

A Unified Framework of Evolutionary Multitasking Graph-based Hyper-heuristic – Supplementary Material

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This document includes nine extra tables and one figure that contains 15 sub-figures. Table SI provides detailed p and h values obtained by conducting the Wilcoxon rank-sum test with 95% confidence level for results shown in Table I. Table SII describes the properties of the Carter benchmarks. Table SIII presents the detailed ranks for the selected algorithms. Table SIV-SVI presents the test sets with three, four, and five tasks, respectively, and the corresponding experimental results are given in Table SVII-SIX, respectively. Finally, Fig.S1 provides the remaining computational cost comparisons between SOHH and EMHH in addition to those shown in Fig.5 in the main document.

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TABLE SI
CHARACTERISTICS OF THE TEST SETS USED IN THIS PAPER

Instance	No. of exams	No. of students	Enrollments	Conflict matrix density	No. of timeslots
car91	682	16925	56877	0.13	35
car92	543	18419	55522	0.14	32
ear83	190	1125	8109	0.27	24
ear83 IIc	189	1108	8057	0.27	24
hec92	81	2823	10632	0.42	18
hec92 II	80	2823	10625	0.42	18
kfu93	461	5349	25113	0.06	20
lse91	381	2726	10918	0.06	18
rye93	486	11483	45051	0.08	23
sta83	139	611	5751	0.14	13
sta83 IIc	138	549	5417	0.19	35
tre92	261	4360	14901	0.18	23
uta92	622	21266	58979	0.13	35
uta92 II	638	21329	59144	0.12	35
ute92	184	2749	11793	0.08	10
yor83	181	941	6034	0.29	21

TABLE SII
DETAILED RANKS FOR EACH COMPARED ALGORITHM ON EACH INSTANCE.

Instance	Asm04	Asm07	Asm09	Bur07	Pil07	Pil09a	Pil09b	Qu09a	Qu09b	Sab12	Qu15	EMHH		
												f_{\min}	$f_{i-\min-\min}$	$f_{i-\text{avg}-\min}$
car91	9	8	10	12	--	--2	--	11	3	5	1	4	6	7
car92	10	4	9	8	--	3	--	11	5	12	1	2	6	7
ear83	10	7	11	14	8	9	12	3	4	13	1	2	5	6
ear83 IIc	--	--	--	--	--	--	--	--	4	--	--	1	3	2
hec92	9	7	10	14	5	12	2	11	8	13	1	3	6	4
hec92 II	--	--	--	--	--	--	--	--	3	--	--	1	4	3
kfu93	13	5	12	10	2	3	--	6	9	11	1	4	7	8
lse91	10	9	11	8	2	4	--	13	7	12	1	3	5	6
rye93	8	7	9		3	6	--	--	--	10	1	2	4	5
sta83	13	9	14	2	3	4	5	11	10	12	1	6	8	7
sta83 IIc	--	--	--	--	--	--	--	--	4	--	--	1	3	2
tre92	10	3	11	13	4	5	--	12	6	2	1	7	9	8
uta92	9	8	10	11		6	--	2	1	12	3	4	5	7
uta92 II	--	--	--	--	--	--	--	--	4	--	--	1	3	2
ute92	4	7	11	10	2	12	5	13	9	14	1	3	6	8
yor83	9	2	4	13	17	12	5	6	11	7	1	3	8	10
INS (12)	9.5	6.3	10.2	10.5	5.1	6.5	5.8	9.0	6.6	10.3	1.2	3.6	6.3	6.9
INS (16)	--	--	--	--	--	--	--	--	3.8	--	--	1	3.3	2.3

TABLE III
H AND P-VALUES OBTAINED FROM THE WILCOXON RANK-SUM TEST.

