The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Modelling and Optimisation of Personnel Scheduling Problems in Healthcare

Dr Rong Qu
Computational Optimisation and Learning (COL) Group
Automated Scheduling, Optimisation and Planning (ASAP) Group
School of Computer Science, The University of Nottingham
rong.qu@nottingham.ac.uk

Collaborators: Prof Bruker, Prof E Burke, Dr T Curtois, Dr F He, Dr J Li, Dr B McCollum, Dr G Post

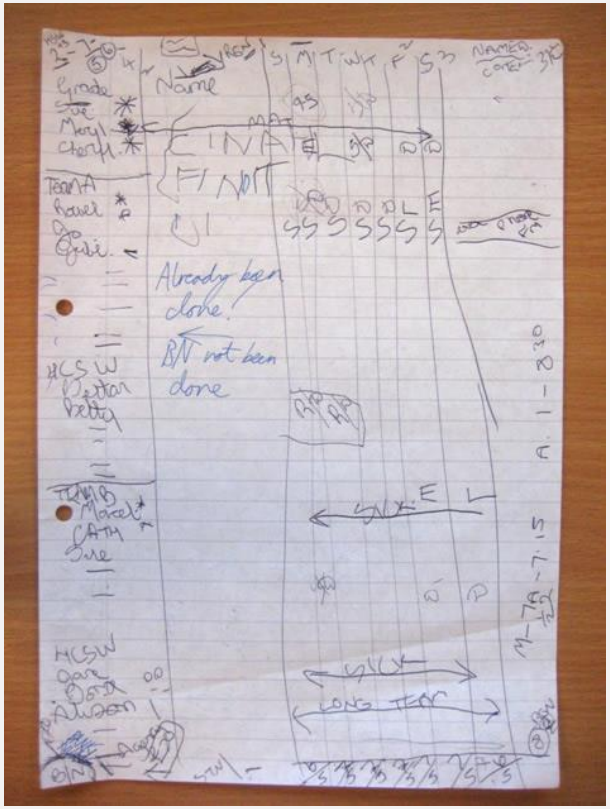
ASAP research group

Computational Optimisation and Learning COL

Nurse Rostering in Healthcare

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

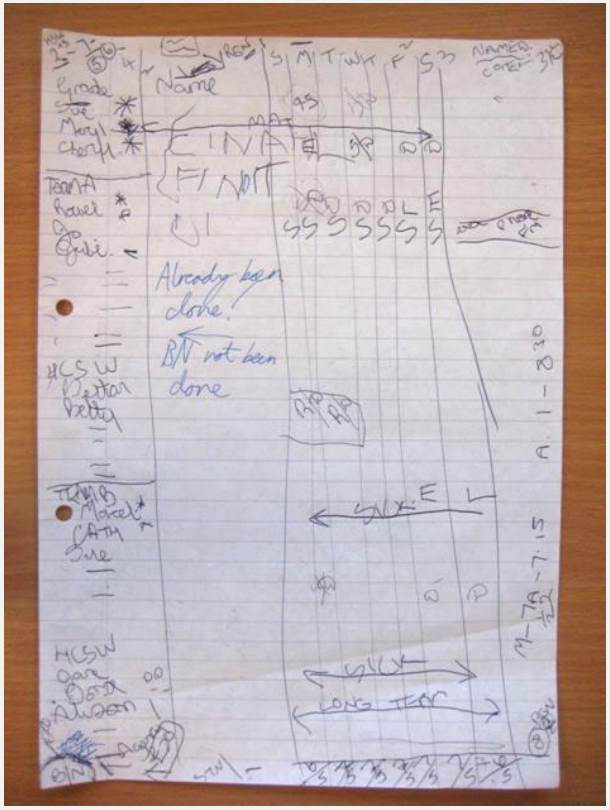
- Schedule a number of shifts to nurses in rosters, satisfying a set of constraints
  - Enough number of shifts (of different types) coverage on each day during the scheduling period
  - Side constraints: working/resting hours limit, complete weekends, skill levels, personal preferences, etc.
- Difficult combinatorial optimisation problem
  - Different grade and skill mixes, Number of shift types (early, day, late, night), Cover requirements can vary
  - Many constraints and objectives



Nurse Rostering – Modelling

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

- Hospitals operate 24/7, introduces constraints related to night shifts and weekends
- Different grade and skill mixes
- Number of shift types (early, day, late, night)
- Cover requirements can vary
- Long scheduling horizons and large no. of employees
- Problems occur in hospital wards worldwide
- Difficult problem with many constraints and objectives
- Time consuming, frustrating and stressful problem
- Regular rescheduling required to cope with absences
- Poor planning and excess workload can cause decrease in quality of healthcare
- …



Nurse Rostering – Modelling

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

	1							2							3							4								
December	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31		
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
1A	D	E	E	E	L			E	E	E	E		D	D	D	N	N	N				L	L	L	L				51	11
A	DH	DH	DH	DH	DH			DH	DH	DH		DH	DH	DH	DH		DH	DH				DH	DH	DH	DH	DH			20	3
B	N	N	N	N				D	D	L	L	L				L	L	L				E	E	E	D	D			0	0
C	D	D	D	D	D				N	N	N		L	L	L				L	L	L		E	E	E	L			25	4
D				L	N	N	N	N			DH	D				E	E	E	DH	E	E		N	N			E	E	13	7
E					D	DH	DH	D					E	E		DH	E	E	E	DH	DH		D	D	E	E	DH	DH	21	10
F	L	L	L			L	L	L	L			N	N	N	N			D	D			D				D	D	D	10	3
G				E	E	E	E			D	D	D			E	E			D	D	D	D			N	N	N	N	10	2
H	E	E	E			D	D		E	E	E	E			D	D	D		N	N	N	N					L	L	26	6

Total Penalty 176

Unassigned Shifts 0

Minimum Cover

E	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1	
D	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	
DH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
N	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

Nurse Rostering web site at http://www.schedulingbenchmarks.org/

Nurse Rostering – Modelling

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

- Problem formulation
  - Hard constraints
    - binding, feasibility, or imperative planning rules
  - Soft constraints
    - floppy, non binding, preference planning rules
  - Weights
    - to specify relative priorities
    - weighted sum objective function

December	1							2							3							4								
	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31		
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
1A	D	E	E	E	L			E	E	E	E		D	D	D	N	N	N					L	L	L	L				51	11
A	DH	DH	DH	DH	DH			DH	DH	DH		DH	DH	DH	DH		DH	DH				DH	DH	DH	DH	DH			20	3
B	N	N	N	N				D	D	L	L	L				L	L	L				E	E	E	D	D			0	0
C	D	D	D	D	D				N	N	N		L	L	L				L	L	L		E	E	E	L			25	4
D				L	N	N	N	N			DH	D				E	E	E	DH	E	E		N	N			E	E	13	7
E					D	DH	DH	D					E	E		DH	E	E	E	DH	DH		D	D	E	E	DH	DH	21	10
F	L	L	L			L	L	L	L			N	N	N	N			D	D			D				D	D	D	10	3
G				E	E	E	E			D	D	D			E	E			D	D	D	D			N	N	N	N	10	2
H	E	E	E			D	D		E	E	E	E			D	D	D		N	N	N	N					L	L	26	6

Total Penalty 176
Unassigned Shifts 0

Minimum Cover																														
E	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1		
D	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1		
DH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
N	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		

Nurse Rostering – Modelling

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

	1							2							3							4							
December	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	
1A	D	E	E	E	L			E	E	E	E		D	D	D	N	N	N				L	L	L	L				51
A	DH	DH	DH	DH	DH			DH	DH	DH		DH	DH	DH	DH		DH	DH				DH	DH	DH	DH	DH			20
B	N	N	N	N				D	D	L	L	L				L	L	L				E	E	E	D	D			0
C	D	D	D	D	D				N	N	N		L	L	L				L	L	L		E	E	E	L			25
D				L	N	N	N	N			DH	D				E	E	E	DH	E	E		N	N			E	E	13
E					D	DH	DH	D					E	E		DH	E	E	E	DH	DH		D	D	E	E	DH	DH	21
F	L	L	L			L	L	L	L			N	N	N	N			D	D			D				D	D	D	10
G				E	E	E	E			D	D	D			E	E			D	D	D	D			N	N	N	N	10
H	E	E	E			D	D		E	E	E	E			D	D	D		N	N	N	N					L	L	26

Too few resting time (10)

Too few consecutive late shifts (5)

Too few consecutive night shifts (5)

Total Penalty 176

Unassigned Shifts 0

Minimum Cover																													
E	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1	
D	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	
DH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
N	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

Nurse Rostering – Literature

- Meta-heuristics heavily used
  - GAs, Memetic Algorithm, Tabu Search, Variable Neighbourhood Search, etc.
- Hyper-heuristics showed to be flexible and effective
- Mathematical programming also reported good results
  - Integer programming hybridised with heuristics / meta-heuristics
- Others
  - Constraint programming
  - Case based reasoning, and
  - Many more

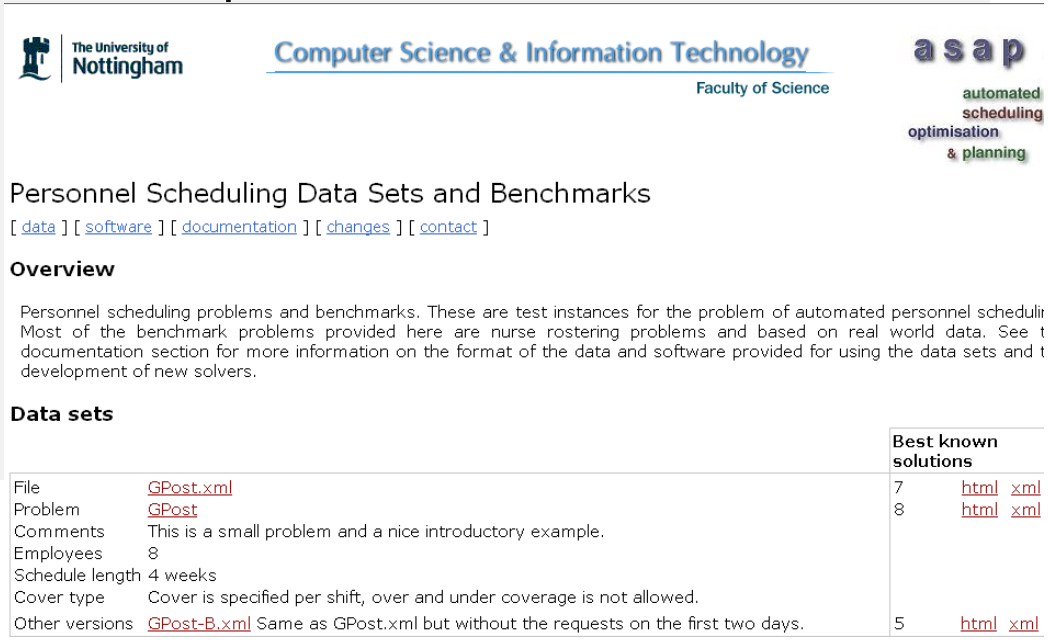
Burke E. K., De Causmaecker P., Vanden Berghe G., and Van Landeghem H. The State of the Art of Nurse Rostering. Journal of Scheduling, 7(6):441-499, Nov-Dec 2004

Nurse Rostering – Literature

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

- Very few benchmark nurse rostering problems
  - No typical nurse rostering problem
  - Each hospital has its own problem specific complicated objective functions and lots of constraints
- Benchmarks would help validate algorithms
  - Real-world problems collected and modelled
  - Encourage collaboration and competition

Nurse Rostering web site at
http://www.schedulingbenchmarks.org/



The University of Nottingham
Computer Science & Information Technology
Faculty of Science
asap automated scheduling optimisation & planning

Personnel Scheduling Data Sets and Benchmarks

[data] [software] [documentation] [changes] [contact]

Overview

Personnel scheduling problems and benchmarks. These are test instances for the problem of automated personnel schedulin
Most of the benchmark problems provided here are nurse rostering problems and based on real world data. See t
documentation section for more information on the format of the data and software provided for using the data sets and t
development of new solvers.

