Recent Developments on Nurse Rostering and Other Ongoing Research

Dr Rong Qu
ASAP Group, School of Computer Science
The University of Nottingham
rxq@cs.nott.ac.uk; http://www.cs.nott.ac.uk/~rxq

Collaborators
Professor **Edmund Burke**, Dr **Jingpeng Li**, Dr **Tim Curtois**, University of Nottingham
Professor **Peter Brucker**, Osnabruck University
Dr **Barry McCollum**, Queens University, Belfast, N. Ireland
Dr **Gehard Post**, University of Twente, The Netherlands

Recent Research on Nurse Rostering and Others

ASAP Group, The University of Nottingham

Content

- PART I: nurse rostering research
  - Description & formulation
  - Brief literature review
  - Benchmarks
  - Approaches

- PART II: other ongoing and previous research
  - …

PART I: Nurse Rostering Research

- Description & formulation
- Brief literature review
- Benchmarks
- Approaches

Nurse Rostering Problems

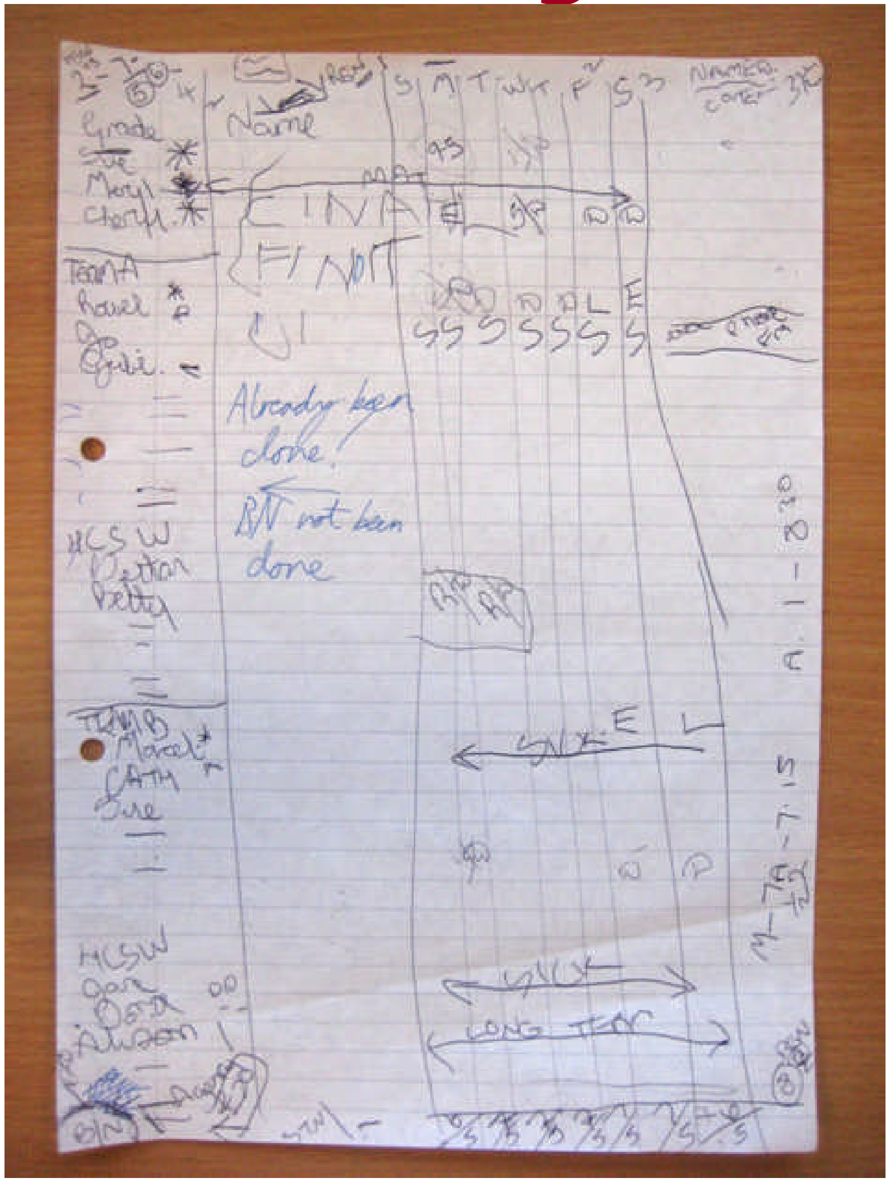
- Hospitals worldwide operate 24/7
  - Number of shift types (early, day, late, night)
  - Cover requirements can vary, every day or weekend
  - Different grades and skill mixes

- Difficult optimisation problem with many constraints and objectives
  - Time consuming, frustrating and stressful
  - Long scheduling horizons and large numbers of employees
  - Regular rescheduling required to cope with absences
  - Poor planning can cause decrease in quality of healthcare

Nurse Rostering Problems

- Schedule a number of shifts to nurses in rosters, satisfying a set of constraints
  - Enough number of shifts (of different types) coverage on each day during the scheduling period
  - Side constraints
    - working/resting hours limit, complete weekends, skill levels, personal preferences, etc

Nurse Rostering Problems



Nurse Rostering Problems

- Automated nurse rostering
  - Satisfying more personal requests and preferences
  - Helps nurses plan their leisure time more effectively
  - Flexible schedules helps recruiting and retaining staff
  - Computers regarded as impartial

Nurse Rostering Problems

- Automated nurse rostering
  - Can ensure legal requirements are not broken
  - Lower costs, e.g. hire less agency nurses to fill gaps in rosters
  - Generate management reports and statistics, connect to payroll systems, less paperwork, etc

Recent Research on Nurse Rostering and Others

Nurse Rostering Problems

	1							2							3							4							
December	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	
1A	D	E	E	E	L			E	E	E	E		D	D	D	N	N	N				L	L	L	L				51
A	DH	DH	DH	DH	DH			DH	DH	DH		DH	DH	DH	DH		DH	DH				DH	DH	DH	DH	DH			20
B	N	N	N	N				D	D	L	L	L				L	L	L				E	E	E	D	D			0
C	D	D	D	D	D				N	N	N		L	L	L				L	L	L		E	E	E	L			25
D				L	N	N	N	N			DH	D				E	E	E	DH	E	E		N	N			E	E	13
E					D	DH	DH	D					E	E		DH	E	E	E	DH	DH		D	D	E	E	DH	DH	21
F	L	L	L			L	L	L	L			N	N	N	N			D	D			D				D	D	D	10
G				E	E	E	E			D	D	D			E	E			D	D	D	D			N	N	N	N	10
H	E	E	E			D	D		E	E	E	E			D	D	D		N	N	N	N					L	L	26