MFOs	f_1		f_2		MFOs	f_1		f_2		MFOs	f_1		f_2		MFOs	f_1		f_2	
	h	p	h	p		h	p	h	p		h	p	h	p		h	p	h	p
car91 car92	1	2.39e-04	0	1.12e-01	ear83 hec92	1	2.21e-03	0	8.77e-02	hec92 tre92	0	5.94e-02	1	2.58e-06	lse91 uta92	1	1.41e-04	1	6.52e-09
car91 ear83	1	2.39e-04	0	2.94e-01	ear83 hec92 II	1	8.29e-06	0	3.95e-01	hec92 uta92	1	1.44e-02	0	5.35e-01	lse91 yor83	1	1.34e-03	1	1.91e-02
car91 ear83 IIc	1	2.89e-03	0	1.22e-01	ear83 kfu93	1	1.20e-03	1	5.44e-03	hec92 uta92 II	0	6.57e-02	0	8.48e-01	rye93 sta83	0	2.40e-01	1	2.94e-11
car91 hec92	1	1.49e-04	1	1.84e-02	ear83 lse91	0	8.77e-02	1	1.95e-03	hec92 ute92	0	1.76e-01	1	5.53e-08	rye93 sta83 IIc	1	4.36e-02	1	7.37e-08
car91 hec92 II	1	6.73e-04	0	1.22e-01	ear83 rye93	1	2.42e-02	0	2.97e-01	hec92 yor83	0	1.22e-01	1	7.29e-03	rye93 tre92	1	3.03e-02	1	7.22e-06
car91 kfu93	1	6.55e-04	1	2.50e-03	ear83 sta83	1	3.92e-02	1	2.94e-11	hec92 II kfu93	0	4.83e-01	0	5.28e-02	rye93 uta92	0	4.12e-01	0	7.73e-01
car91 lse91	1	3.37e-04	1	1.03e-02	ear83 sta83 IIc	1	8.31e-03	1	1.02e-07	hec92 II lse91	0	1.26e-01	1	1.11e-06	rye93 uta92 II	1	3.15e-02	0	9.12e-01
car91 rye93	1	2.13e-04	0	3.95e-01	ear83 tre92	1	3.18e-03	1	4.33e-04	hec92 II rye93	0	1.69e-01	0	5.01e-02	rye93 ute92	0	1.54e-01	1	6.01e-08
car91 sta83	1	2.27e-03	1	2.95e-11	ear83 uta92	1	8.12e-04	0	3.75e-01	hec92 II sta83	0	8.50e-02	1	2.95e-11	rye93 yor83	0	1.49e-01	0	2.12e-01
car91 sta83 IIc	1	9.52e-04	1	1.87e-07	ear83 uta92 II	1	8.87e-03	0	5.30e-01	hec92 II sta83 IIc	0	9.35e-01	1	1.41e-07	sta83 sta83 IIc	1	2.95e-11	1	7.21e-06
car91 tre92	1	1.17e-02	1	5.56e-04	ear83 ute92	1	4.64e-05	1	2.19e-0	hec92 II tre92	0	2.40e-01	1	9.88e-03	sta83 tre92	1	2.94e-11	1	1.03e-06
car91 uta92	1	8.79e-04	0	5.69e-01	ear83 yor83	1	7.66e-05	1	1.44e-02	hec92 II uta92	0	3.63e-01	0	8.77e-01	sta83 uta92	1	2.94e-11	0	7.01e-01
car91 uta92 II	1	6.91e-04	0	8.19e-01	ear83 IIc hec92	0	9.19e-02	1	1.91e-02	hec92 II uta92 II	0	1.19e-01	0	8.19e-01	sta83 uta92 II	1	2.94e-11	0	1.09e-01
car91 ute92	1	2.92e-04	1	9.75e-10	ear83 IIc hec92 II	1	4.22e-03	0	1.96e-01	hec92 II ute92	0	4.12e-01	1	1.85e-08	sta83 ute92	1	2.94e-11	1	1.46e-10
car91 yor83	1	3.76e-03	0	1.58e-01	ear83 IIc kfu93	1	2.16e-03	1	3.39e-02	hec92 II yor83	0	1.30e-01	1	4.59e-02	sta83 yor83	1	2.94e-11	0	5.94e-02