Data sets

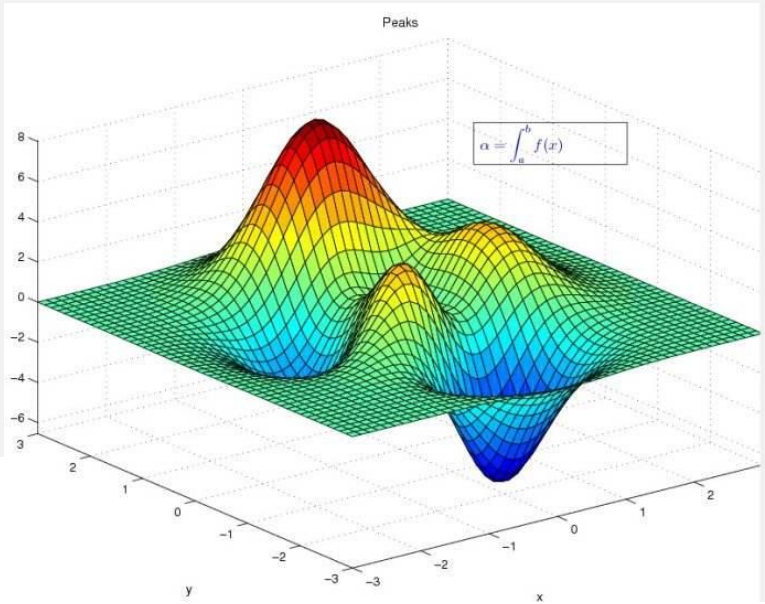
		Best known solutions		
File	GPost.xml	7	html	xml
Problem	GPost	8	html	xml
Comments	This is a small problem and a nice introductory example.			
Employees	8			
Schedule length	4 weeks			
Cover type	Cover is specified per shift, over and under coverage is not allowed.			
Other versions	GPost-B.xml Same as GPost.xml but without the requests on the first two days.	5	html	xml

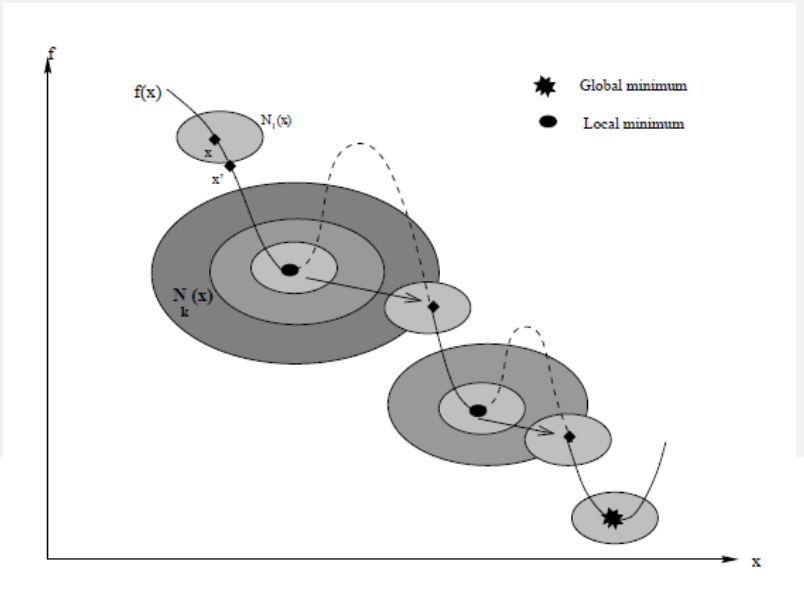
Hybrid Variable Neighbourhood Search (Hybrid VNS)

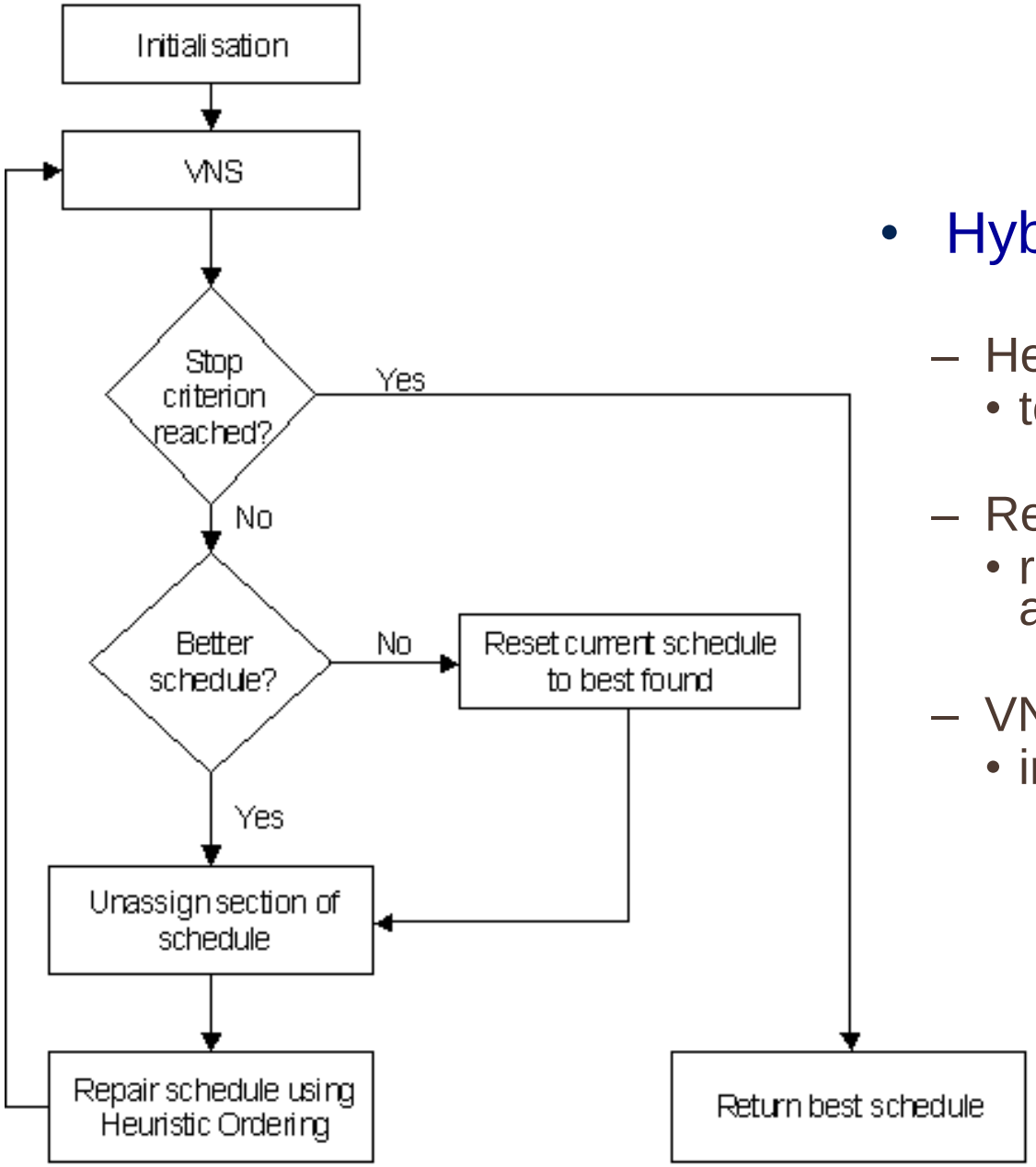
Nurse Rostering – Hybrid VNS

- Most meta-heuristics use only one neighbourhood operator

- Variable neighbourhood search (VNS)
  - Employ at least two neighbourhood operators
  - Theory: local optimal solutions of one neighbourhood operator might not be the local optimum of another neighborhood
  - Effective on escaping from local optimum

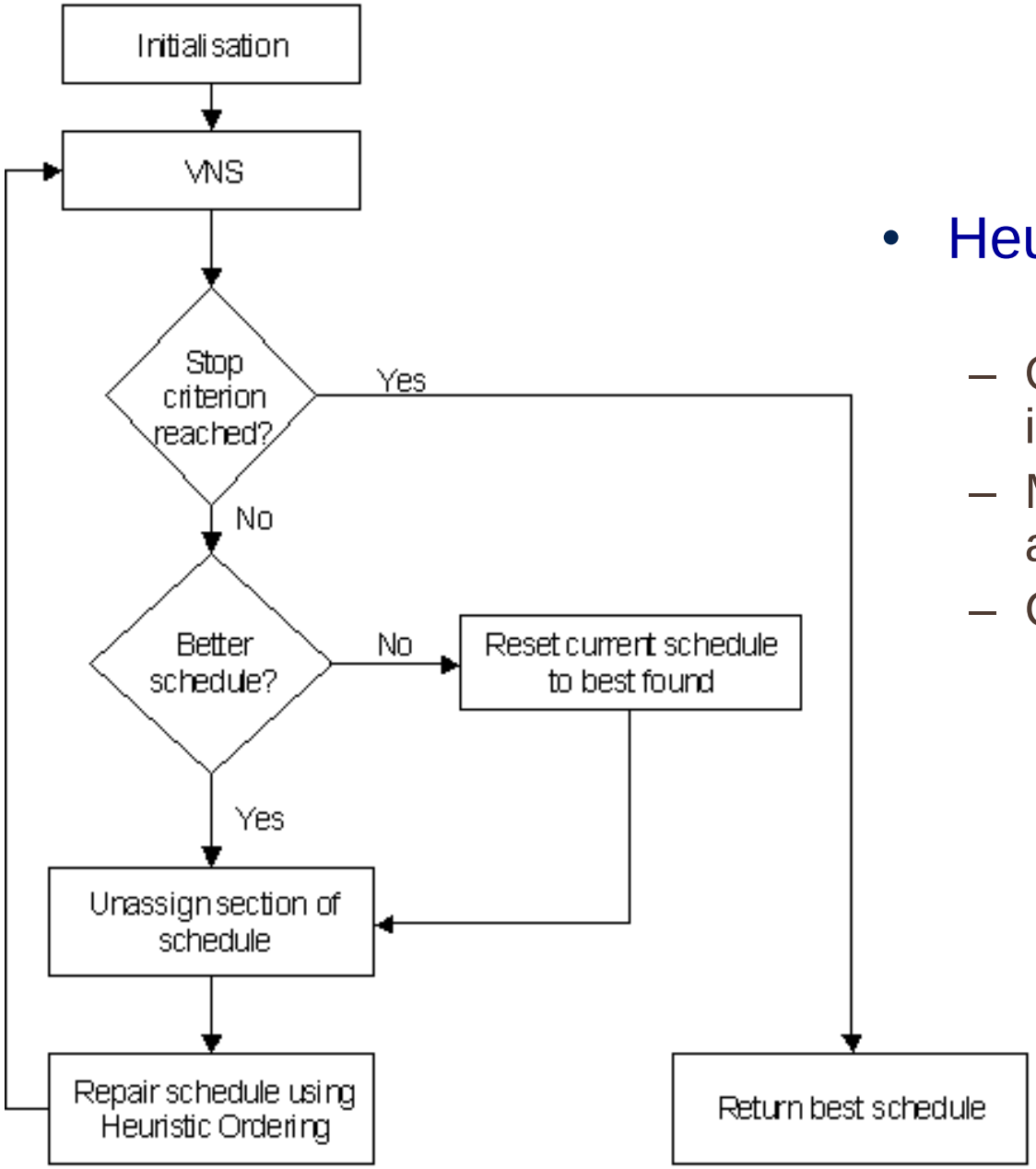






- Hybrid VNS
  - Heuristic ordering
    - to order shifts for construction
  - Repairing method
    - remove worse part of roster and re-construct
  - VNS
    - improvement upon rosters

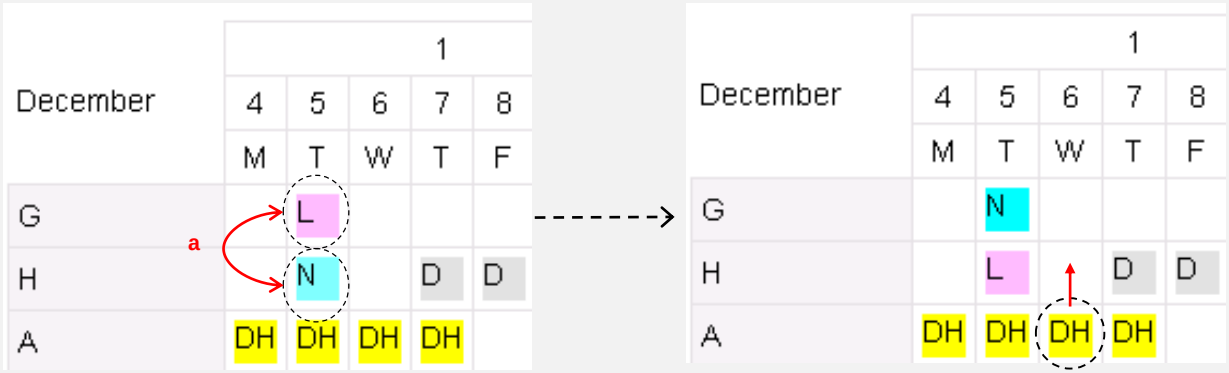
Burke E. K., Curtois T. E., Post G., Qu R., and Veltman B. A Hybrid Heuristic Ordering and Variable Neighbourhood Search for the Nurse Rostering Problem. ***European Journal of Operational Research***, 2: 330-341, 2008.



