1 best	2	2	1	1	0	310	419	332	324	162	0.8/1162
GHH1 avg	2.6	2.8	1	3	2.6	323	428	345	335	182	1162
GHH1 time	155	218	240	171	260	62115	50403	57387	65821	36955	81148

Research issues

- Hybridisation in the framework with simple greedy search
  - Hybrid GHH vs. Memetic Algorithms
    - Diversification vs. intensification



Research issues

- Search in two search spaces
  - Diversification of the high level search in the framework in the heuristic space
  - Intensification by the local search in the solution space

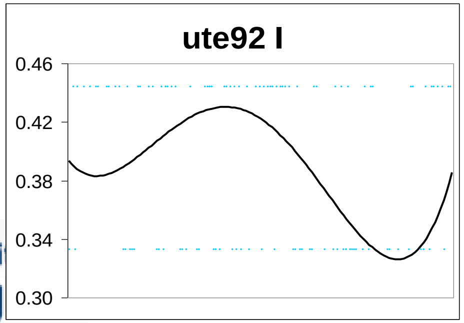
- Role of high level search methods

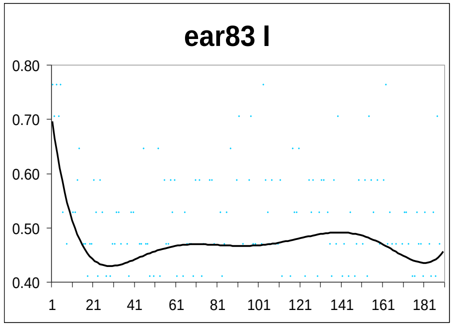
  To explore diversified solutions in the solution space

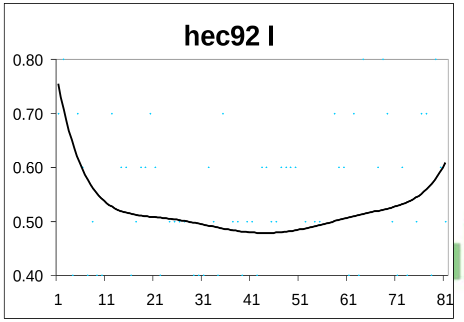
  by searching in the high level heuristic space

Hybridisations

- How to (adaptively) hybridise heuristics?
  - Knowledge / lesson learnt from the offline heuristic hybridisations?
  - I – Random (SD+LWD, SD+LE, SD+LD)
    - A large collection of different heuristic sequences
    - Systematically produce heuristic sequences
    - Full coverage of different amount of hybridisations
  - II – Analyze the best/worst 5% heuristic sequences
    - Rates of hybridisation at different positions of heuristic sequences
    - Trends of hybridizations in the best sequences