car92 ear83	0	2.39e-04	1	2.94e-01	ear83 IIc lse91	1	7.62e-03	1	3.56e-04	kfu93 lse91	1	1.50e-02	1	4.42e-06	sta83 IIc tre92	1	2.67e-07	1	3.95e-05
car91 ear83 IIc	1	2.89e-03	1	1.22e-01	ear83 IIc rye93	1	1.08e-02	0	5.74e-01	kfu93 rye93	1	4.06e-02	0	1.09e-01	sta83 IIc uta92	1	1.47e-07	0	4.96e-01
car92 hec92	1	8.13e-03	1	1.84e-02	ear83 IIc sta83	0	1.76e-01	1	2.94e-11	kfu93 sta83	1	1.91e-02	1	2.95e-11	sta83 IIc uta92 II	1	3.38e-07	0	6.73e-01
car92 hec92 II	1	1.80e-02	1	2.51e-02	ear83 IIc sta83 IIc	1	6.98e-05	1	1.44e-06	kfu93 sta83 IIc	1	2.19e-02	1	7.67e-08	sta83 IIc ute92	1	2.77e-06	1	4.00e-09
car92 kfu93	1	1.60e-02	0	6.57e-02	ear83 IIc tre92	0	1.33e-01	1	3.37e-05	kfu93 tre92	1	4.51e-02	1	5.83e-03	sta83 IIc yor83	1	1.24e-06	0	1.30e-01
car92 lse91	0	5.55e-02	1	1.15e-06	ear83 IIc uta92	1	1.54e-04	0	9.94e-01	kfu93 uta92	0	6.15e-02	0	8.36e-01	tre92 uta92	1	2.43e-05	0	9.18e-01
car92 rye93	1	1.41e-02	1	1.99e-02	ear83 IIc uta92 II	1	1.86e-03	0	5.64e-01	kfu93 uta92 II	1	2.98e-02	0	6.20e-01	tre92 uta92 II	1	4.64e-03	0	9.82e-01
car92 sta83	0	8.11e-02	1	2.95e-11	ear83 IIc ute92	1	6.10e-03	1	9.43e-08	kfu93 ute92	0	6.35e-02	1	1.93e-06	tre92 ute92	1	1.89e-04	1	1.01e-08
car92 sta83 IIc	1	5.57e-03	1	2.49e-08	ear83 IIc yor83	1	3.34e-03	1	1.50e-02	kfu93 yor83	1	2.76e-02	1	2.15e-02	tre92 yor83	1	2.43e-05	1	2.05e-03
car92 tre92	1	2.07e-02	1	2.25e-04	hec92 hec92 II	1	8.50e-03	0	7.98e-02	lse91 rye93	1	1.54e-04	0	6.10e-01	uta92 uta92 II	0	8.07e-01	0	6.52e-01
car92 uta92	1	5.83e-03	0	8.30e-01	hec92 kfu93	1	4.13e-02	1	2.38e-03	lse91 sta83	1	3.52e-07	1	2.95e-11	uta92 ute92	0	6.79e-01	1	4.62e-10
car92 uta92 II	1	3.34e-03	0	4.12e-01	hec92 lse91	0	3.48e-01	1	5.46e-06	lse91 sta83 IIc	1	6.77e-05	1	6.27e-07	uta92 yor83	0	5.11e-01	1	2.24e-02
car92 ute92	0	7.48e-02	1	1.60e-07	hec92 rye93	0	1.19e-01	0	6.57e-02	lse91 tre92	1	1.19e-06	1	4.21e-05	uta92 II ute92	0	6.52e-01	1	1.69e-09
car92 yor83	1	2.56e-02	1	3.03e-02	hec92 sta83	0	5.89e-01	1	2.95e-11	lse91 uta92	1	1.14e-03	0	9.71e-01	uta92 II yor83	0	8.30e-01	1	1.56e-03
ear83 ear83 IIc	1	1.14e-04	1	3.37e-05	hec92 sta83 IIc	0	5.15e-01	1	7.73e-06	lse91 uta92 II	1	2.76e-04	0	8.19e-01	ute92 yor83	1	1.70e-08	0	6.90e-02