- Heuristic ordering
  - Order shifts for construction in initialisation and repair
  - More ***troublesome*** shifts assigned first
  - Criteria to evaluate the shifts
    - Type of shifts, number of employees able to cover it, etc.

Nurse Rostering – Hybrid VNS

- Variable Neighbourhood Search
  - Neighbours of a solution
    - those schedules that can be obtained by making a “move” e.g. single shifts swapped between any two nurses
  - Two neighbourhood operators
    - Assign a shift to another nurse
    - Swap shifts between nurses



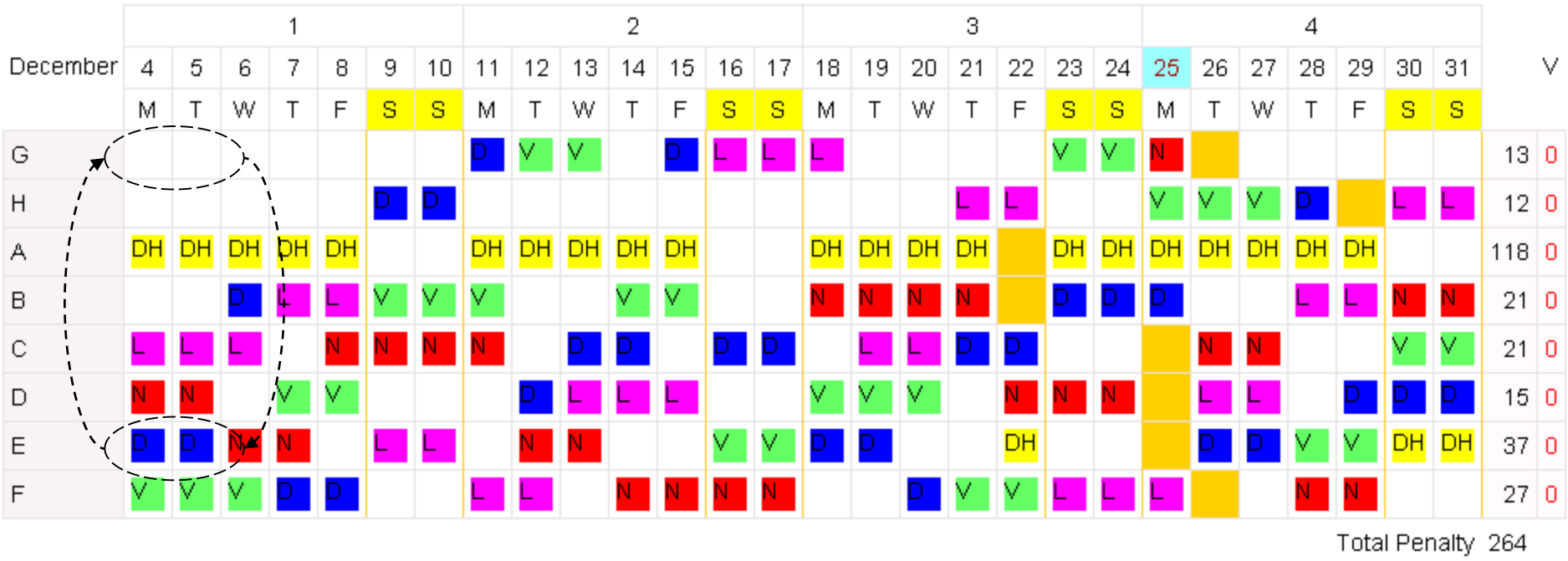
Nurse Rostering – Hybrid VDS

- Variable Depth Search
  - Form chains of moves/swaps
  - Each neighbour of the best solution: a possible starting point for the chain of moves
  - The 2nd nurse in the last move: the 1st nurse in the next move
  - New best solution: start another set of moves
  - Restart a new chain: if the chain cannot be continued for the current best solution
  - Algorithm terminates: no untried starting points of the current best solution

E.K. Burke, T. Curtois, R. Qu and G. Vanden Berghe. A Time Pre-defined Variable Depth Search for Nurse Rostering. **Journal of Heuristics**, 2007.

Nurse Rostering – Hybrid VDS

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

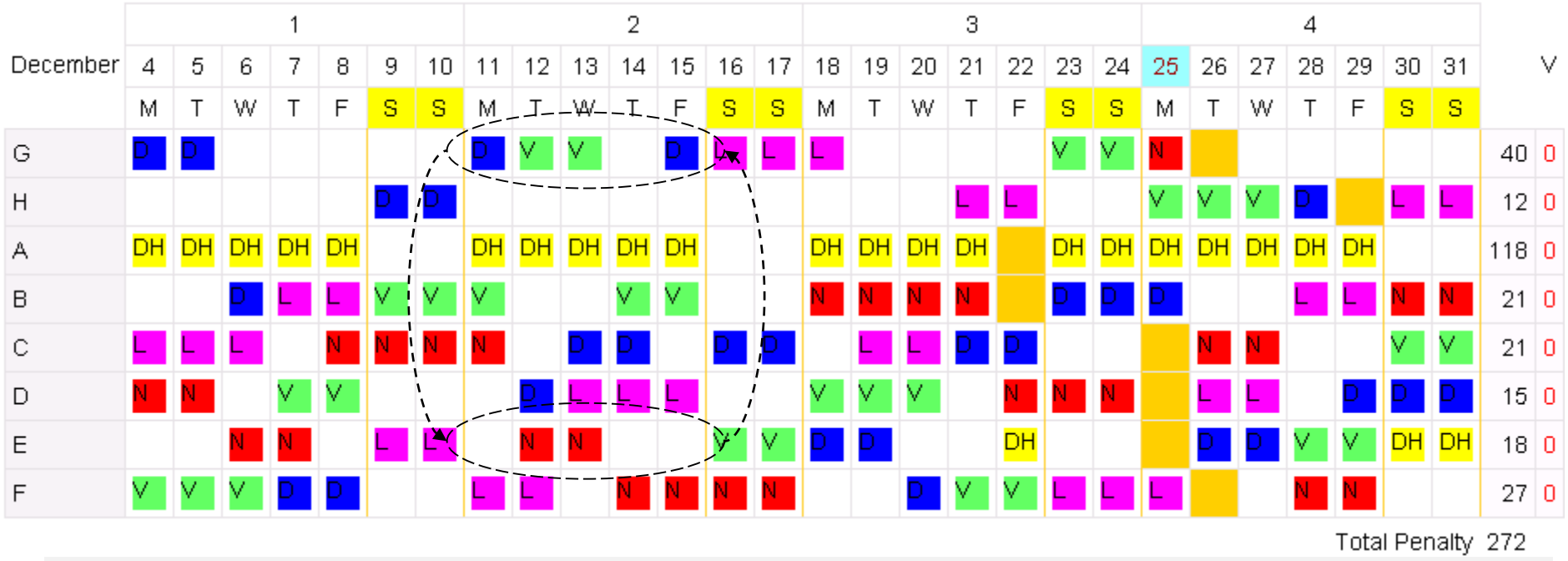


	1							2							3							4								
December	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31		V
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
G								D	V	V		D	L	L	L					V	V	N							13	0
H						D	D											L	L			V	V	V	D		L	L	12	0
A	DH	DH	DH	DH	DH			DH	DH	DH	DH	DH			DH	DH	DH	DH		DH	DH	DH	DH	DH	DH	DH			118	0
B			D	L	L	V	V	V			V	V			N	N	N	N		D	D	D			L	L	N	N	21	0
C	L	L	L		N	N	N	N		D	D		D	D		L	L	D	D				N	N			V	V	21	0
D	N	N		V	V				D	L	L	L			V	V	V		N	N	N		L	L		D	D	D	15	0
E	D	D	N	N		L	L		N	N			V	V	D	D			DH				D	D	V	V	DH	DH	37	0
F	V	V	V	D	D			L	L		N	N	N	N			D	V	V	L	L	L			N	N			27	0

Total Penalty 264

Nurse Rostering – Hybrid VDS

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

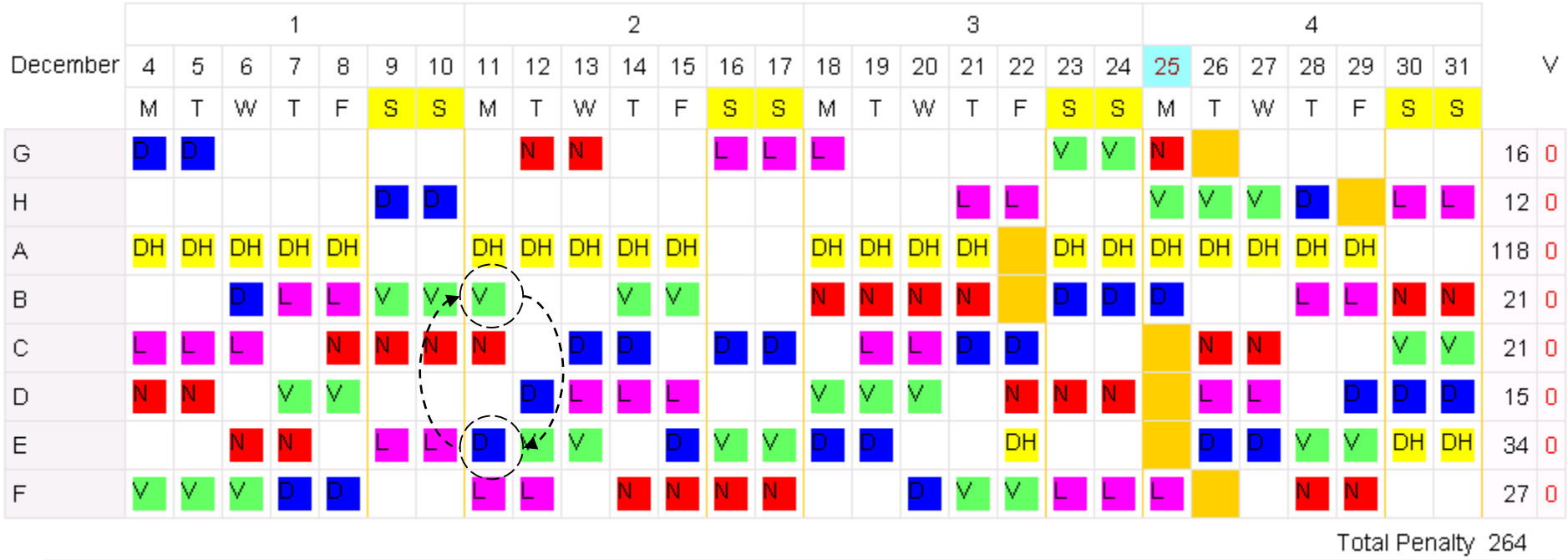


	1							2							3							4								
December	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31		V
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
G	D	D						D	V	V		D	L	L	L					V	V	N							40	0
H						D	D											L	L			V	V	V	D		L	L	12	0
A	DH	DH	DH	DH	DH			DH	DH	DH	DH	DH			DH	DH	DH	DH		DH	DH	DH	DH	DH	DH	DH			118	0
B			D	L	L	V	V	V			V	V			N	N	N	N		D	D	D			L	L	N	N	21	0
C	L	L	L		N	N	N	N		D	D		D	D		L	L	D	D				N	N			V	V	21	0
D	N	N		V	V				D	L	L	L			V	V	V		N	N	N		L	L		D	D	D	15	0
E			N	N		L	L		N	N			V	V	D	D			DH				D	D	V	V	DH	DH	18	0
F	V	V	V	D	D			L	L		N	N	N	N			D	V	V	L	L	L			N	N			27	0