Total Penalty 176

Unassigned Shifts 0

Minimum Cover

E	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1
D	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1
DH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
N	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Nurse Rostering web site at http://www.asap.cs.nott.ac.uk/projects/nmhpr/data

Nurse Rostering Problems

- Problem formulation
  - Hard constraints
    - binding, feasibility, or imperative planning rules
  - Soft constraints
    - floppy, non binding, preference planning rules
  - Weights
    - to specify relative priorities
    - weighted sum objective function

Nurse Rostering Problems

	1							2							3							4							
December	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	
1A	D	E	E	E	L			E	E	E	E		D	D	D	N	N	N				L	L	L	L				51
A	DH	DH	DH	DH	DH			DH	DH	DH		DH	DH	DH	DH		DH	DH				DH	DH	DH	DH	DH			20
B	N	N	N	N				D	D	L	L	L				L	L	L				E	E	E	D	D			0
C	D	D	D	D	D				N	N	N		L	L	L				L	L	L		E	E	E	L			25
D				L	N	N	N	N			DH	D				E	E	E	DH	E	E		N	N			E	E	13
E					D	DH	DH	D					E	E		DH	E	E	E	DH	DH		D	D	E	E	DH	DH	21
F	L	L	L			L	L	L	L			N	N	N	N			D	D			D				D	D	D	10
G				E	E	E	E			D	D	D			E	E			D	D	D	D			N	N	N	N	10
H	E	E	E			D	D		E	E	E	E			D	D	D		N	N	N	N					L	L	26

Too few resting time (10)

Too few consecutive late shifts (5)

Too few consecutive night shifts (5)

Total Penalty 176

Unassigned Shifts 0

Minimum Cover

E	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1	1	2	2	2	1	1	1
D	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1
DH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
N	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

PART I: Nurse Rostering Research

- Description & formulation
- Brief literature review
- Benchmarks
- Approaches

Nurse Rostering Literature

- Meta-heuristics heavily used [BUR04]
  - GAs[AIC04,07], Memetic Algorithm[VAN01,OZC07], Tabu Search[DOW98], Variable Neighbourhood Search [BUR07]

- Hyper-heuristics showed to be flexible and effective
  - Tabu Search Hyper-heuristic[BUR03], Rule-Based Hyper-heursitic[AIC07a], Memetic Algorithm hyper-heuristics[OZC07a]

Nurse Rostering Literature

- Mathematical programming also report good results
  - Hybridised with meta-heuristics[BUR07]

- Others
  - Case based reasoning[BED06]
  - Multi-objective[BUR07a]

Nurse Rostering Literature

- Heuristics
  - Advantages
    - Can exploit problem specific information
    - Do not require expensive software packages
  - Disadvantages
    - More programming involved
    - Can be inconsistent

PART I: Nurse Rostering Research

- Description & formulation
- Brief literature review
- Benchmarks
- Approaches

Nurse Rostering Benchmarks

- Very few benchmark nurse rostering problems
  - No typical nurse rostering problem
  - Each hospital has its own problem with a variety of complicated objective functions and lots of constraints
- Benchmarks would help validate algorithms
  - We are collecting real-world problems at http://www.asap.cs.nott.ac.uk/projects/nmhpr/data
  - Encourage collaborations and competition

Recent Research on Nurse Rostering and Others

The University of Nottingham

Computer Science & Information Technology

Faculty of Science

asap
automated scheduling optimisation & planning

Personnel Scheduling Data Sets and Benchmarks

[data] [software] [documentation] [changes] [contact]

Overview

Personnel scheduling problems and benchmarks. These are test instances for the problem of automated personnel schedulir Most of the benchmark problems provided here are nurse rostering problems and based on real world data. See t documentation section for more information on the format of the data and software provided for using the data sets and t development of new solvers.

Data sets

		Best known solutions		
File	GPost.xml	7	html	xml
Problem	GPost	8	html	xml
Comments	This is a small problem and a nice introductory example.			
Employees	8			
Schedule length	4 weeks			
Cover type	Cover is specified per shift, over and under coverage is not allowed.			
Other versions	GPost-B.xml Same as GPost.xml but without the requests on the first two days.	5	html	xml

Nurse Rostering Benchmarks

- Collected from real hospitals firstly by KaHo Sint-Lieven, Belgium
  - Anonymized, removed confidential information and country specific constraints
- Updated frequently by ASAP Group
  - More recent data from UK, The Netherlands and Canada
- XML
  - flexible, extendible
  - simple representation of different problems
- API evaluation function
  - Standard measure for scientific comparisons

PART I: Nurse Rostering Research

- Description & formulation
- Brief literature review
- Benchmarks
- Approaches
  - A decomposition approach
  - A sequence based hybrid approach
  - A hybrid variable neighbourhood search
  - Other recent work

A Decomposition Approach

- The problem
  - To create monthly schedules for wards
  - Different types of nurses (PT, FT)
  - 4 shift types and demand in a week

  - Derived from real-world problems in ORTEC, Netherlands

Brucker P., Qu R, Burke E.K. and Post G. A Decomposition, Construction and Post-processing Approach for a Specific Nurse Rostering Problem. **MISTA'05**, 397-406. New York, USA, Jul 2005

A Decomposition Approach

- The problem

12 Full-time nurses	36 hours/week
1 Part-time nurse	32 hours/week
3 Part-time nurses	20 hours/week

			Demand						
Shift type	Start time	End time	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Early	07:00	16:00	3	3	3	3	3	2	2
Day	08:00	17:00	3	3	3	3	3	2	2
Late	14:00	23:00	3	3	3	3	3	2	2
Night	23:00	07:00	1	1	1	1	1	1	1

A Decomposition Approach

- Hard constraints
  - HC1: daily coverage requirement of each shift type
  - HC2: for each day, a nurse works at most one shift
  - HC3: max number of working days per month
  - HC4: max number of on-duty weekends per month
  - HC5: max number of *night* shifts per month
  - HC6: no *night* shift between two non-*night* shifts
  - HC7: min two free days after a series of *night* shifts
  - HC8: max number of consecutive *night* shifts
  - HC9: max number of consecutive working days
  - HC10: no *late* shifts for one particular nurse

A Decomposition Approach

- Soft constraints

SC1	either no shifts or two shifts in weekends	1000
SC2	avoiding a single day between two days off	1000
SC3	length of a series of night shifts	1000
SC4	Min number of free days after a series of shifts	100
SC5	Max/Min number of consecutive assignments of a specific shift type	10
SC6	Max/Min number of weekly working days	10
SC7	Max number of consecutive working days for part-time nurses	10
SC8	avoiding certain shift type successions (e.g. a *day* shift followed by an *early* one, etc)	5