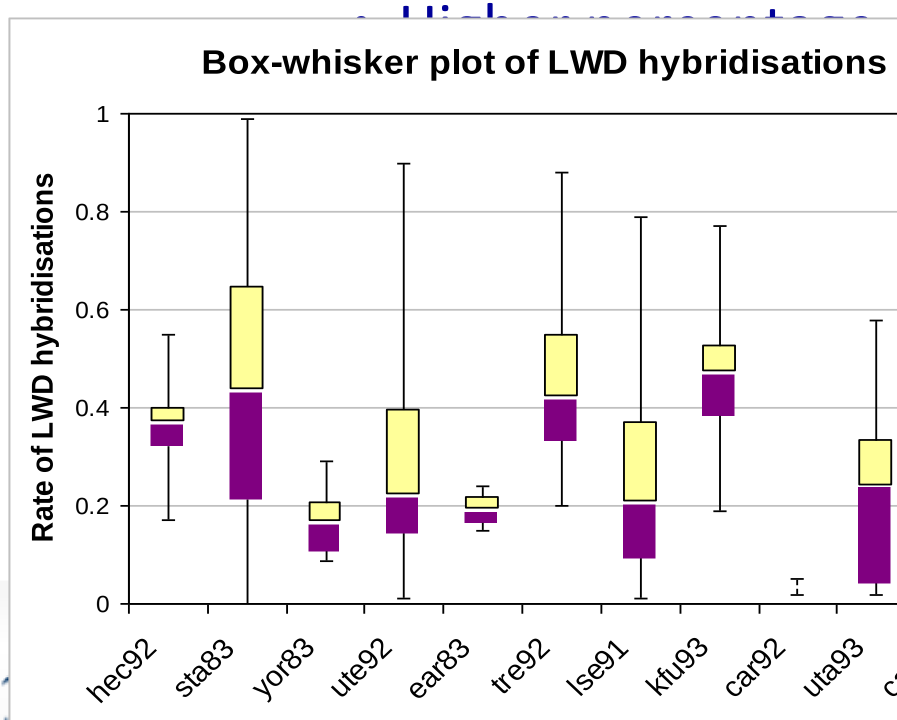


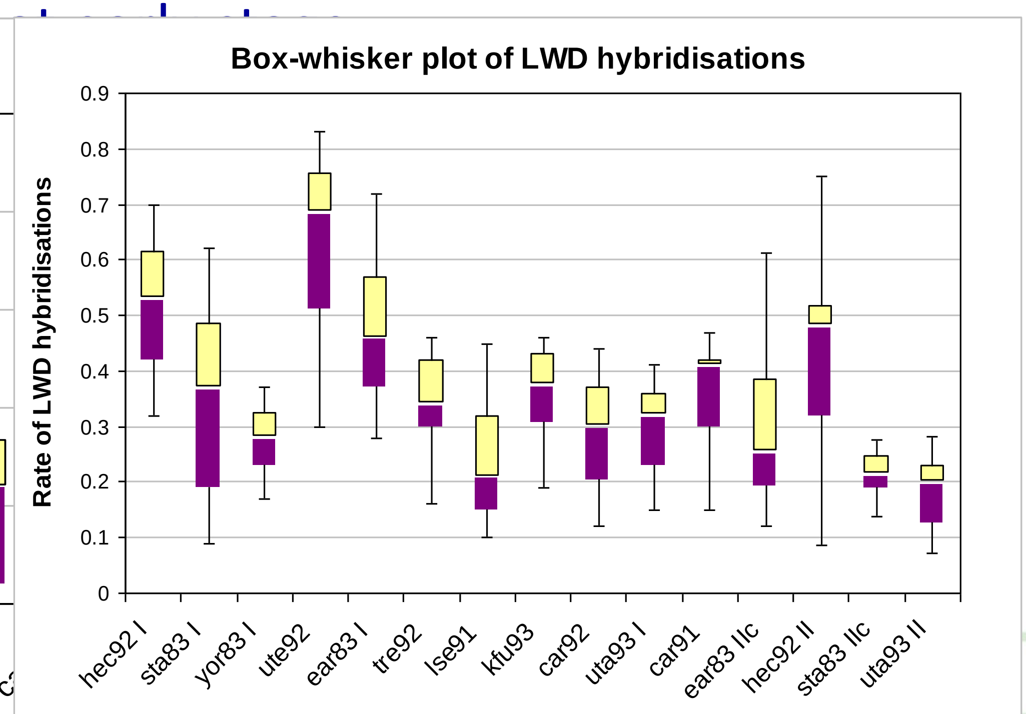




Hybridisations

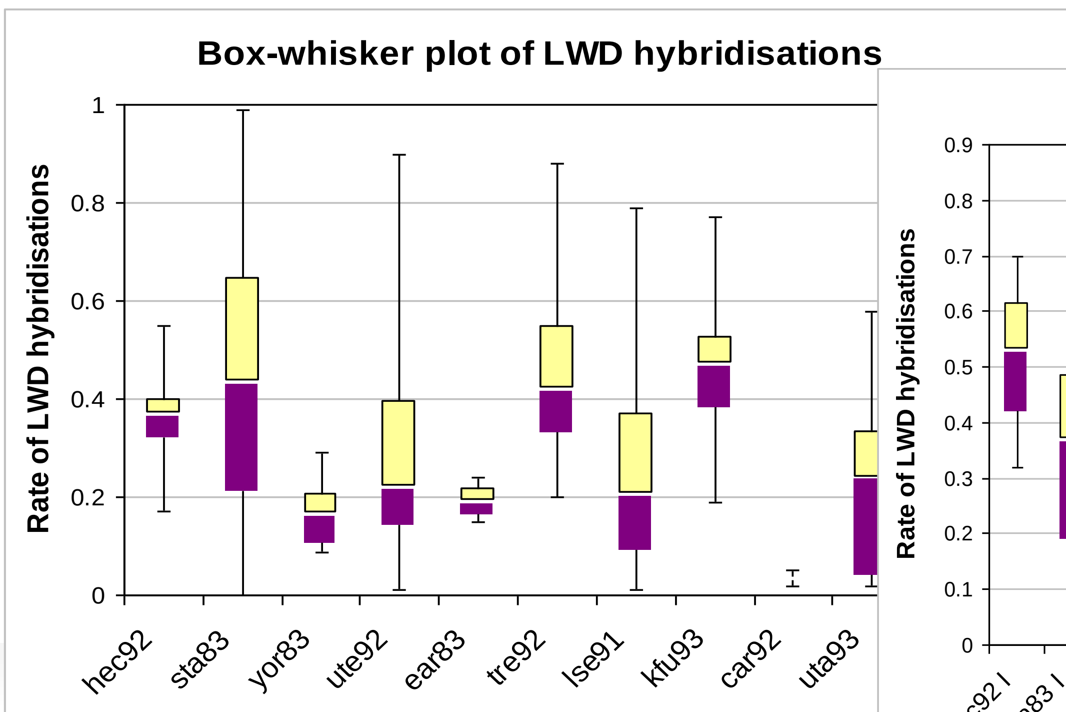
- Results of analysis
  - Hybridising SD with LWD obtained better results compared with LE or LD
  - In the best 5% sequences