Total Penalty 272

Nurse Rostering – Hybrid VDS

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

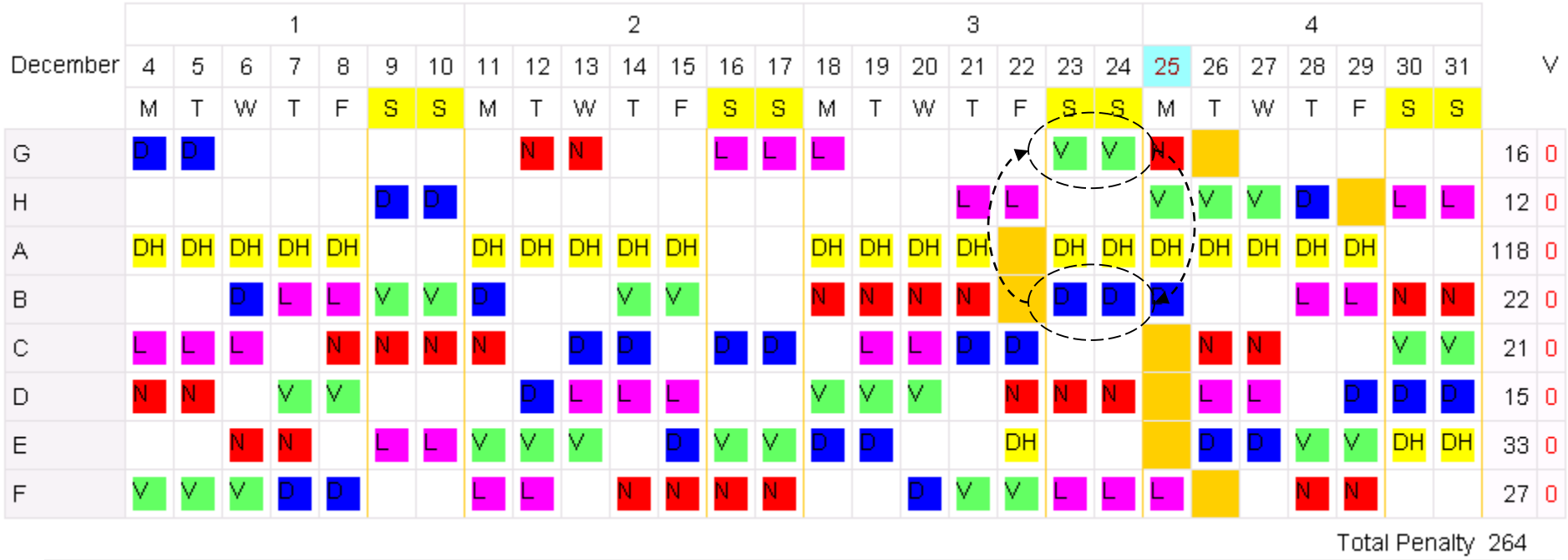


	1							2							3							4								
December	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31		V
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
G	D	D							N	N			L	L	L					V	V	N							16	0
H						D	D											L	L			V	V	V	D		L	L	12	0
A	DH	DH	DH	DH	DH			DH	DH	DH	DH	DH			DH	DH	DH	DH		DH	DH	DH	DH	DH	DH	DH			118	0
B			D	L	L	V	V	V			V	V			N	N	N	N		D	D	D			L	L	N	N	21	0
C	L	L	L		N	N	N	N		D	D		D	D		L	L	D	D				N	N			V	V	21	0
D	N	N		V	V				D	L	L	L			V	V	V		N	N	N		L	L		D	D	D	15	0
E			N	N		L	L	D		V		D	V	V	D	D			DH				D	D	V	V	DH	DH	34	0
F	V	V	V	D	D			L	L		N	N	N	N			D	V	V	L	L	L			N	N			27	0

Total Penalty 264

Nurse Rostering – Hybrid VDS

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA



	1							2							3							4								
December	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31		V
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
G	D	D							N	N			L	L	L					V	V	N							16	0
H						D	D											L	L			V	V	V	D		L	L	12	0
A	DH	DH	DH	DH	DH			DH	DH	DH	DH	DH			DH	DH	DH	DH		DH	DH	DH	DH	DH	DH	DH			118	0
B			D	L	L	V	V	D			V	V			N	N	N	N		D	D	D			L	L	N	N	22	0
C	L	L	L		N	N	N	N		D	D		D	D		L	L	D	D				N	N			V	V	21	0
D	N	N		V	V				D	L	L	L			V	V	V		N	N	N		L	L		D	D	D	15	0
E			N	N		L	L	V	V	V		D	V	V	D	D			DH				D	D	V	V	DH	DH	33	0
F	V	V	V	D	D			L	L		N	N	N	N			D	V	V	L	L	L			N	N			27	0

Total Penalty 264

More complicated to implement compared with VNS
Very effective compared with previous approaches

Nurse Rostering – Hybrid VNS

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

- HARMONY^{TM}
  - Automated workforce management software
  - Developed by ORTEC®, The Netherlands: an international consultancy company on planning, scheduling and optimisation
  - This work improved the algorithm in HARMONY^{TM}

	1							2							3							4								
December	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31		
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
1A	D	E	E	E	L			E	E	E	E		D	D	D	N	N	N				L	L	L	L				51	11
A	DH	DH	DH	DH	DH			DH	DH	DH		DH	DH	DH	DH		DH	DH				DH	DH	DH	DH	DH			20	3
B	N	N	N	N				D	D	L	L	L				L	L	L				E	E	E	D	D			0	0
C	D	D	D	D	D				N	N	N		L	L	L				L	L	L		E	E	E	L			25	4
D				L	N	N	N	N			DH	D				E	E	E	DH	E	E		N	N			E	E	13	7
E					D	DH	DH	D					E	E		DH	E	E	E	DH	DH		D	D	E	E	DH	DH	21	10
F	L	L	L			L	L	L	L			N	N	N	N			D	D			D				D	D	D	10	3
G				E	E	E	E			D	D	D			E	E			D	D	D	D			N	N	N	N	10	2
H	E	E	E			D	D		E	E	E	E			D	D	D		N	N	N	N					L	L	26	6

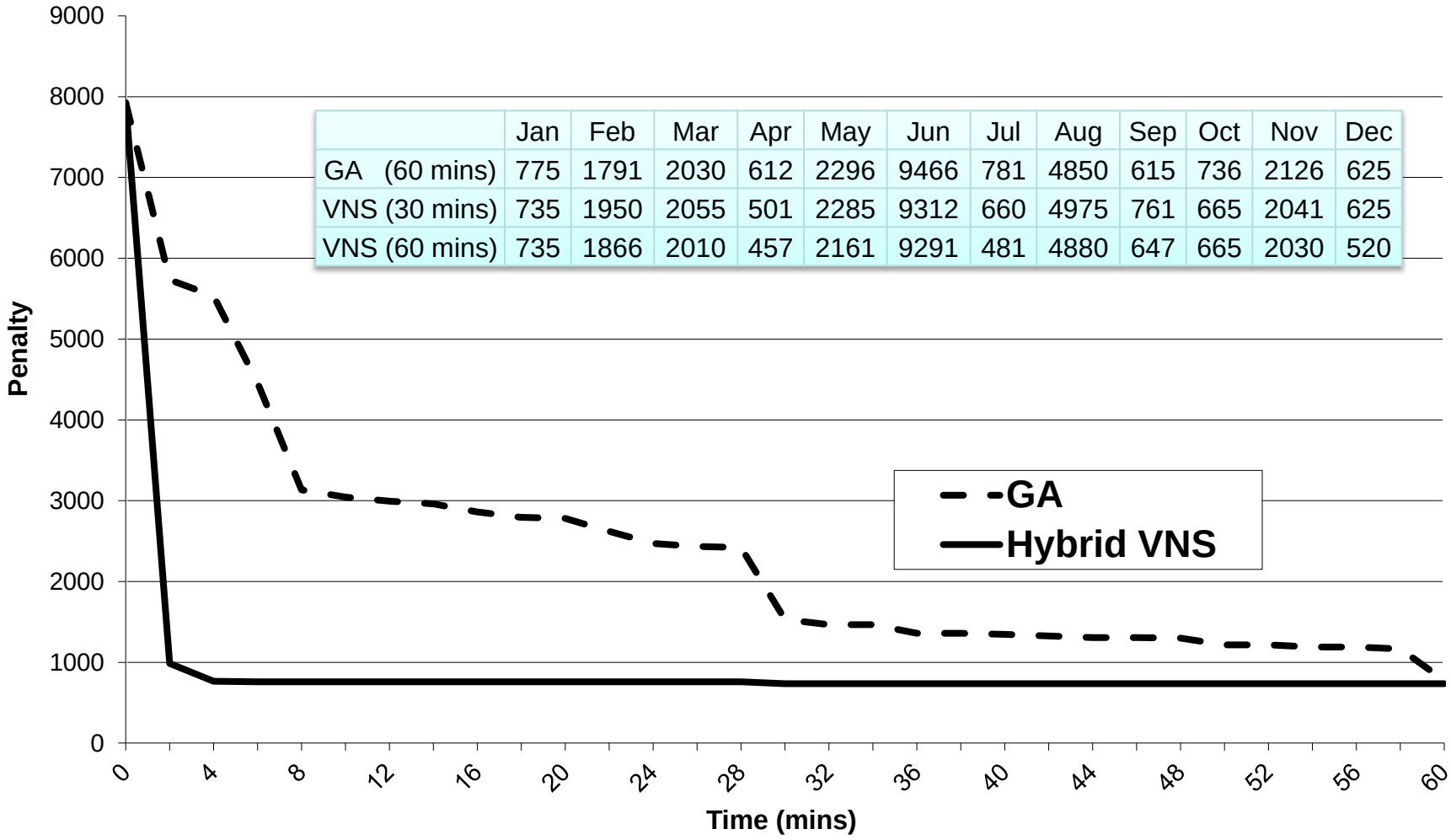
Total Penalty 176
Unassigned Shifts 0

Minimum Cover																													
E	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1	
D	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	
DH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
N	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