A Decomposition Approach

- The main idea
  - to decompose the problem into cyclic schedules for groups of nurses
  - add workload of remaining nurses

  - in a second step a Variable Neighbourhood Search (VNS) is applied for further improvement

A Decomposition Approach

	Week 1						
	M	T	W	T	F	S	S
Nurse 1	D	D	D			E	E
Nurse 2	L	L	L				
Nurse 3	E	E	E	L	L		
Nurse 4				E	E	L	L
Nurse 5	N	N			D	D	D

A Decomposition Approach

	Week 1							Week 2						
	M	T	W	T	F	S	S	M	T	W	T	F	S	S
Nurse 1	D	D	D			E	E	D	D	D			E	E
Nurse 2	L	L	L					L	L	L				
Nurse 3	E	E	E	L	L			E	E	E	L	L		
Nurse 4				E	E	L	L				E	E	L	L
Nurse 5	N	N			D	D	D	N	N			D	D	D

A Decomposition Approach

	Week 1							Week 2						
	M	T	W	T	F	S	S	M	T	W	T	F	S	S
Nurse 1	D	D	D			E	E							
Nurse 2	L	L	L					L	L	L				
Nurse 3	E	E	E	L	L			E	E	E	L	L		
Nurse 4				E	E	L	L				E	E	L	L
Nurse 5	N	N			D	D	D	N	N			D	D	D

D	D	D			E	E

A Decomposition Approach

	Week 1							Week 2						
	M	T	W	T	F	S	S	M	T	W	T	F	S	S
Nurse 1	D	D	D			E	E	L	L	L				
Nurse 2	L	L	L					E	E	E	L	L		
Nurse 3	E	E	E	L	L						E	E	L	L
Nurse 4				E	E	L	L	N	N			D	D	D
Nurse 5	N	N			D	D	D							

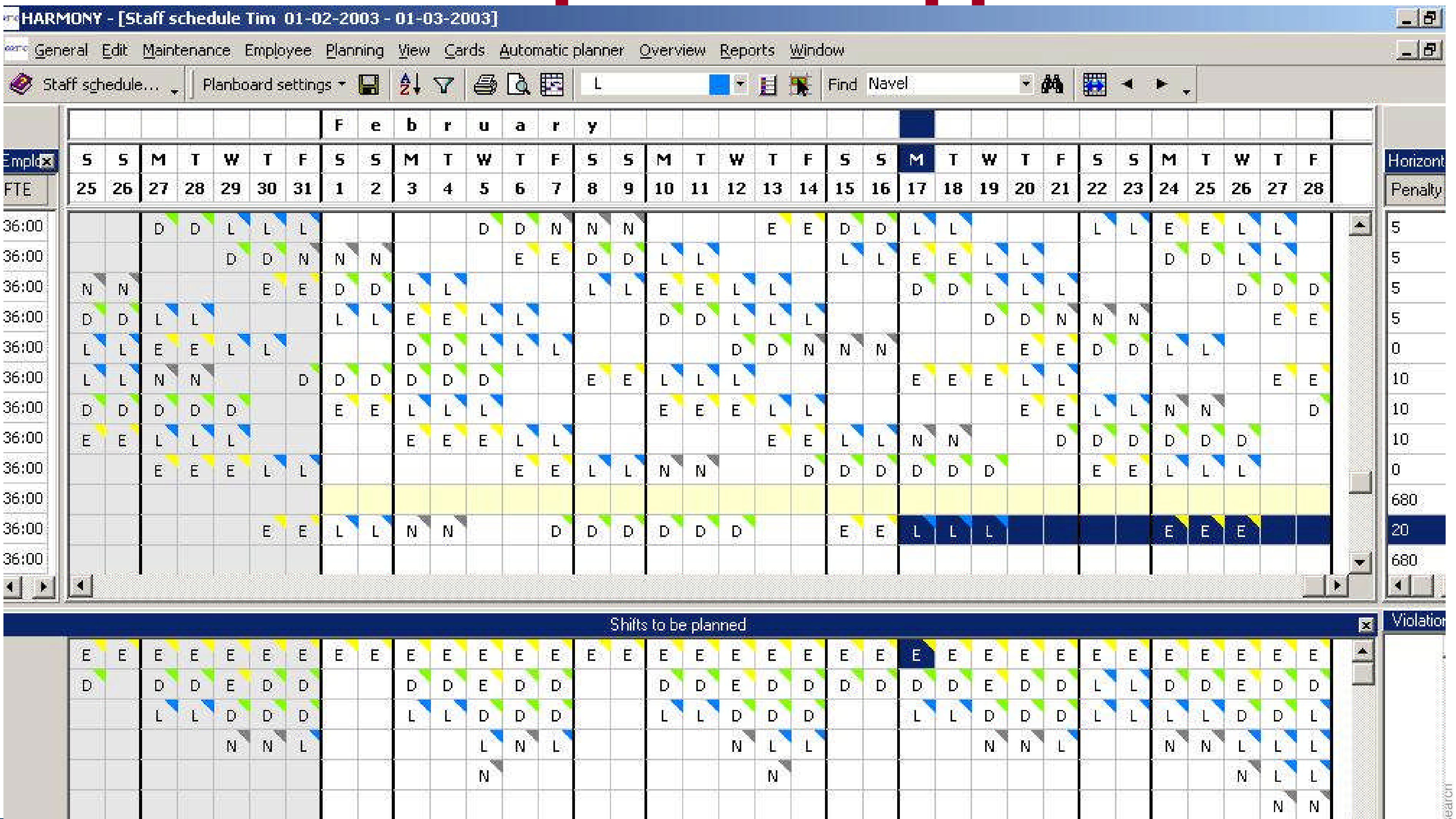
D	D	D			E	E

A Decomposition Approach

	Week 1							Week 2						
	M	T	W	T	F	S	S	M	T	W	T	F	S	S
Nurse 1	D	D	D			E	E	L	L	L				
Nurse 2	L	L	L					E	E	E	L	L		
Nurse 3	E	E	E	L	L						E	E	L	L
Nurse 4				E	E	L	L	N	N			D	D	D
Nurse 5	N	N			D	D	D	D	D	D			E	E