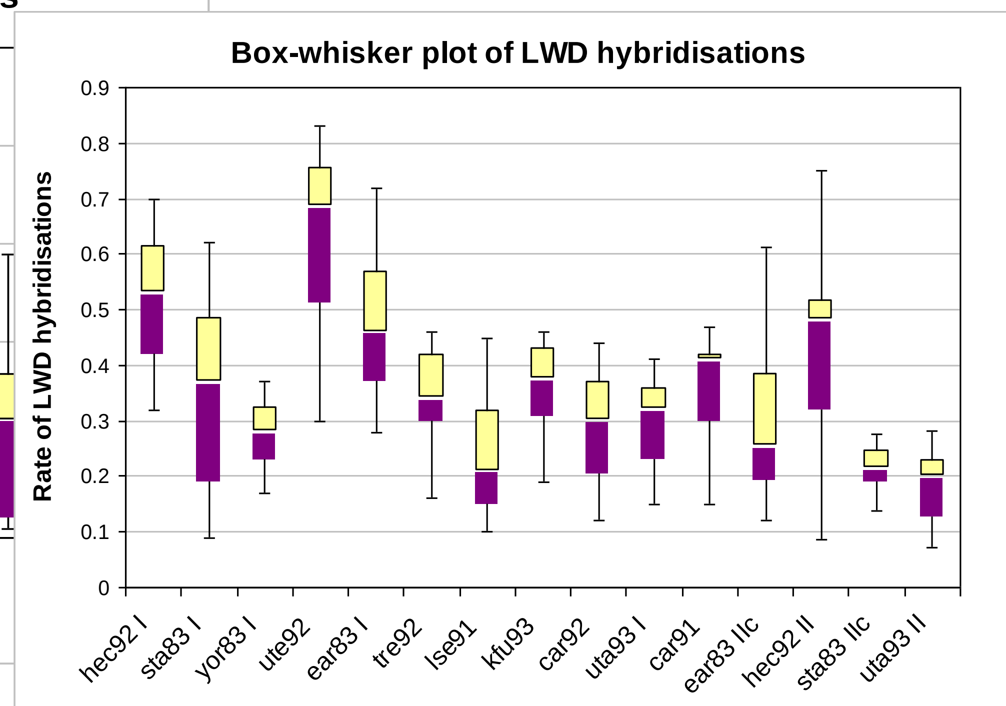




Hybridisations

- Adaptive online heuristic hybridization
  - Focus on early stage of heuristic hybridization
  - Rate of LWD hybridisation adaptively adjusted