TABLE SIV
THREE-TASK TEST SETS.

	f_1	f_2	f_3
Set 1	hec92	hec92 II	sta83
Set 2	hec92	hec92 II	lse91
Set 3	hec92	hec92 II	car91
Set 4	hec92	lse91	kfu93
Set 5	hec92	lse91	car91
Set 6	hec92	uta92	car91

TABLE SV
FOUR-TASK TEST SETS.

	f_2 (uETTP)	f_2 (GCP)	f_3 (uETTP)	f_4 (GCP)
Set 7	hec92	hec92	lse91	lse91
Set 8	hec92	hec92	car91	car91

TABLE SVI
FIVE-TASK TEST SETS.

	f_1	f_2	f_3	f_4	f_5
Set 9	hec92	sta83	yor83	ute92	ear83
Set 10	hec92	yor83	tre92	car92	car91

TABLE SVII
THE MINIMUM, MEAN AND STANDARD DEVIATIONS OBTAINED BY EMHH AND SOHH ON THREE-TASK TEST SETS.

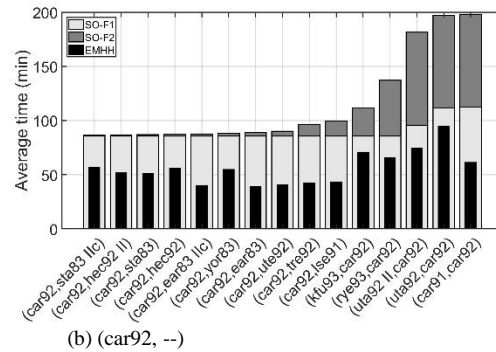
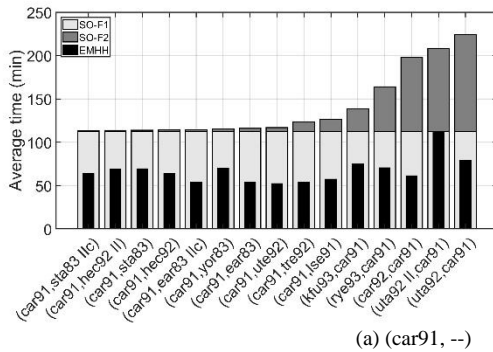
Tasks	EMHH			SOHH			Tasks	EMHH			SOHH				
	F1	F2	F3	F1	F2	F3		F1	F2	F3	F1	F2	F3		
Set 1	hec92	11.57	11.67	159.03	11.69	11.67	169.77	Set 4	hec92	11.51	11.16	15.40	11.69	11.65	15.37
	hec92 II	12.09	12.07	159.66	12.12	12.10	169.86		lse91	12.09	11.70	15.63	12.12	11.85	15.68
	sta83	0.25	0.24	0.42	0.27	0.29	0.09		kfu93	0.22	0.24	0.15	0.27	0.16	0.15
Set 2	hec92	11.78	11.50	8.67	11.69	11.67	8.78	Set 5	hec92	11.80	11.37	5.19	11.69	11.65	5.18
	hec92 II	12.08	12.05	8.81	12.12	12.10	8.88		lse91	12.13	11.71	5.23	12.12	11.85	5.26
	tre92	0.20	0.25	0.078	0.27	0.29	0.06		car91	0.16	0.20	0.03	0.27	0.16	0.04
Set 3	hec92	11.82	11.60	5.19	11.69	11.67	5.18	Set 6	hec92	11.60	3.38	5.14	11.69	3.39	5.18
	hec92 II	12.09	12.05	5.25	12.12	12.10	5.26		uta92	12.04	3.42	5.24	12.12	3.43	5.26
	car91	0.18	0.25	0.03	0.27	0.29	0.04		car91	0.20	0.02	0.04	0.27	0.02	0.04