• • •

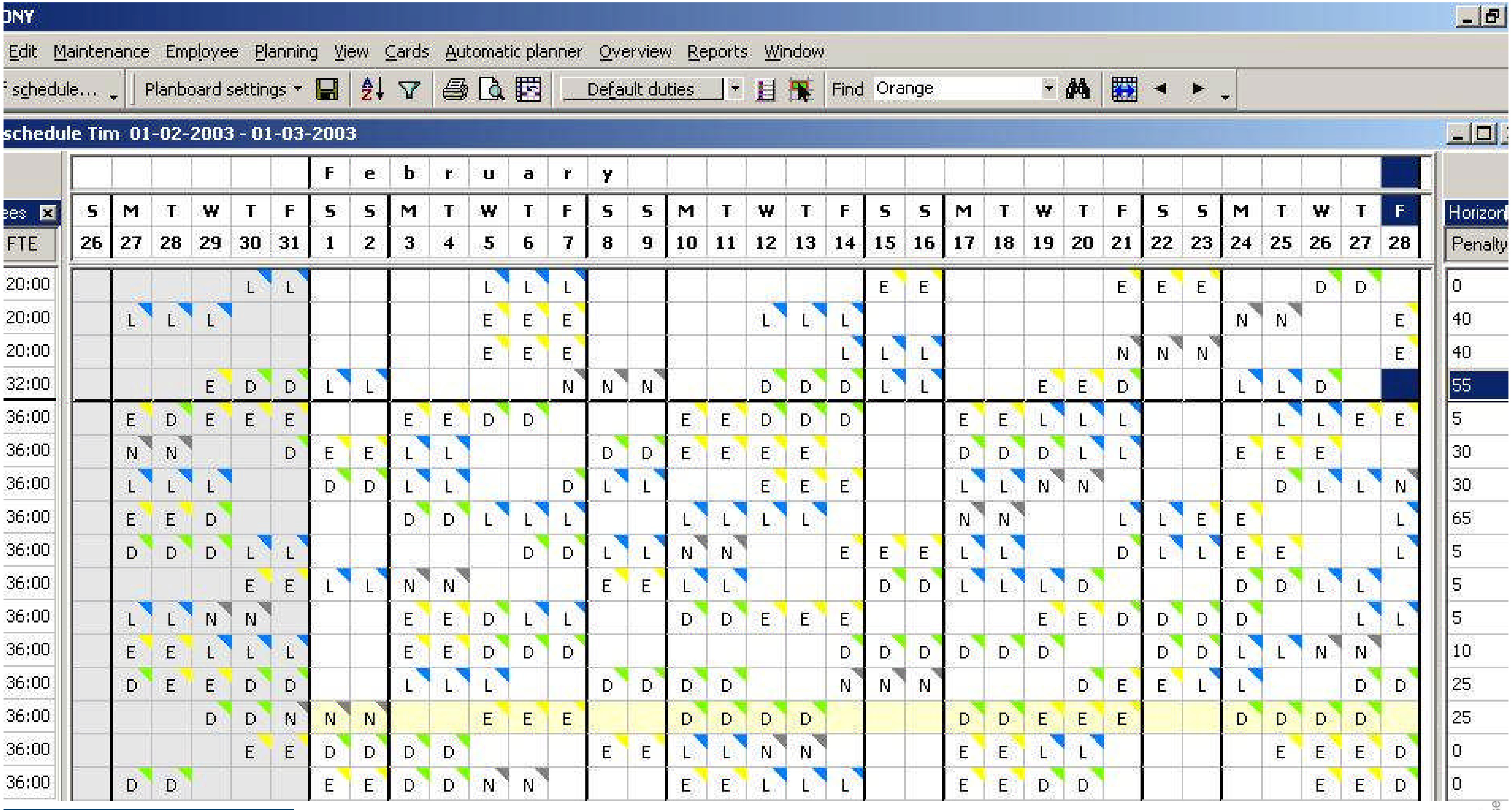
A Decomposition Approach



A Decomposition Approach

- Add the remaining shifts by using a heuristic ordering method
  - More ***troublesome*** shifts assigned first
  - Criteria to evaluate the shifts
    - Type of shifts, number of employees able to cover it, etc

A Decomposition Approach



ees	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	Horizon
FTE	26	27	28	29	30	31	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	Penalty
20:00					L	L					L	L	L								E	E					E	E	E			D	D		0
20:00		L	L	L							E	E	E					L	L	L										N	N			E	40
20:00											E	E	E							L	L	L					N	N	N					E	40
32:00				E	D	D	L	L					N	N	N			D	D	D	L	L			E	E	D			L	L	D			55
36:00		E	D	E	E	E			E	E	D	D				E	E	D	D	D			E	E	L	L	L				L	L	E	E	5
36:00		N	N			D	E	E	L	L				D	D	E	E	E	E				D	D	D	L	L			E	E	E			30
36:00		L	L	L			D	D	L	L			D	L	L			E	E	E			L	L	N	N					D	L	L	N	30
36:00		E	E	D					D	D	L	L	L			L	L	L	L				N	N			L	L	E	E				L	65
36:00		D	D	D	L	L						D	D	L	L	N	N			E	E	E	L	L			D	L	L	E	E			L	5
36:00					E	E	L	L	N	N				E	E	L	L				D	D	L	L	L	D				D	D	L	L		5
36:00		L	L	N	N				E	E	D	L	L			D	D	E	E	E					E	E	D	D	D	D			L	L	5
36:00		E	E	L	L	L			E	E	D	D	D							D	D	D	D	D	D			D	D	L	L	N	N		10
36:00		D	E	E	D	D			L	L	L			D	D	D	D			N	N	N				D	E	E	L	L			D	D	25
36:00				D	D	N	N	N			E	E	E			D	D	D	D				D	D	E	E	E			D	D	D	D		25
36:00					E	E	D	D	D	D				E	E	L	L	N	N				E	E	L	L					E	E	E	D	0
36:00		D	D				E	E	D	D	N	N				E	E	L	L	L			E	E	D	D						E	E	D	0

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automated scheduling optimisation & planning

A Decomposition Approach

- **Hybrid GA**
  - 630 (5 min) → 505 (40 min) → 411 (6 hours)
- **Hybrid VNS**
  - 466 (1 min)

- **Decomposition + construction**
  - 340
- **VNS after Decomposition + construction**
  - 170 (< 1 min)

Hybrid Variable Neighbourhood Search

- Meta-heuristics are the state-of-the-art in nurse rostering research
  - Most algorithms use only one neighbourhood operator
- Variable neighbourhood search (VNS) showed to be very effective on a number of scheduling problems
  - Employ at least two neighbourhood operators
  - Effective on escaping from local optimum

Burke E. K., Curtois T. E., Post G., Qu R., and Veltman B. A Hybrid Heuristic Ordering and Variable Neighbourhood Search for the Nurse Rostering Problem. ***European Journal of Operational Research***, 2: 330-341, 2008.