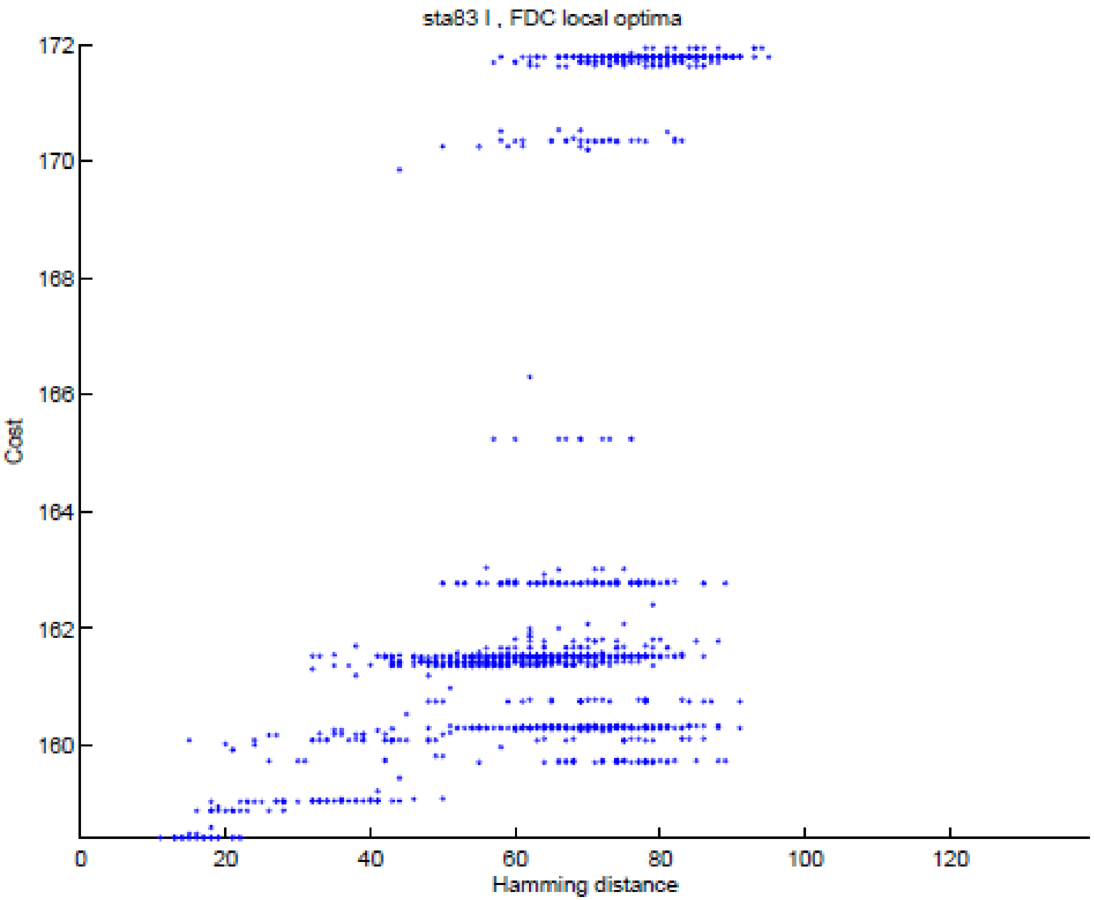


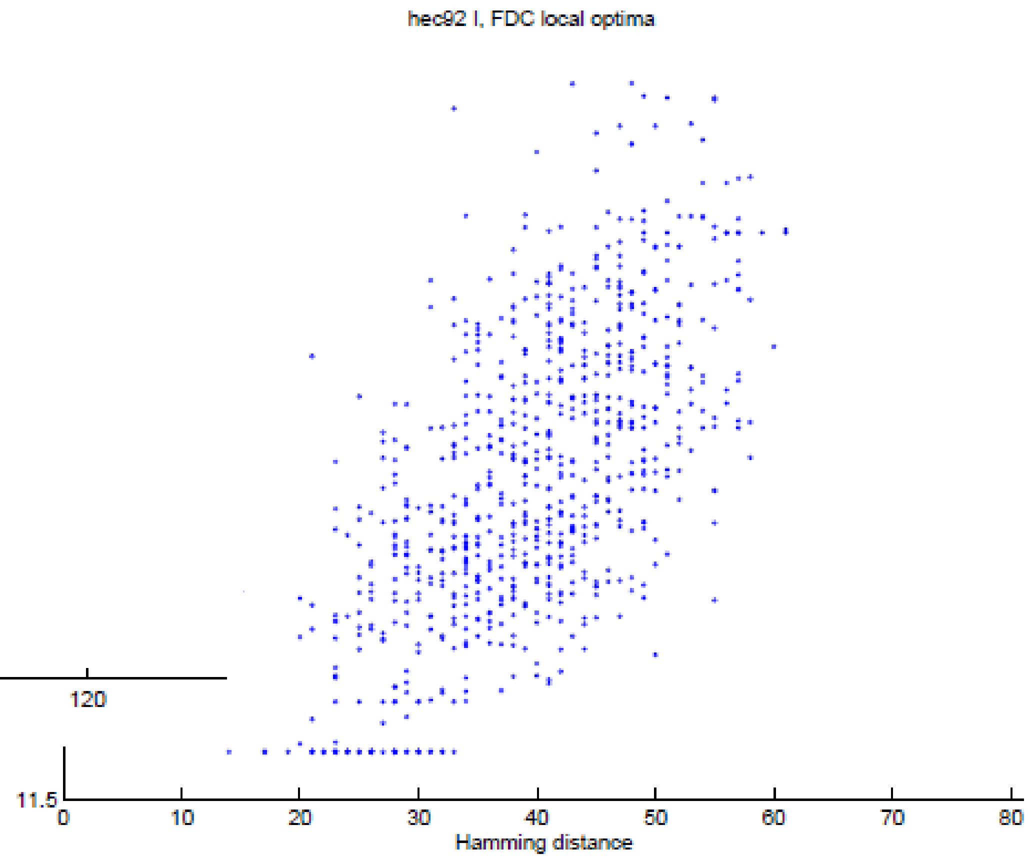


Landscape analysis

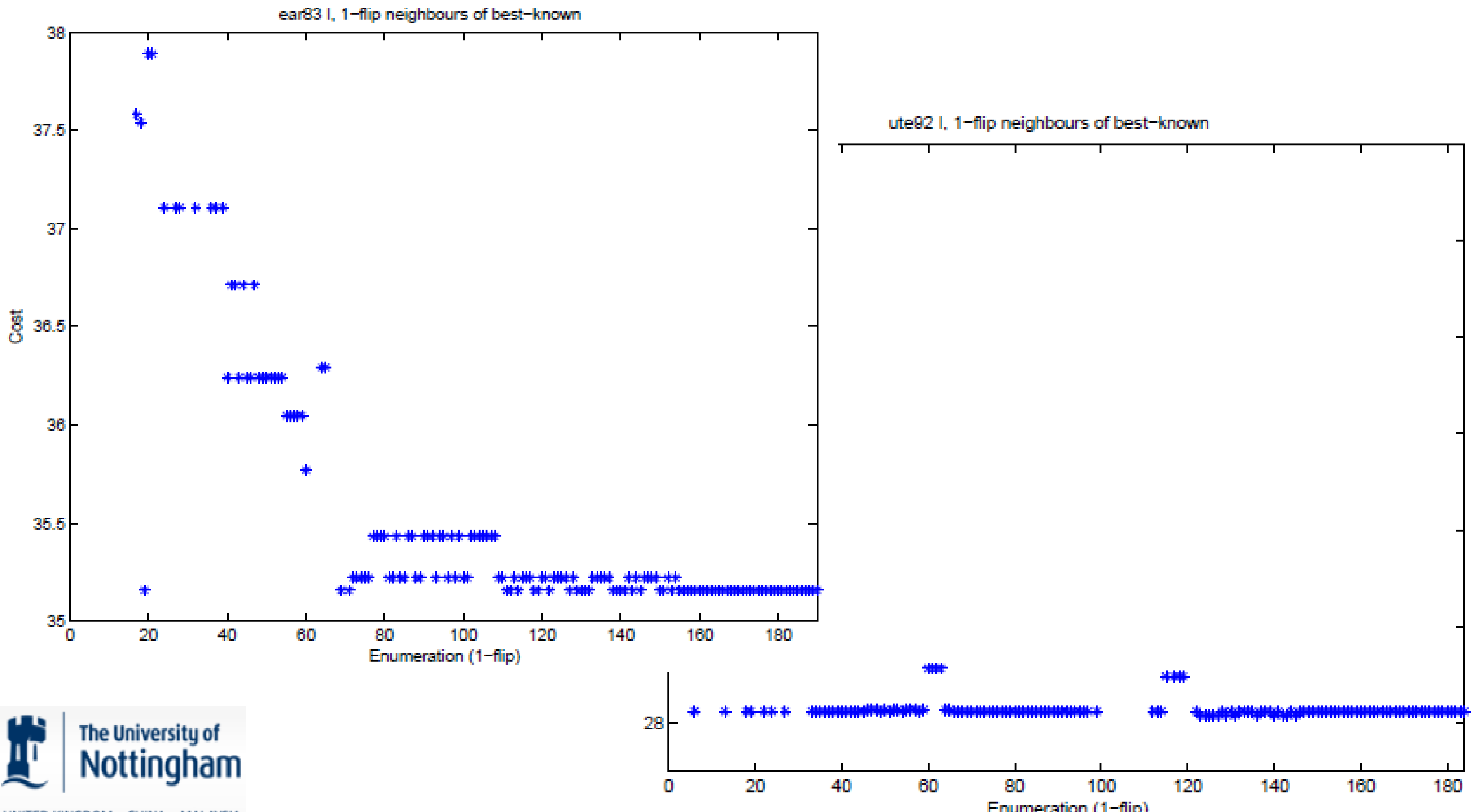
- Understanding the structure of heuristic search spaces, i.e. heuristic sequences vs. solutions
- Fitness landscape analysis on constructive hyper-heuristics
  - Fitness distance correlation (*fdc*) of local optima to the global optimum
  - One-flip of global optimum
  - Correlation length
- Although rugged, the encouraging feature of a globally big valley structure
- A high level of neutrality and positional bias

Landscape analysis





Landscape analysis



OTHER RESEARCH TOPICS

Nurse Rostering Problems

December	04	05	06	07	08	0	23	24	25	26	27	28	29	30	31	
	1								4							
	M	T	W	T	F	S	S	S	M	T	W	T	F	S	S	
1A	D	E	E	E	L				L	L	L	L				51
A	DH	DH	DH	DH	DH				DH	DH	DH	DH	DH			20
B	N	N	N	N					E	E	E	D	D			0
C	D	D	D	D	D		L	L		E	E	E	L			25
D				L	N	N	E	E		N	N			E	E	13
E					D	D	DH	DH		D	D	E	E	DH	DH	21
F	L	L	L			L			D				D	D	D	10
G				E	E	E	D	D	D			N	N	N	N	10
H	E	E	E			D	N	N	N					L	L	26