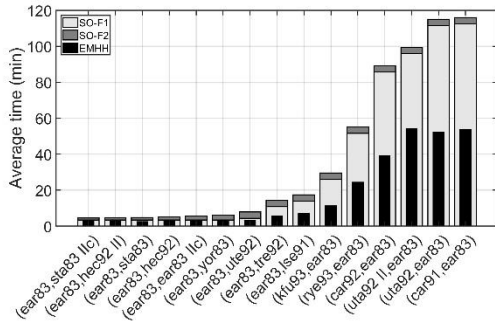
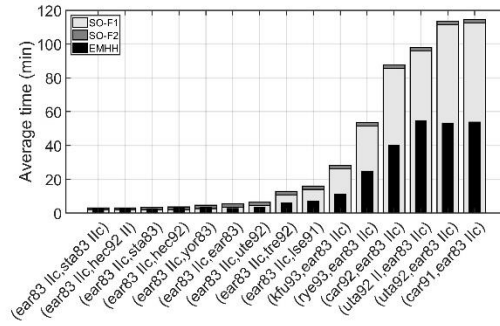
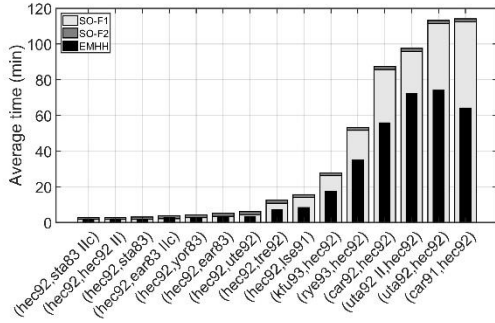
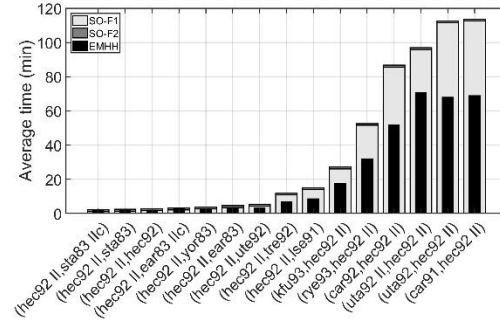
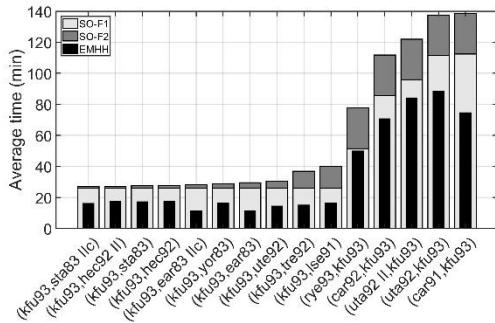
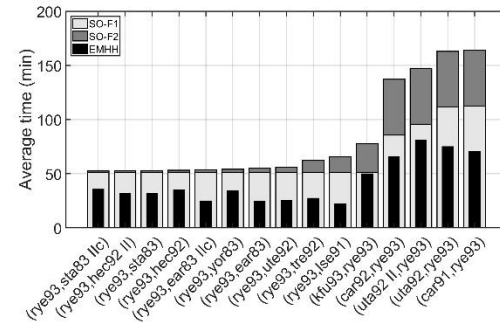
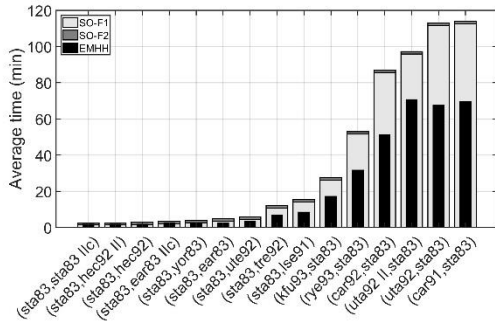
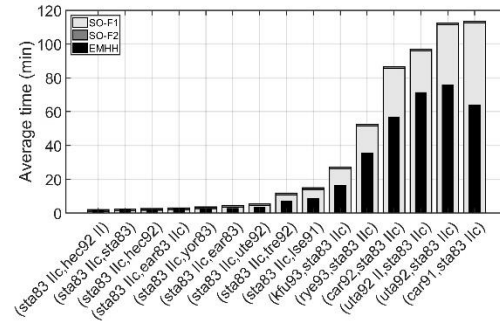
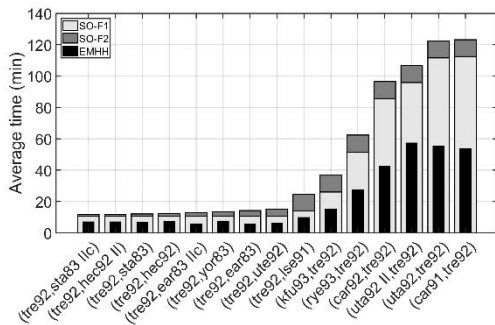
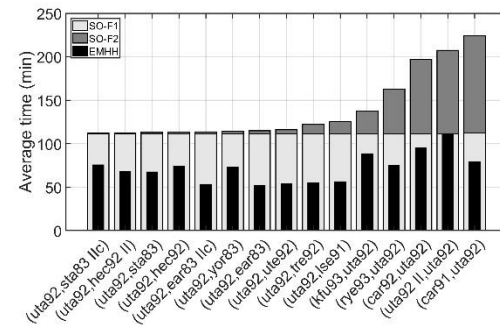
TABLE SVIII
COMPARISONS OF EMHH AND SOHH ON FOUR-TASK CROSS-DOMAIN TEST SETS.