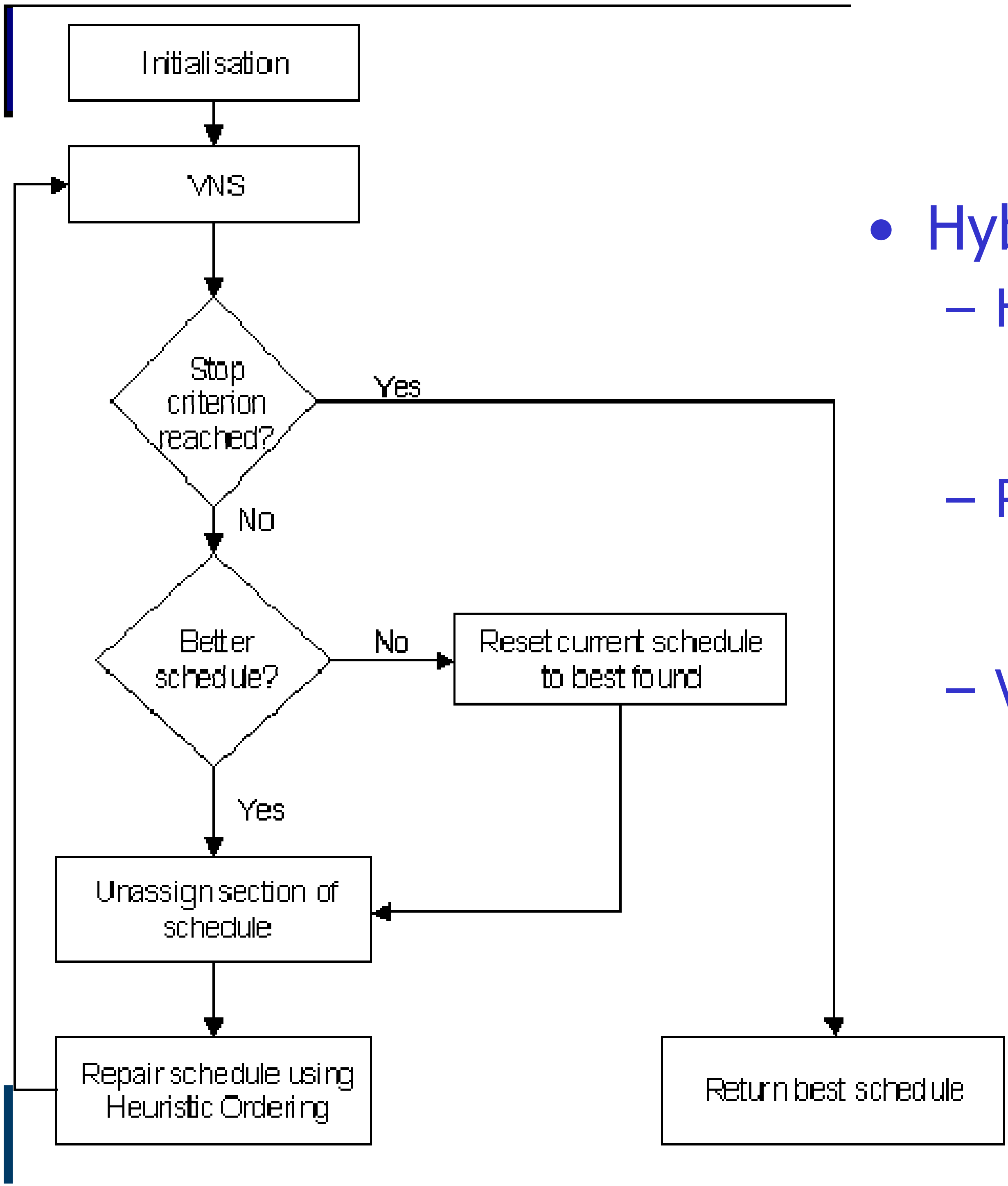
Hybrid Variable Neighbourhood Search

- HARMONY™
  - Automated workforce management software
  - Developed by ORTEC, The Netherlands
    - an international consultancy company on planning, scheduling, optimisation and decision support

- This work improved the algorithm in the previous version of the commercial software HARMONY™

Hybrid Variable Neighbourhood Search

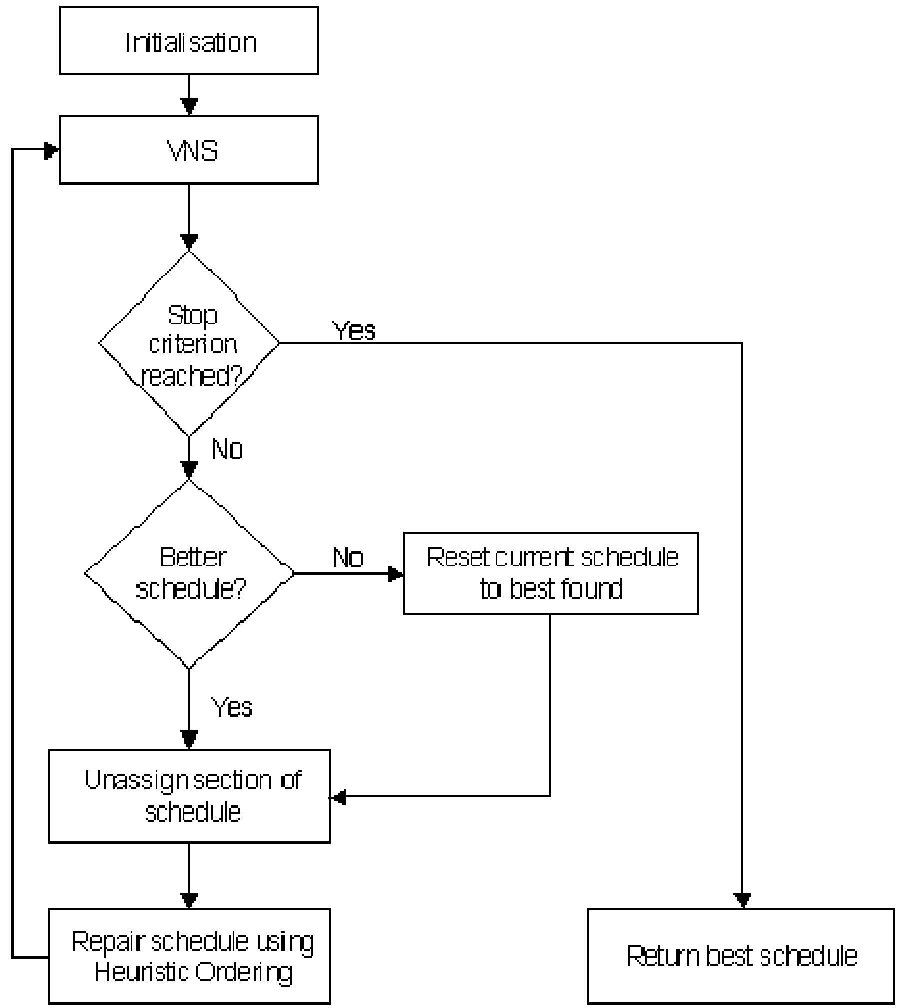
- In this work
  - Heuristic ordering
    - to order shifts for construction
  - Repairing method
    - remove worse part of roster and re-construct
  - VNS
    - improvement upon rosters