Nurse Rostering – Hybrid VNS

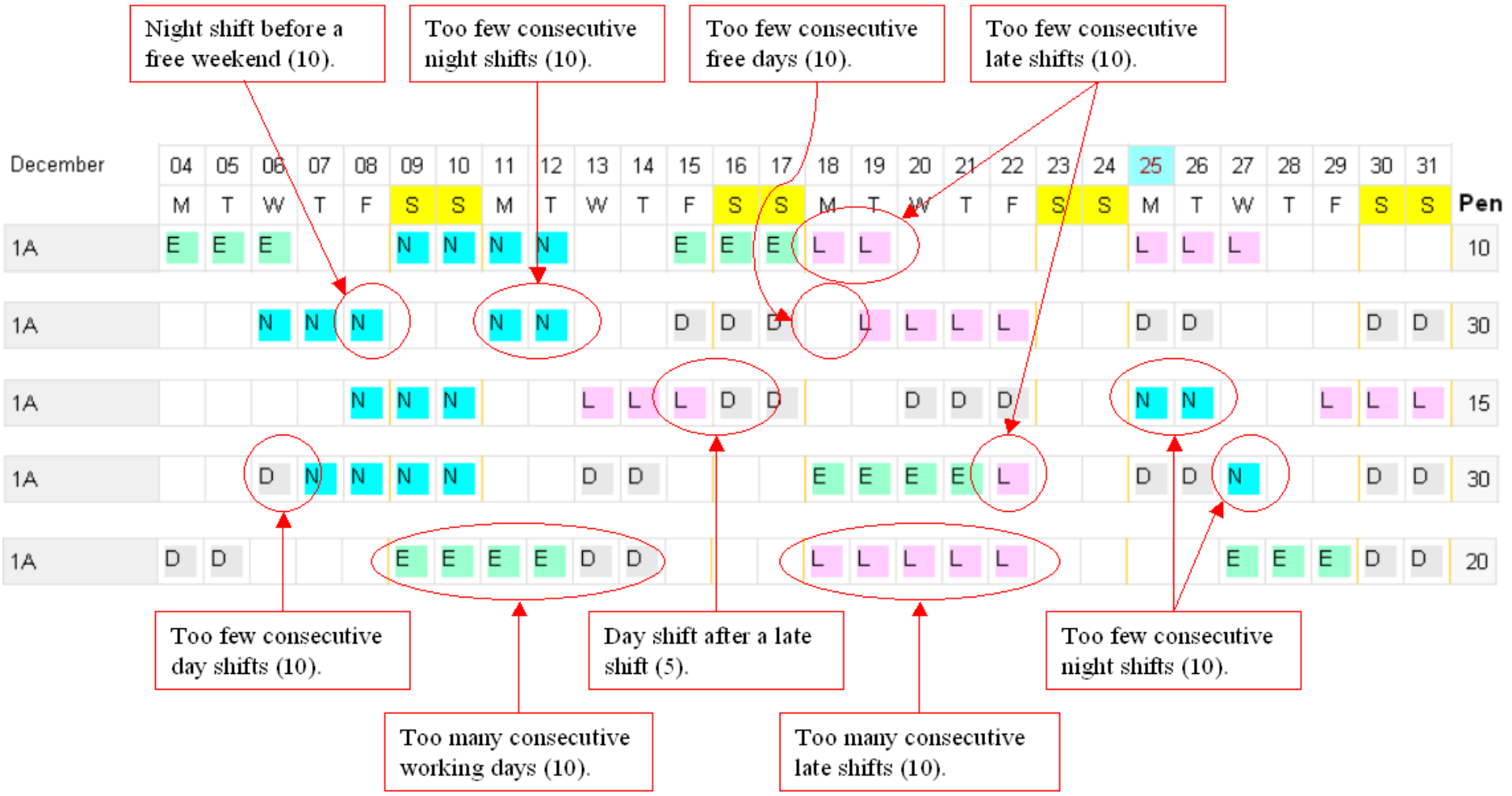
The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
GA (60 mins)	775	1791	2030	612	2296	9466	781	4850	615	736	2126	625
VNS (30 mins)	735	1950	2055	501	2285	9312	660	4975	761	665	2041	625
VNS (60 mins)	735	1866	2010	457	2161	9291	481	4880	647	665	2030	520



Sequence Based Adaptive Approach

Sequences / Work Stretches



P. Brucker, E.K. Burke, T. Curtois, R. Qu. Adaptive Construction of Nurse Schedules: A Shift Sequence Based Approach. ***European Journal of Operational Research***, 2008.

Sequences / Work-Stretches

- In literature
  - A few work consider feasible patterns (or work-stretches) of one/two weeks, associated with pre-assigned costs
- In this work
  - Problems are firstly modelled by categorising constraints into 3 types, *Sequence*, *Schedule* and *Roster* related
  - Penalties of *sequences*, *schedules* and *roster* are calculated by corresponding constraints

Sequences	A series of shifts for nurses i.e. EEELL
Schedules	Ordered list of sequences and days off
Roster	Overall solution consisting of same length schedules of the scheduling period

Sequences / Work-Stretches

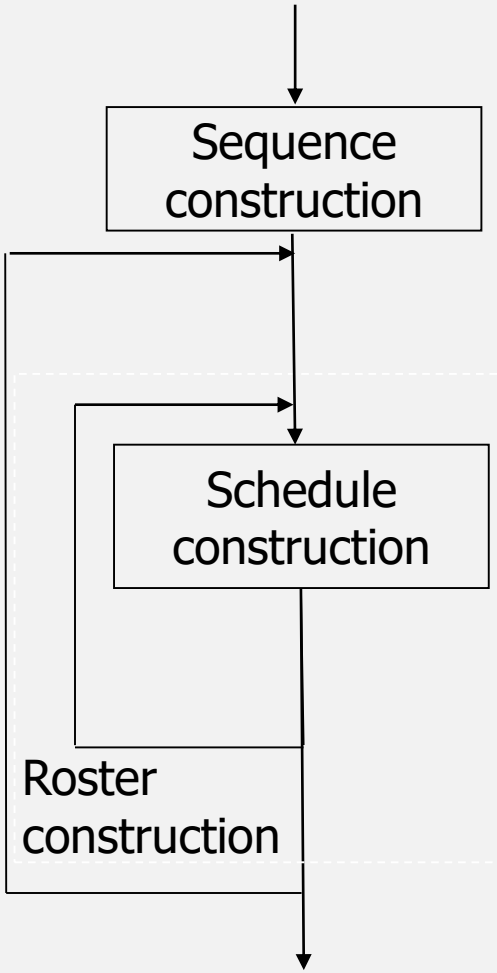
The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

	Hard Constraints	Type
1	Shifts which require certain skills can only be taken by (or assigned to) nurses who have those skills	sequence
2	The shift coverage requirements must be fulfilled	roster

	Soft Constraints	Type
1	Minimum rest time between shifts	sequence
2	Alternative skill (if a nurse is able to cover a shift but prefers not to as it does not require his/her primary skill)	sequence
3	Maximum number of shift assignments	schedule
4	Maximum number of consecutive working days	sequence
5	Minimum number of consecutive working days	sequence
...	...	...

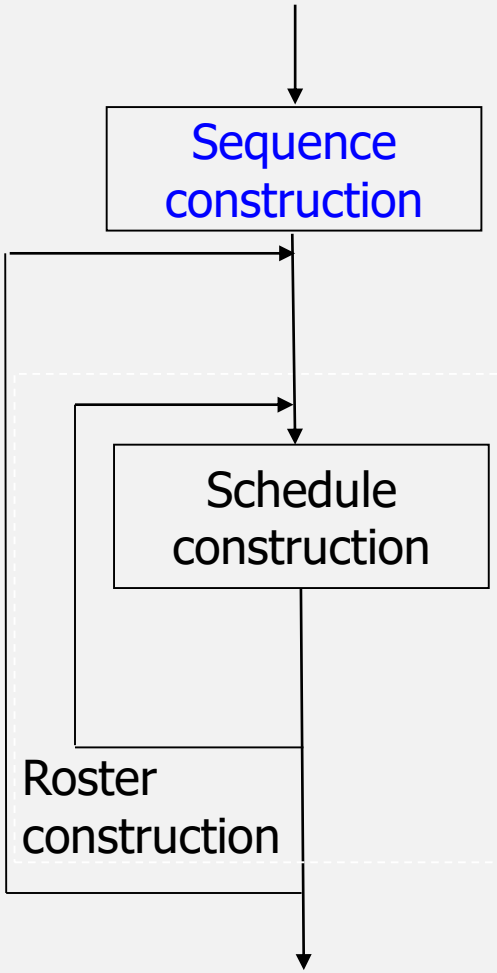
Sequence Based Adaptive Approach

- Two stage approach
  - Construct high quality sequences for each nurse considering only *sequence* related constraints
  - Construct schedules and roster considering only *schedule* and *roster* related constraints



Sequence Based Adaptive Approach

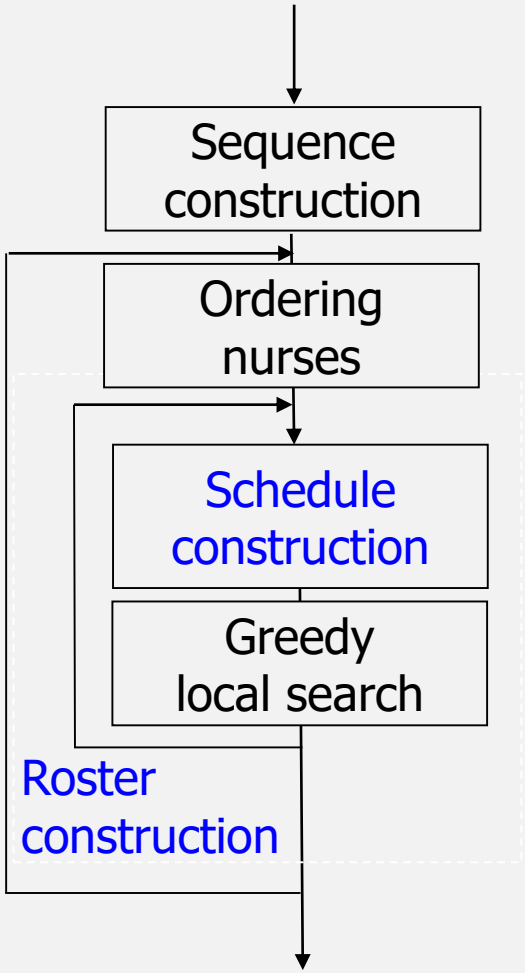
- Stage I: sequence construction for each nurse
  - Construct sequences by considering
    - *sequence* related hard constraints
    - *sequence* related soft constraints
    - length of up to 5
  - Best 50 are ranked



Sequences	Penalty	Comment
E, E, E	0	
D, D, E, E, E	5	E not preferred to follow D.
L, L, L, D, D	5	D not allo preferred wed to follow L.
N, N	10	Two N's not preferred.
E, D, D	10	One E not preferred.

Sequence Based Adaptive Approach

- Stage II: schedule and roster construction
  - Build schedules based on sequences for each nurse considering only *schedule* related constraints
  - Iteratively combine schedules of nurses to construct rosters considering only *roster* related constraints
  - Hybridise different techniques with this simple and fast approach
    - Adaptive ordering: nurses with worse schedules scheduled first
    - Greedy local search: improvement during and after roster construction



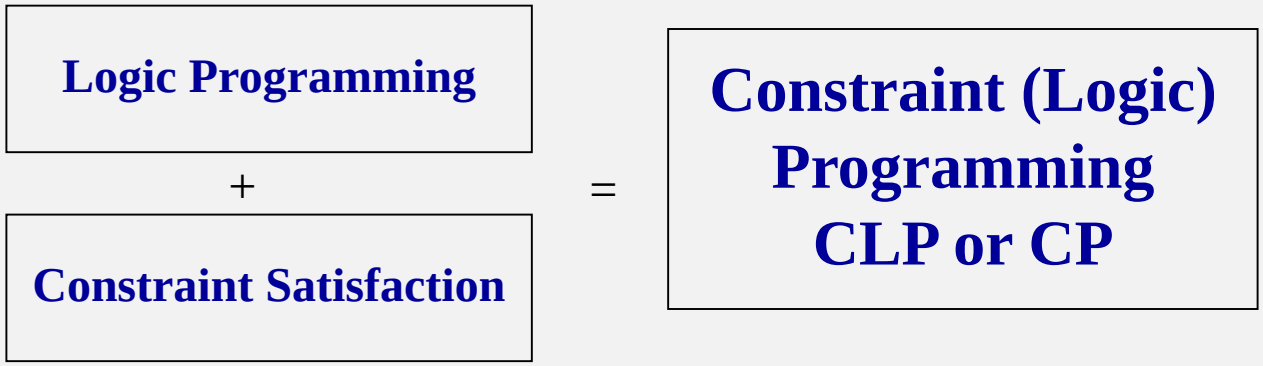
The University of Nottingham

UNITED KINGDOM · CHINA · MALAYSIA

Constraint Programming

Constraint Programming

- Constraint satisfaction
  - Solve problems by exploring constraints which must be satisfied by the solution
  - Set of variables taking values within domains satisfying **constraints**
  - Arose from AI & computer graphics in 1960s, 1970s



Constraint Programming

- Constraint satisfaction
- Tree search

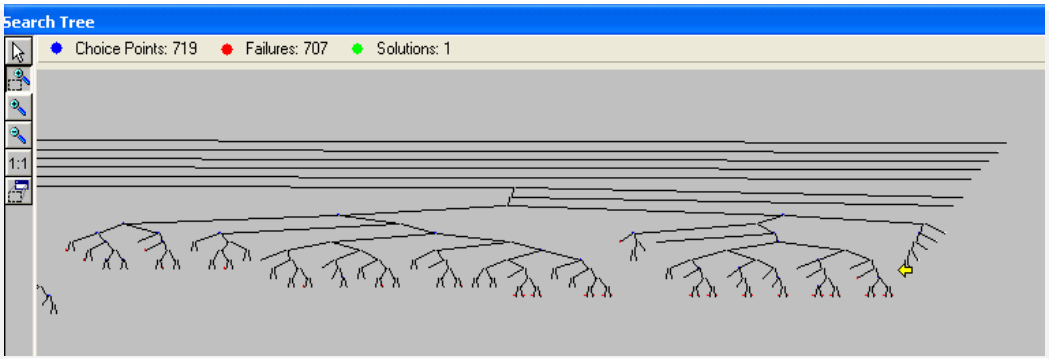
- Simply model / define the problem i.e. constraints / relationship between decision variables

```
    SEND
  + MORE
  -------
   MONEY
```

```
enum Letters {S, E, N, D, M, O, R, Y};

var int l[Letters] in 0..9;

solve {
   alldifferent(l) onDomain;
   //l[S] <> 0;
   //l[M] <> 0;

      1000*l[S] + 100*l[E] + 10*l[N] + l[D]
   +  1000*l[M] + 100*l[O] + 10*l[R] + l[E]
   =  10000*l[M] + 1000*l[O] + 100*l[N] + 10*l[E] + l[Y];
};
```

Constraint Programming

- CP vs.
  Integer Programming



Decision variables:

x_{idt}	1 if employee i is assigned shift type t on day d, 0 otherwise
k_{iw}	1 if employee i works on weekend w, 0 otherwise
y_{dt}	total below the preferred cover for shift type t on day d.
z_{dt}	total above the preferred cover for shift type t on day d.