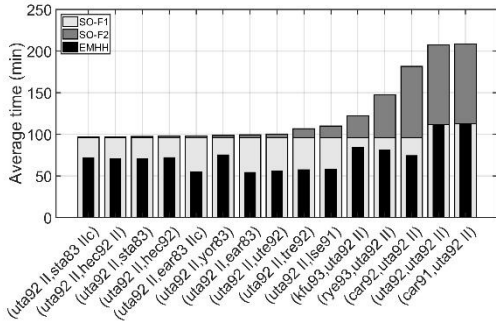
Tasks	EMHH				SOHH			
	F1	F2	F3	F4	F1	F2	F3	F4
Set 7	hec92				11.69			
	hec92	11.68		11.50	17		11.65	17
	lse91	12.14	17	11.76	17	12.12	11.85	17
	lse91	0.22	18	0.13	17	0.267	0.16	17
Set 8	hec92				11.69			
	hec92	11.73		5.16	17		5.18	30
	car91	12.11	17	5.24	31	12.12	5.26	31
	car91	0.24	18	0.04	31	0.267	0.04	31

TABLE SIX
THE MINIMUM, MEAN AND STANDARD DEVIATIONS OBTAINED BY EMHH AND SOHH ON FIVE-TASK TEST SETS

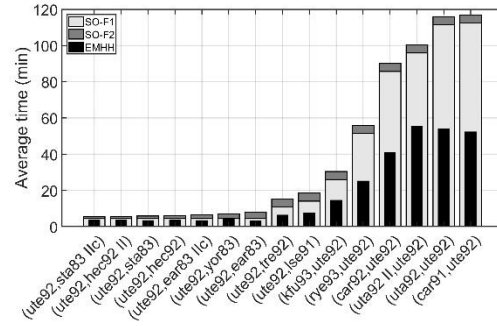
Tasks	EMHH					SOHH					
	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5	
Set 9	hec92					11.69					
	sta83	11.49	158.73	40.22	27.57	36.25	11.69	169.77	41.05	28.31	36.62
	yor83	12.15	159.94	41.98	28.47	37.10	12.12	169.86	42.87	28.75	37.12
	ute92	0.25	0.55	0.71	0.40	0.52	0.267	0.09	2.91	0.14	0.25
	ear83										
Set 10	hec92					11.69					
	yor83	11.42	40.48	8.58	4.33	5.16	11.69	41.05	8.78	4.31	5.18
	tre92	12.11	42.42	8.85	4.42	5.25	12.12	42.87	8.88	4.41	5.26
	car92	0.30	2.14	0.09	0.04	0.04	0.267	2.91	0.06	0.04	0.04
	car91										