- Hybrid VNS
  - Heuristic ordering
    - to order shifts for construction
  - Repairing method
    - remove worse part of roster and re-construct
  - VNS
    - improvement upon rosters

Hybrid Variable Neighbourhood Search

- Heuristic ordering
  - Order shifts for construction in initialisation and repair
  - More ***troublesome*** shifts assigned first
  - A number of criteria to evaluate the shifts



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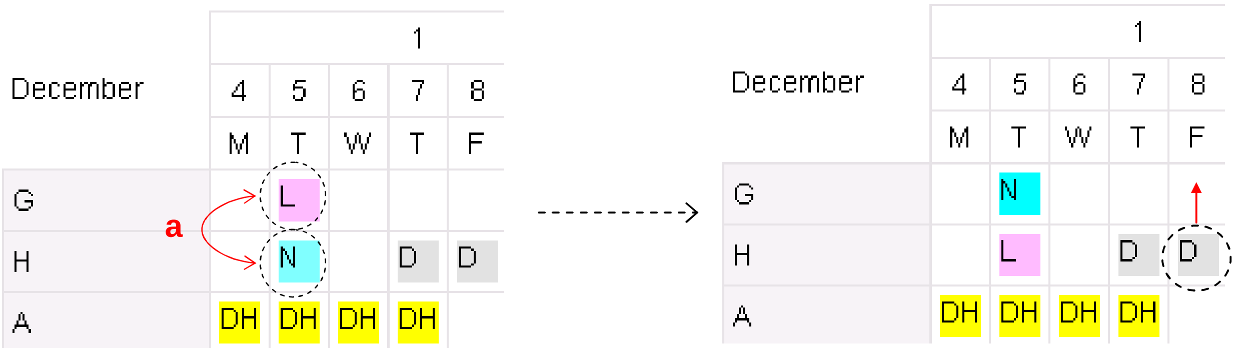
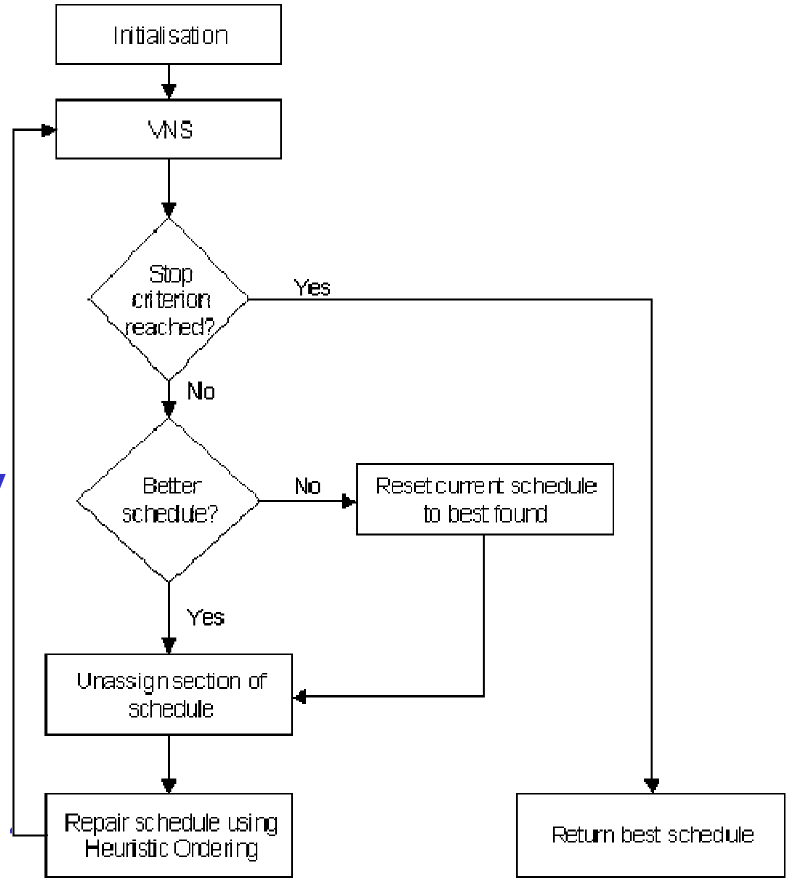
Hybrid Variable Neighbourhood Search

- Variable Neighbourhood Search (VNS)
  - Neighbours of a solution
    - those schedules that can be obtained by making a "move" e.g. single shifts swapped between any two nurses
  - Two neighbourhood operators
    - Assign a shift to another nurse
    - Swap shifts between nurses