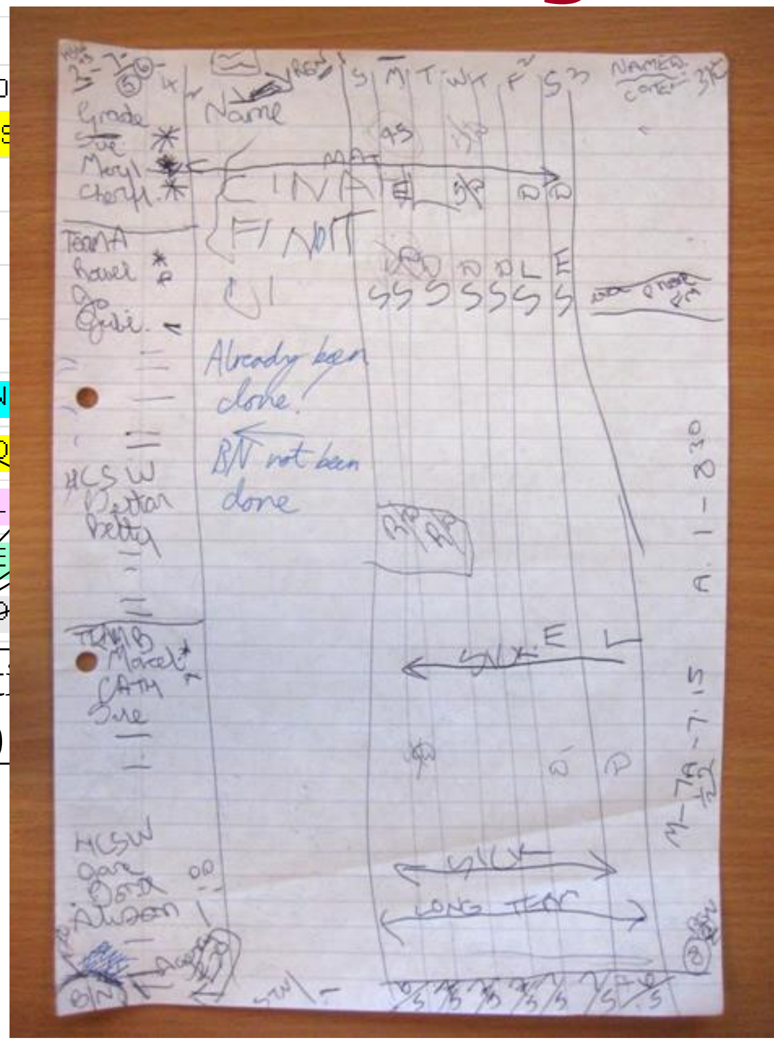
Too few resti... time (10)

...consecutive shifts (5)

Total Penalty 176

Unassigned Shifts 0

Minimum Cover	04	05	06	07	08	0	23	24	25	26	27	28	29	30	31
E	1	2	2	2	1	1	1	1	1	2	2	2	1	1	1
D	2	1	1	1	2	1	1	1	2	1	1	1	2	1	1
DH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
N	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1



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Nurse Rostering web site at
http://www.asap.cs.nott.ac.uk/projects/nmhpr/data

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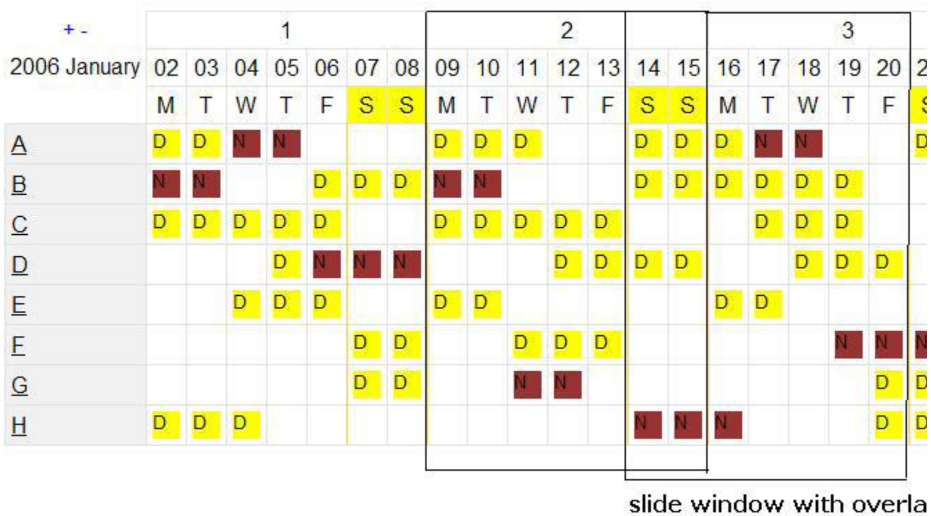
Nurse Rostering Problems