Constraints:

1. An employee cannot be assigned more than one shift on a single day.

$$\sum_{t \in T} x_{idt} \leq 1, \quad \forall i \in I, d \in D$$

2. Shift rotation. A minimum amount of rest is required after each shift. Therefore certain shifts cannot follow others. For example, an early shift cannot follow a late shift.

$$x_{idt} + x_{i(d+1)u} \leq 1, \quad \forall i \in I, d \in \{1 \ldots h-1\}, t \in T, u \in R_t$$

T Curtois, R Qu, Computational results on new staff scheduling benchmark instances, Technical report, School of Computer Science, University of Nottingham, 2014.

Nurse Rostering – Modelling

- Decision variables: *sij*, the shift type assigned to nurse *i* on day *j*;

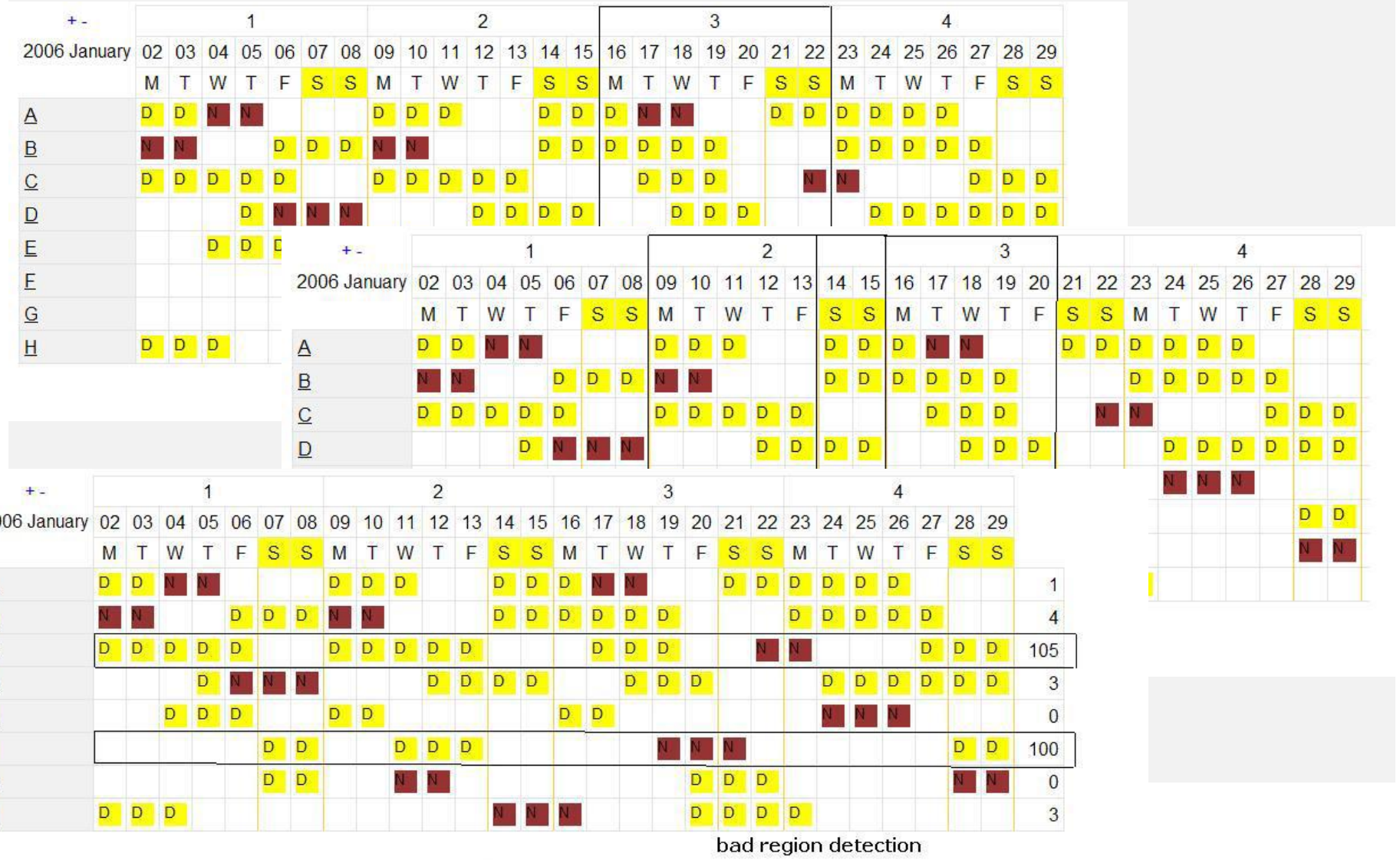
- CSP model which only consists of hard constraint is used to generate initial feasible solution and re-construct partial solution:
  - H1: Cardinality(*sij*, *K*, *Djk*, *Djk*),
  - H5: Cardinality(*sij*, Night, 0, *n1*),
  - H7: Stretch(*sij*, Night, 2, 3, P), P = (Night, Off),
  - …

HC1: daily coverage requirement of each shift type
HC5: max number of *night* shifts per month
HC7: min two free days after a series of *night* shifts

F. He and R. Qu. "Constraint-directed Local Search to Nurse Rostering Problems“, 6th Int. Workshop on Local Search Techniques in Constraint Satisfaction CP'09, 20 Sept 2009, Lisbon, Portugal.

Nurse Rostering – Search

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

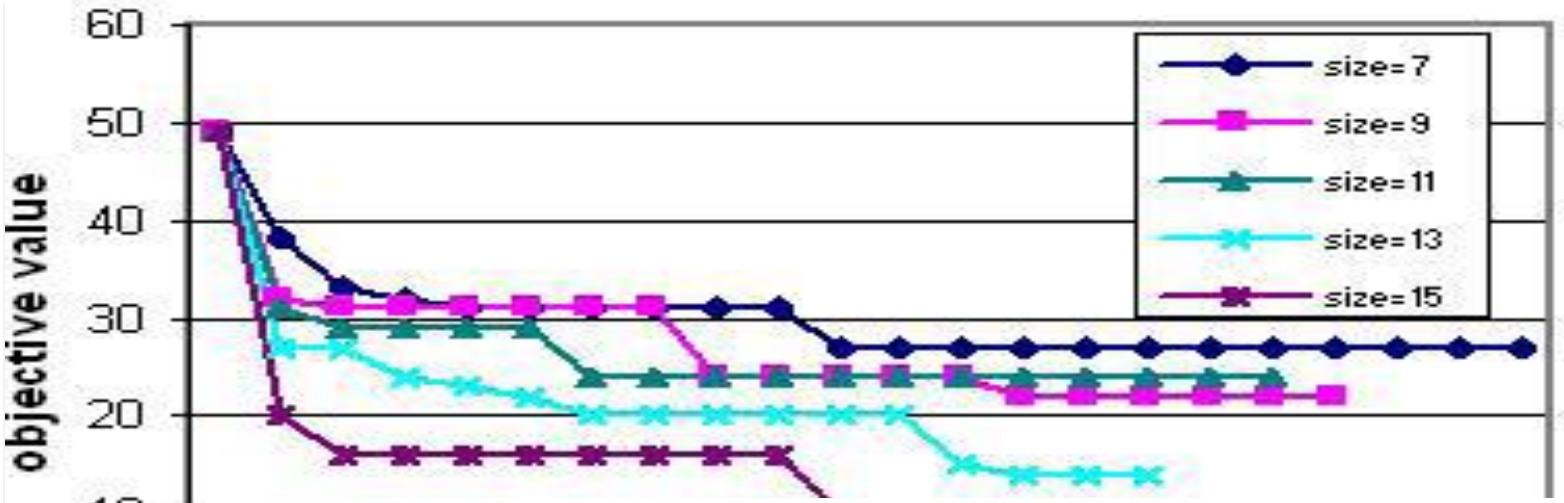


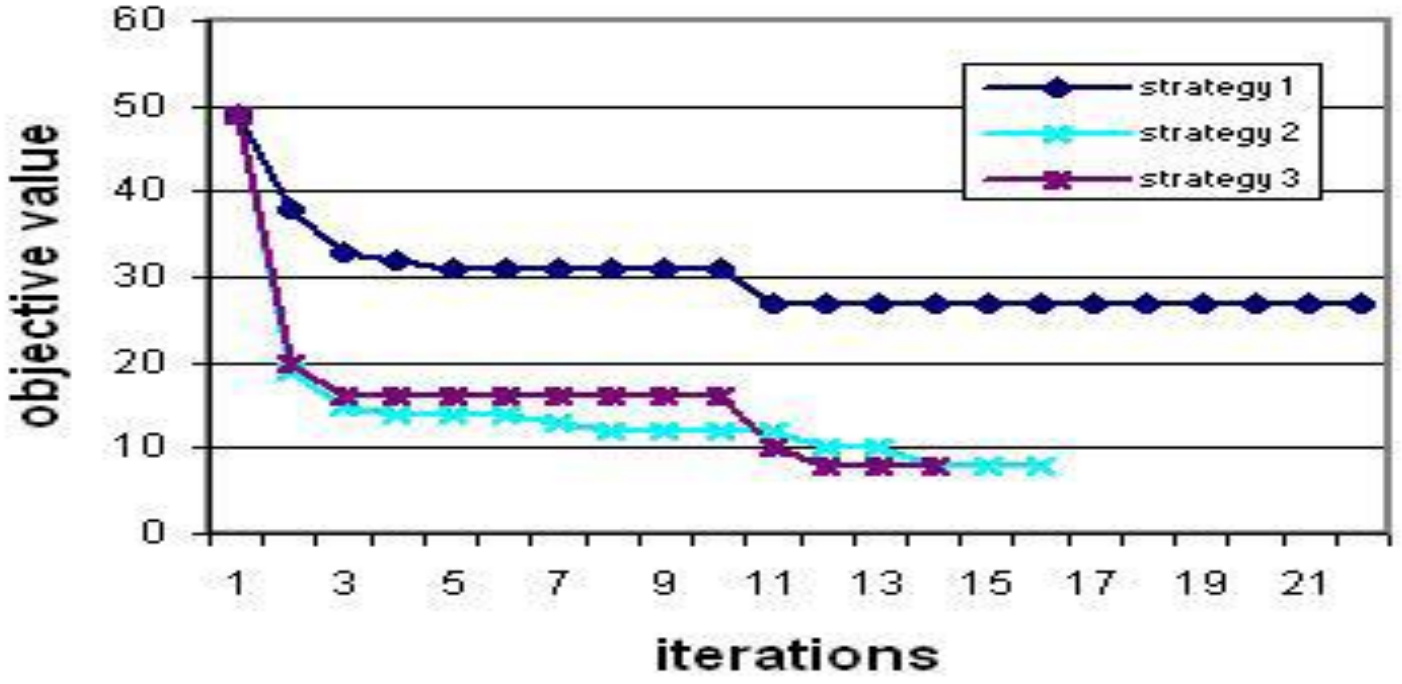
Nurse Rostering – Search

- Large neighborhood search (LNS)
  - more general neighborhood moves performed by using a complete search solver (CP) concerning all constraints involved