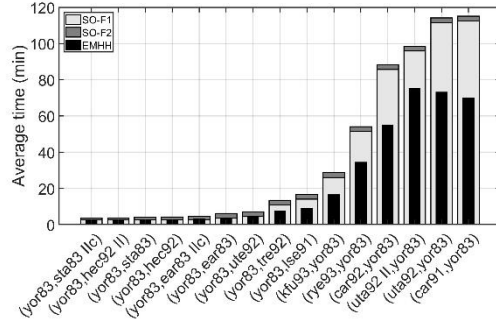
(c) (*ear83*, --)(d) (*ear83 IIc*, --)(e) (*hec92*, --)(f) (*hec92 II*, --)(g) (*kfu93*, --)(h) (*rye93*, --)(i) (*sta83*, --)(j) (*sta83 IIc*, --)(k) (*tre92*, --)(l) (*uta92*, --)



(m) (uta92 II, --)



(n) (ute, --)



(o) (yor83, --)

Fig.S1. Time comparisons between SOHH and EMHH.



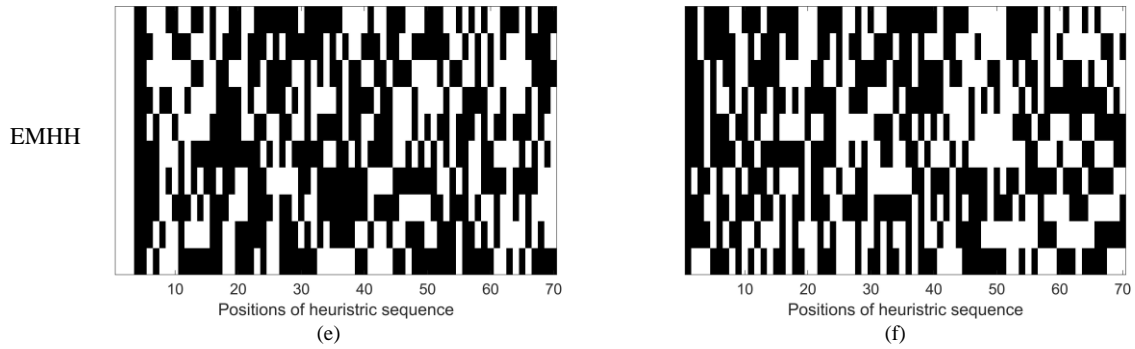


Fig.S2. Best 10 heuristic sequences for ear83 (left) and sta83 (right) ordered from the best at the top to the worst. (a) and (b) are the best 10 heuristic sequences selected from the results obtained by both SOHH and EMHH, (c) and (d) are those obtained by SOHH, and (e) and (f) are obtained by EMHH. Each row represents a heuristic sequence. LWD and SD is replaced with 1 (white block) and 0 (black block), respectively. The length of sta83's heuristic sequences is 70, thus we only show the left 70 heuristics of ear83 for clearer comparison. This does not affect the conjecture, because transfers within the left 70 heuristics actually matters in this case.