- Hybrid variable neighbourhood search
  - HARMONY™, ORTEC, The Netherlands
- Constraint programming
- Sequence based adaptive approach



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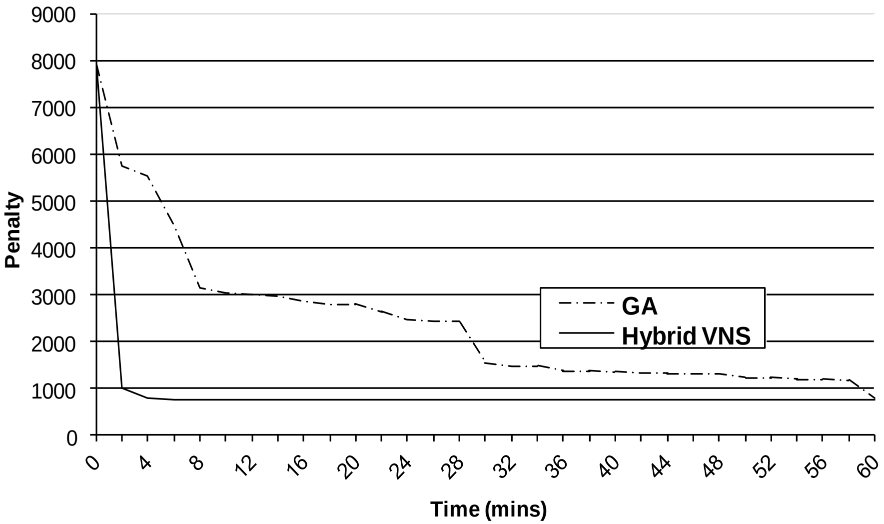
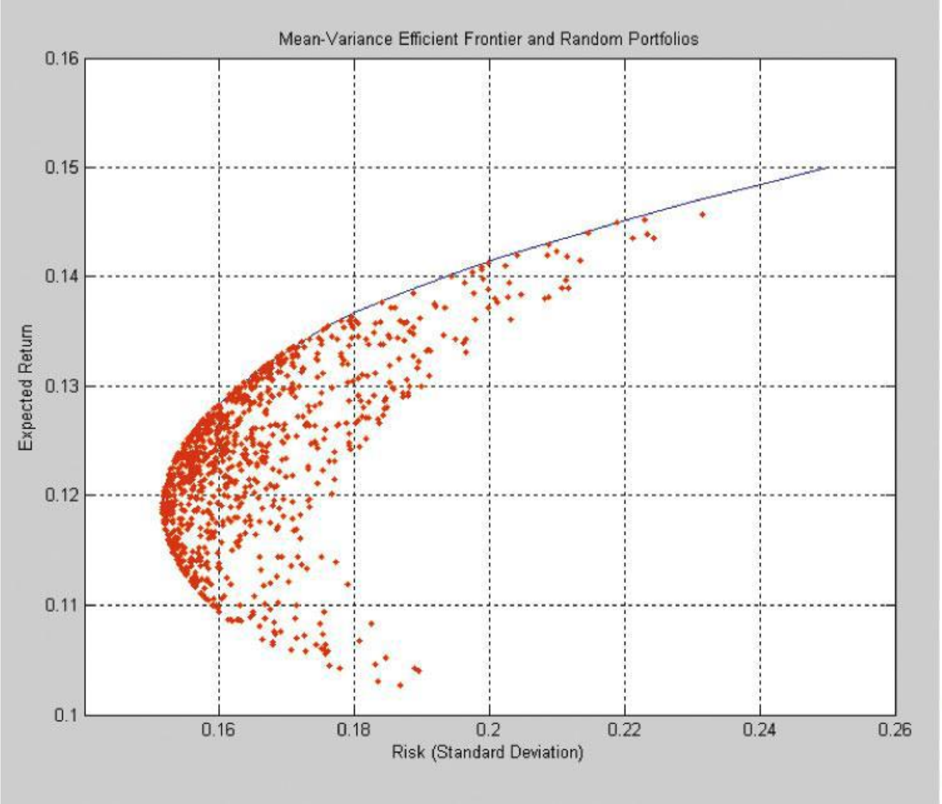
slide window with overla



Portfolio Optimisation

- Allocation of capital of budget to selected assets, aiming to minimise risk and maximise return
- Markowitz's modern portfolio theory
  - Mean-Variance model
  - Efficient frontier
  - Risk vs. return