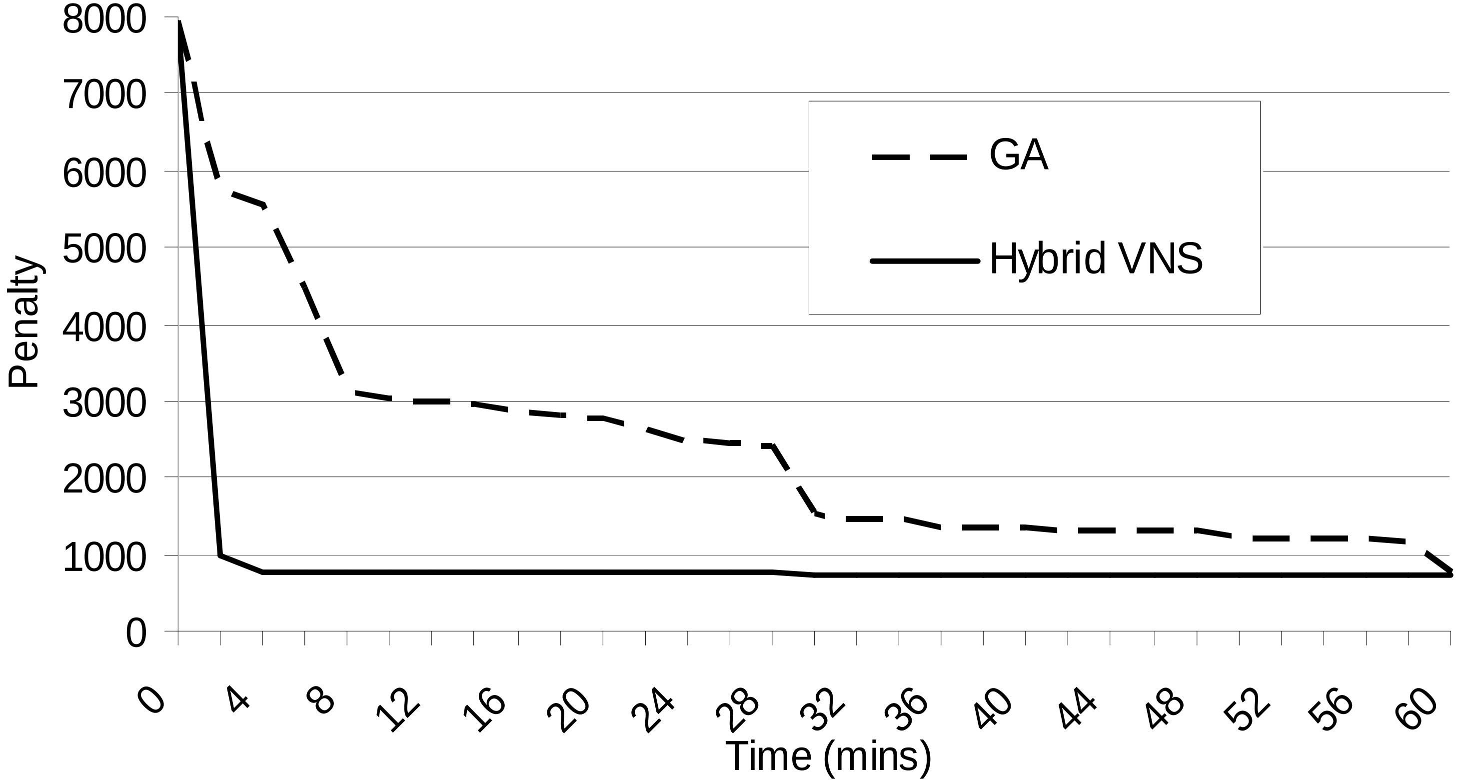
Hybrid Variable Neighbourhood Search

- Repairing method
  - After VNS reached to a local optimum
  - Un-assign a section of roster for further possible improvement operators
  - Re-assign shifts ordered by heuristic ordering



Hybrid Variable Neighbourhood Search

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
GA (60 mins)	775	**1791**	2030	612	2296	9466	781	**4850**	**615**	736	2126	625
VNS (30 mins)	735	1950	2055	501	2285	9312	660	4975	761	665	2041	625
VNS (60 mins)	**735**	1866	**2010**	**457**	**2161**	**9291**	**481**	4880	647	**665**	**2030**	**520**



Hybrid Variable Neighbourhood Search

Algorithm	Penalty
Hybrid VNS after 30 minutes	736
Hybrid VNS after 60 minutes	706
Best ever G.A. (24 hours)	681
Previous best known (made by manual improvements)	587
Hybrid VNS after 12 hours	541

Sequence Based Adaptive Approach

Night shift before a free weekend (10).

Too few consecutive night shifts (10).

Too few consecutive free days (10).

Too few consecutive late shifts (10).

December	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	**Pen**
1A	E	E	E			N	N	N	N			E	E	E	L	L						L	L	L					10
1A			N	N	N			N	N			D	D	D		L	L	L	L			D	D				D	D	30
1A					N	N	N			L	L	L	D	D			D	D	D			N	N			L	L	L	15
1A			D	N	N	N	N			D	D				E	E	E	E	L			D	D	N			D	D	30
1A	D	D				E	E	E	E	D	D				L	L	L	L	L					E	E	E	D	D	20

Too few consecutive day shifts (10).

Day shift after a late shift (5).

Too few consecutive night shifts (10).

Too many consecutive working days (10).

Too many consecutive late shifts (10).

P. Brucker, E.K. Burke, T. Curtois, R. Qu. Adaptive Construction of Nurse Schedules: A Shift Sequence Based Approach. accepted by ***European Journal of Operational Research***, 2008.

Sequence Based Adaptive Approach

- Problems derived from real-world
  - Large number of constraints of different types, and different importance
  - Time consuming when searching for good rosters

	Hard Constraints
1	Shifts which require certain skills can only be taken by (or assigned to) nurses who have those skills
2	The shift coverage requirements must be fulfilled

Sequence Based Adaptive Approach

	Soft Constraint
1	Minimum rest time between shifts
2	Alternative skill (if a nurse is able to cover a shift but prefers not to as it does not require his/her primary skill)
3	Maximum number of shift assignments
4	Maximum number of consecutive working days
5	Minimum number of consecutive working days
6	Maximum number of consecutive non-working days
7	Minimum number of consecutive non-working days
8	Maximum number of hours worked
9	Minimum number of hours worked

Sequence Based Adaptive Approach

	Soft Constraint
10	Maximum total number of assignments for all Mondays, Tuesdays, Wednesdays, etc
11	Maximum number of a certain shift type worked (e.g. maximum seven night shifts for the planning period)
12	Maximum number of a certain shift type worked per week (same as above but for each individual week)
13	Valid number of consecutive shifts of the same type
14	Free days after night shifts
15	Complete weekends (i.e. shifts on both Saturday and Sunday, or no shift over the weekend)
16	No night shifts before free weekends

Sequence Based Adaptive Approach

	Soft Constraint
17	Identical shift types during the weekend
18	Maximum number of consecutive working weekends
19	Maximum number of working weekends in four weeks
20	Maximum number of working bank holidays
21	Shift type successions (e.g. Is shift type A allowed to follow B the next day, etc)
22	Requested days on or off
23	Requested shifts on or off
24	Tutorship (employee X present when employee Y is working)
25	Working separately (employee X not present when employee Y is working)

Sequence Based Adaptive Approach

- In literature
  - Constraints are usually grouped as *hard* and *soft* constraints in most work
  - A few work consider feasible patterns (or work-stretch) of one week, or two weeks, associated with pre-assigned costs

Sequence Based Adaptive Approach

- In our work
  - Problems are firstly modelled by categorising constraints into 3 types, *Sequence*, *Schedule* and *Roster* related
  - Penalties of *sequences*, *schedules* and *roster* are calculated by corresponding constraints

Sequences	A series of shifts for nurses i.e. EEELL
Schedules	Ordered list of sequences and days off
Roster	Overall solution consisting of same length schedules of the scheduling period

Sequence Based Adaptive Approach

December	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	Pen
1A	E	E	E			N	N	N	N			E	E	E	L	L						L	L	L					10
1A			N	N	N			N	N			D	D	D		L	L	L	L			D	D				D	D	30
1A					N	N	N			L	L	L	D	D			D	D	D			N	N			L	L	L	15
1A			D	N	N	N	N			D	D				E	E	E	E	L			D	D	N			D	D	30
1A	D	D				E	E	E	E	D	D				L	L	L	L	L					E	E	E	D	D	20

Night shift before a free weekend (10).