Construct initial solution: solve (H) // H: set of hard constraints
While optimal solution not found or stopping criteria not met **do**
    Choose the low quality fragment to be relaxed
    Freeze the remaining variables
    Re-optimize the fragment
    If found (a) First improved solution, or (b) Best improved solution
        Update solution
    End if
End while

Nurse Rostering – Search





A Decomposition Approach

Decomposition – Modelling

- The main idea
  - decompose the problem into cyclic schedules for groups of nurses
  - add workload of remaining nurses
  - a VNS is then applied for further improvement

Brucker P., Qu R, Burke E.K. and Post G. A Decomposition, Construction and Post-processing Approach for a Specific Nurse Rostering Problem. **MISTA'05**, 397-406. New York, USA, Jul 2005

Decomposition – Modelling

- The problem: derived from real-world problems in ORTEC, Netherlands
  - To create monthly schedules for wards
  - Different types of nurses (PT, FT)
  - 4 shift types and demand in a week

12 Full-time nurses	36 hours/week
1 Part-time nurse	32 hours/week
3 Part-time nurses	20 hours/week

			Demand						
Shift type	Start time	End time	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Early	07:00	16:00	3	3	3	3	3	2	2
Day	08:00	17:00	3	3	3	3	3	2	2
Late	14:00	23:00	3	3	3	3	3	2	2
Night	23:00	07:00	1	1	1	1	1	1	1

Decomposition – Modelling

- The problem: hard constraints
  - HC1: daily coverage requirement of each shift type
  - HC2: for each day, a nurse works at most one shift
  - HC3: max number of working days per month
  - HC4: max number of on-duty weekends per month
  - HC5: max number of *night* shifts per month
  - HC6: no *night* shift between two non-*night* shifts
  - HC7: min two free days after a series of *night* shifts
  - HC8: max number of consecutive *night* shifts
  - HC9: max number of consecutive working days
  - HC10: no *late* shifts for one particular nurse

Decomposition – Modelling

- The problem: soft constraints

SC1	either no shifts or two shifts in weekends	1000
SC2	avoiding a single day between two days off	1000
SC3	length of a series of night shifts	1000
SC4	Min number of free days after a series of shifts	100
SC5	Max/Min number of consecutive assignments of a specific shift type	10
SC6	Max/Min number of weekly working days	10
SC7	Max number of consecutive working days for part-time nurses	10
SC8	avoiding certain shift type successions (e.g. a *day* shift followed by an *early* one, etc)	5

Decomposition – Modelling

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

	Week 1						
	M	**T**	**W**	**T**	**F**	**S**	**S**
Nurse 1	**D**	**D**	**D**			**E**	**E**
Nurse 2	**L**	**L**	**L**				
Nurse 3	**E**	**E**	**E**	**L**	**L**		
Nurse 4				**E**	**E**	**L**	**L**
Nurse 5	**N**	**N**			**D**	**D**	**D**

Decomposition – Modelling

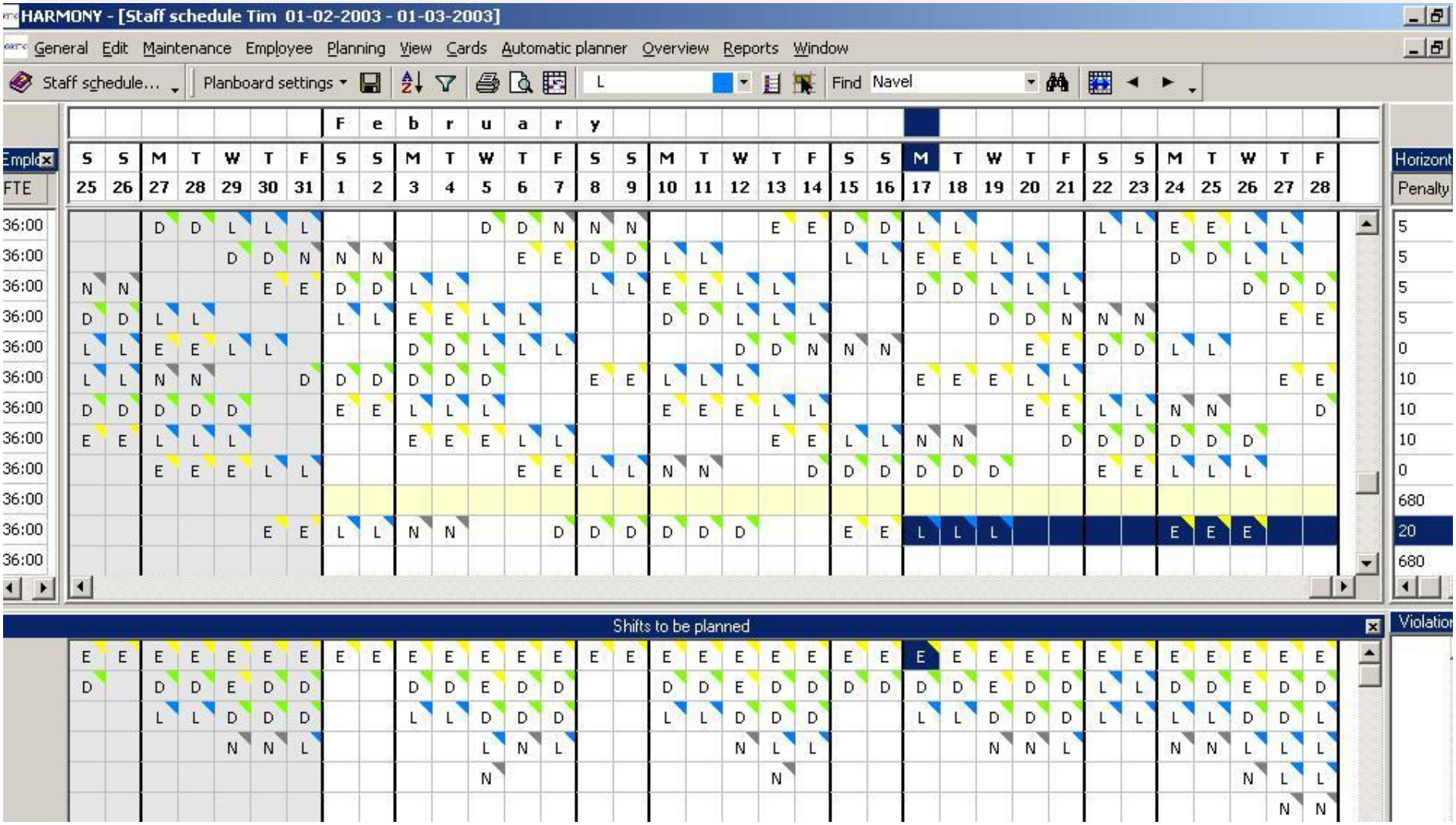
	Week 1							Week 2						
	M	T	W	T	F	S	S	M	T	W	T	F	S	S
Nurse 1	D	D	D			E	E							
Nurse 2	L	L	L					L	L	L				
Nurse 3	E	E	E	L	L			E	E	E	L	L		
Nurse 4				E	E	L	L				E	E	L	L
Nurse 5	N	N			D	D	D	N	N			D	D	D
								D	D	D			E	E

Decomposition – Modelling

	Week 1							Week 2							
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	
Nurse 1	D	D	D			E	E	L	L	L					
Nurse 2	L	L	L					E	E	E	L	L			
Nurse 3	E	E	E	L	L						E	E	L	L	...
Nurse 4				E	E	L	L	N	N			D	D	D	
Nurse 5	N	N			D	D	D	D	D	D			E	E	
								D	D	D			E	E	