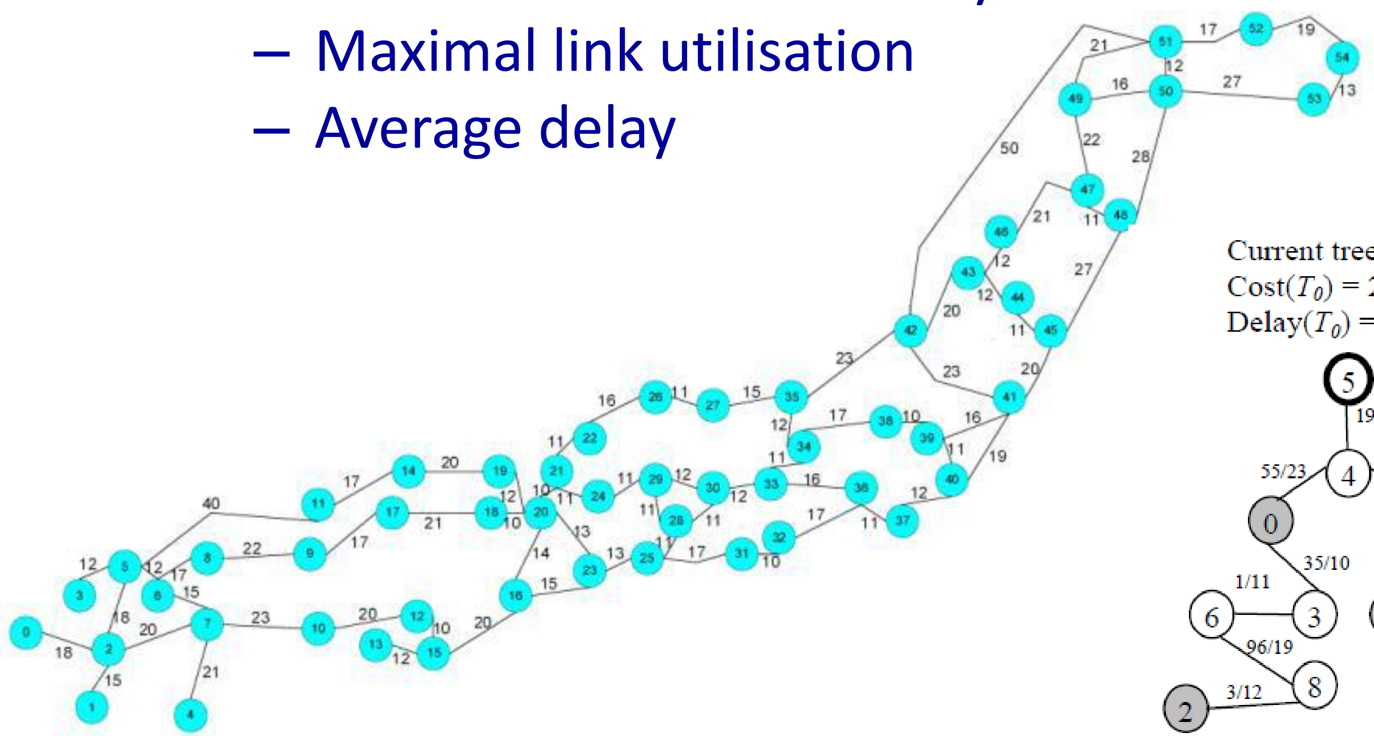
Matlab™

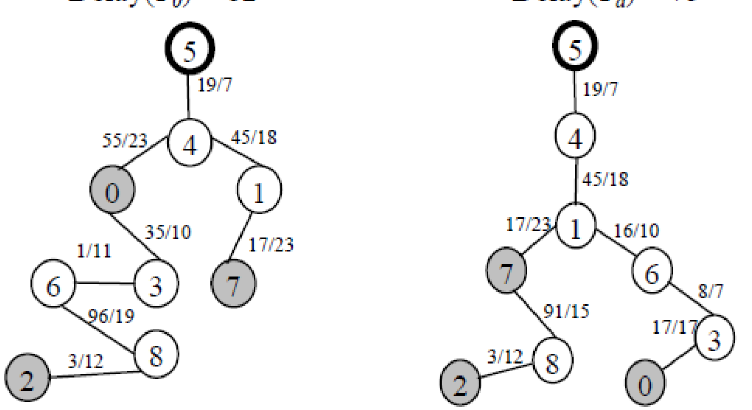
Freight Transport Routing

- Design of routing plan, starting from a depot, to serve all customers within a network
- Constraints
  - Capacity
  - Time window
  - Pick-up vs. drop
- Objectives
  - Cost
  - Empty load
  - …

Multicast Routing

- Finding the multicast tree serving all terminals with the minimal cost while satisfying delay bound
- Multiple objectives
  - Maximal end-to-end delay
  - Maximal link utilisation
  - Average delay





Questions?

Thank you!

– More details at: **http://www.cs.nott.ac.uk/~rxq/publications.htm**

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- More references at http://www.asap.cs.nott.ac.uk/?q=bibliography