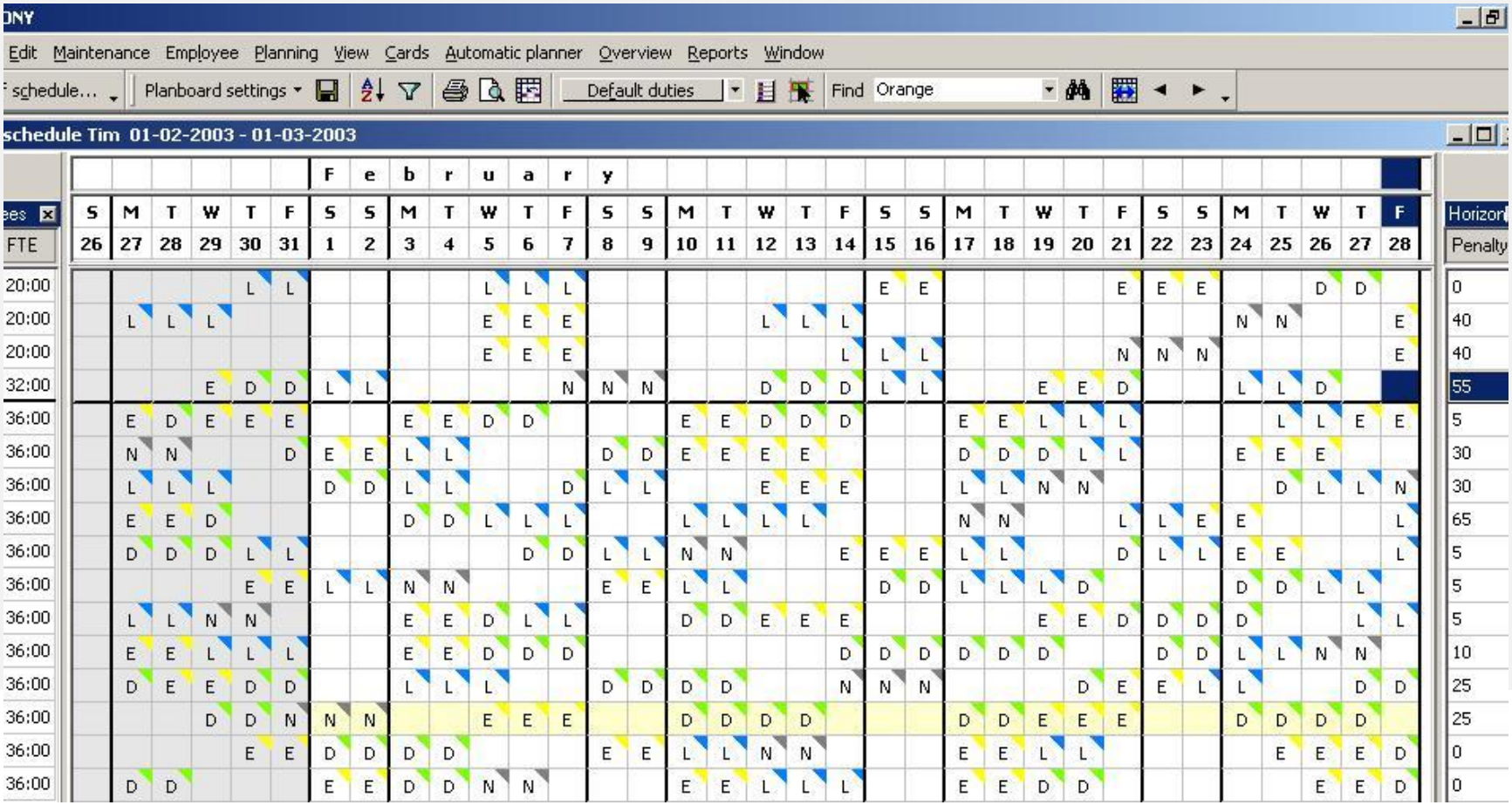
Decomposition – Approach

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA



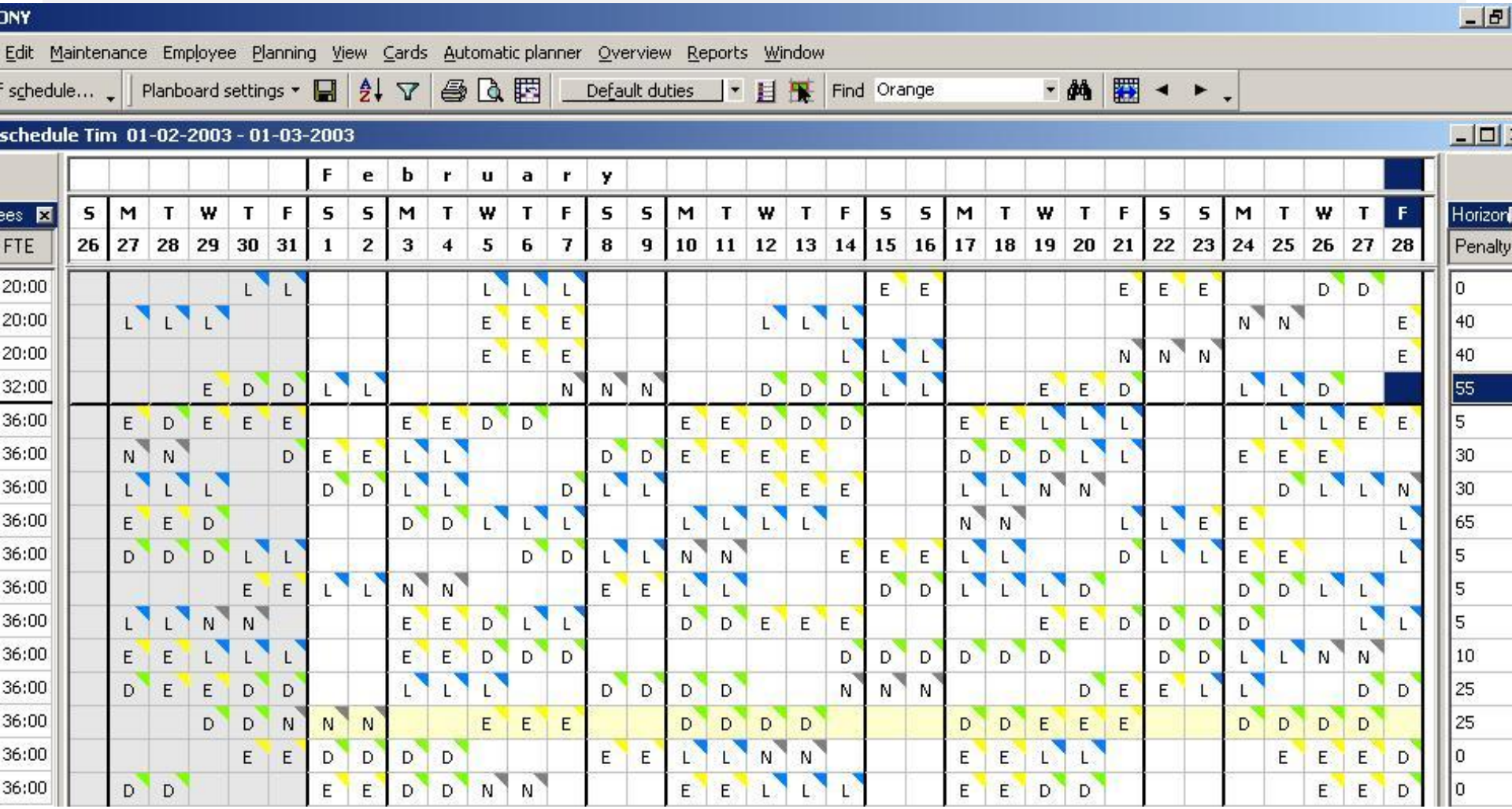
Decomposition – Approach

- Add the remaining shifts by using a heuristic ordering method
  - More ***troublesome*** shifts assigned first
  - Criteria to evaluate the shifts
    - Type of shifts, number of employees able to cover it, etc.



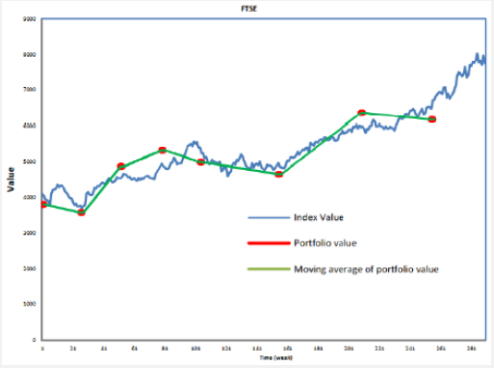
Decomposition – Approach

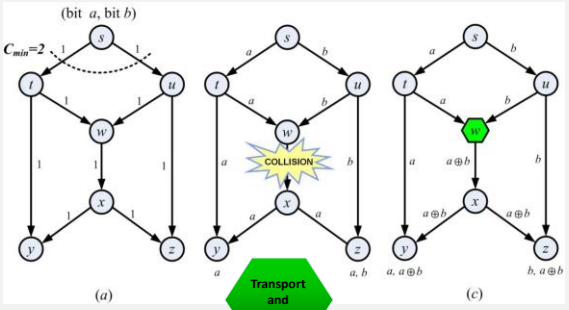
The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

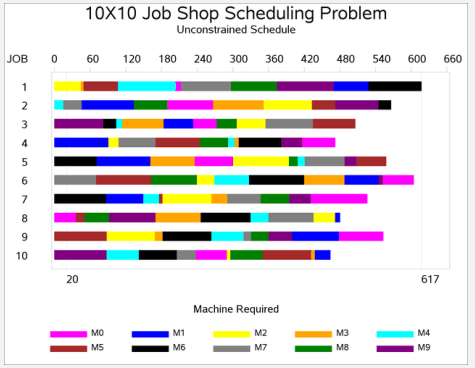


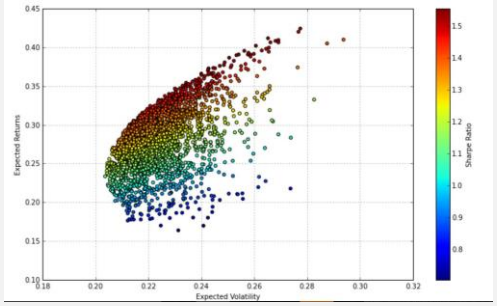
Other Applications

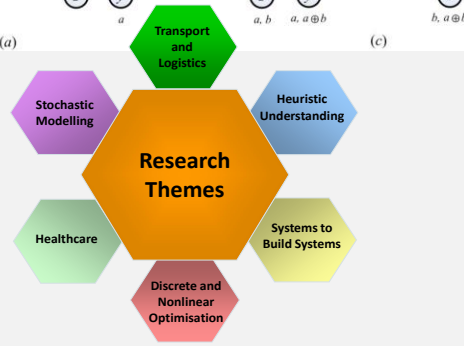
The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

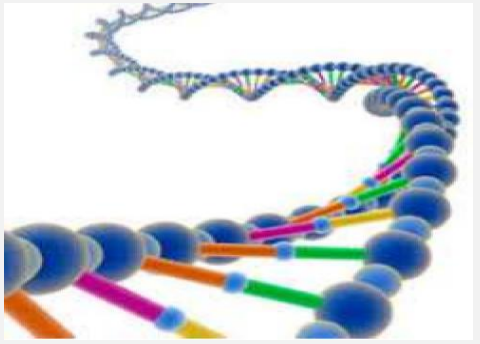


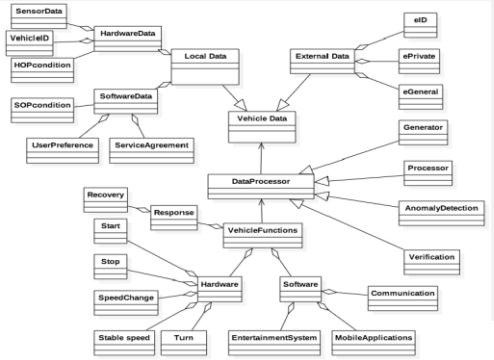


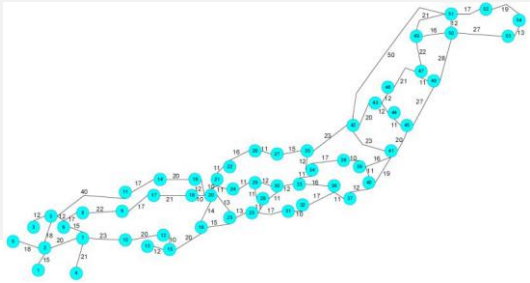






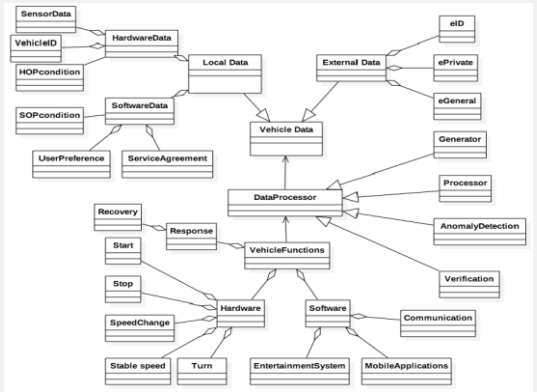


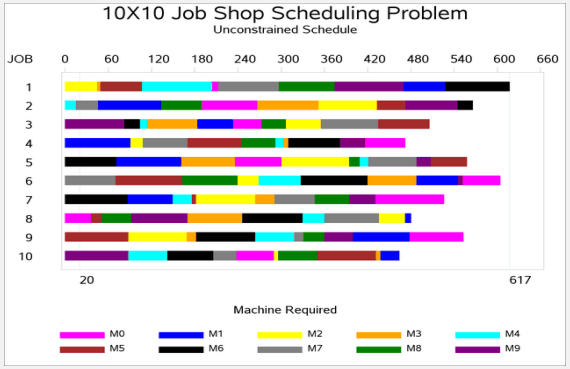


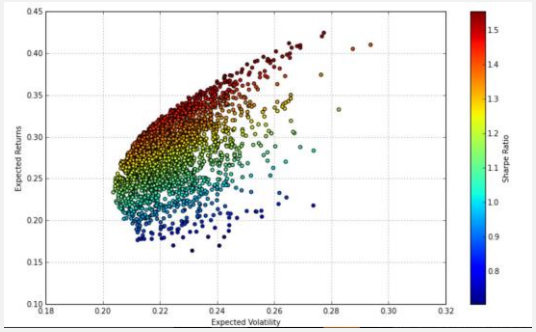


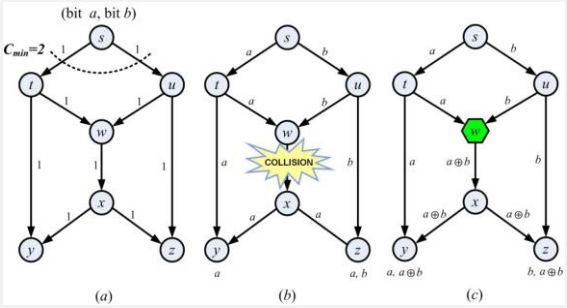
Other Problem Applications

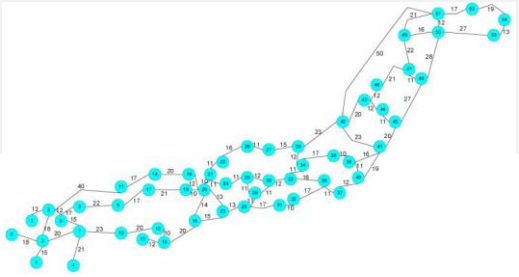
- IEEE Computational Intelligence Society Task Force on Intelligent Systems in Healthcare

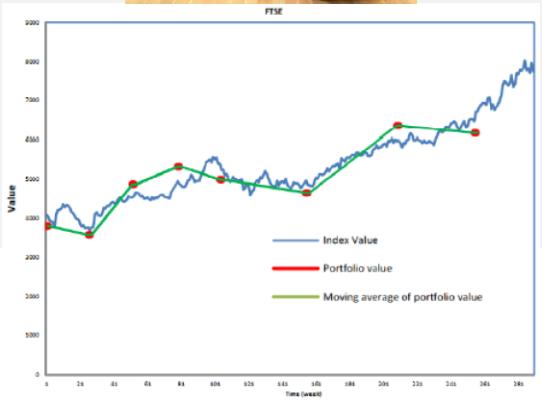












Other Recent Developments

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

- Automatic Algorithm Design
  - General search algorithm framework
  - Theory: landscape analysis
  - Machine learning: patterns of algorithm design / configurations

  - N. Pillay, R. Qu, ≪ Hyper-heuristics: Theory and Applications≫, Springer, ISBN 978-3-319-96514-7, December, 2018
  - N. Pillay, R. Qu (ed.), Automated Design of Machine Learning and Search Algorithms, Special Issue at Computational Intelligence Magazine, 13(2), June 2018.
  - IEEE Computational Intelligence Society Task Force on Hyper-heuristics
  - IEEE Computational Intelligence Society Task Committee on Evolutionary Computation
  - ≪Automated Machine Learning and Search≫, Springer, under consideration