Too few consecutive night shifts (10).

Too few consecutive free days (10).

Too few consecutive late shifts (10).

Too few consecutive day shifts (10).

Day shift after a late shift (5).

Too few consecutive night shifts (10).

Too many consecutive working days (10).

Too many consecutive late shifts (10).

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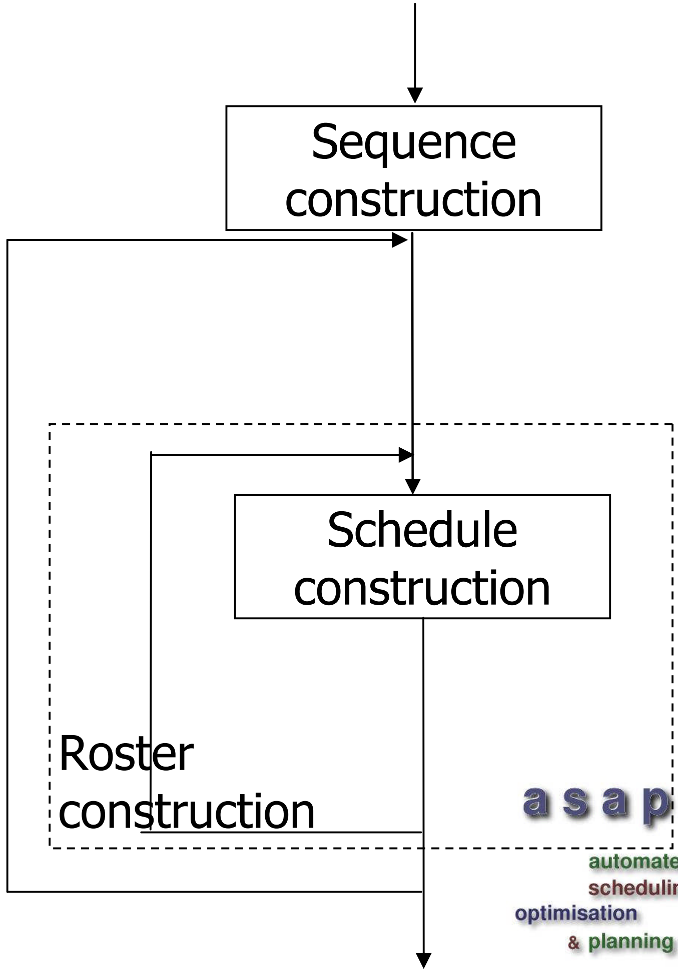
Sequence Based Adaptive Approach

- Two stage approach
  - Construct high quality sequences for each nurse considering only *sequence* related constraints
  - Construct schedules and roster considering only *schedule* and *roster* related constraints



Sequence Based Adaptive Approach

	Hard Constraints	Type
1	Shifts which require certain skills can only be taken by (or assigned to) nurses who have those skills	sequence
2	The shift coverage requirements must be fulfilled	roster

	Soft Constraints	Type
1	Minimum rest time between shifts	sequence
2	Alternative skill (if a nurse is able to cover a shift but prefers not to as it does not require his/her primary skill)	sequence
3	Maximum number of shift assignments	schedule
4	Maximum number of consecutive working days	sequence
5	Minimum number of consecutive working days	sequence
…	…	

Sequence Based Adaptive Approach

- Decomposition on complex problems
  - Our previous work decompose the problem by considering sub-groups of nurses
  - This work decompose the problem in a different way
    - Constraints are dealt with in different stages
  - Overall aim is to reduce the complexity of the problem and size of the search space

Sequence Based Adaptive Approach

- Stage I: sequence construction for each nurse
  - Construct sequences by considering
    - *sequence* related hard constraints
    - *sequence* related soft constraints
    - length of up to 5
  - Best 50 are ranked

Shift Sequences	Penalty	Comment
E, E, E	0	
D, D, E, E, E	5	E not preferred to follow D.
L, L, L, D, D	5	D not allo preferred wed to follow L.
N, N	10	Two N's not preferred.
E, D, D	10	One E not preferred.

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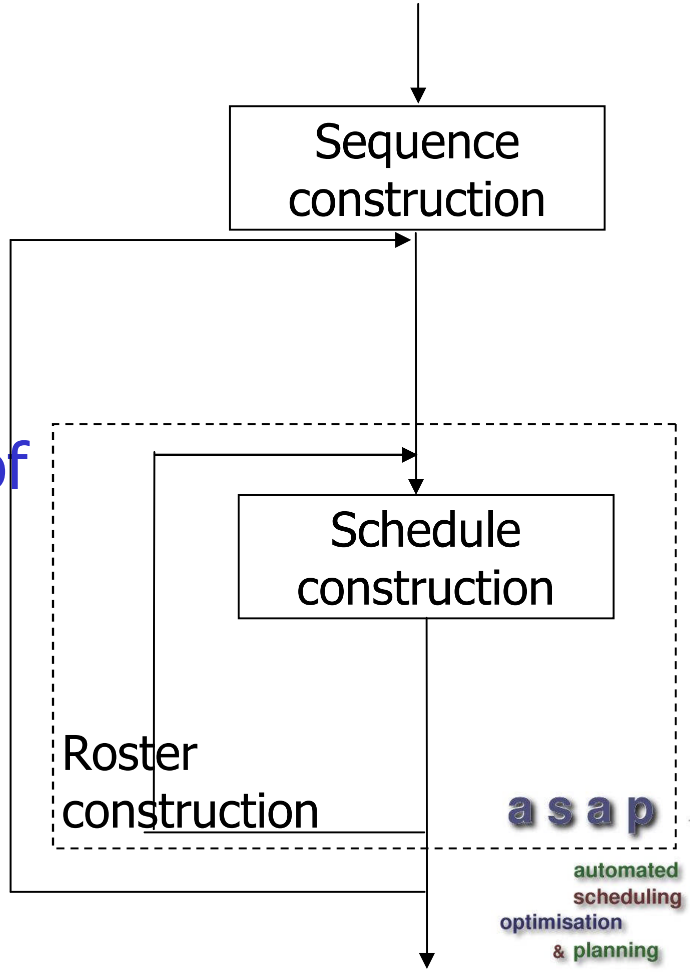
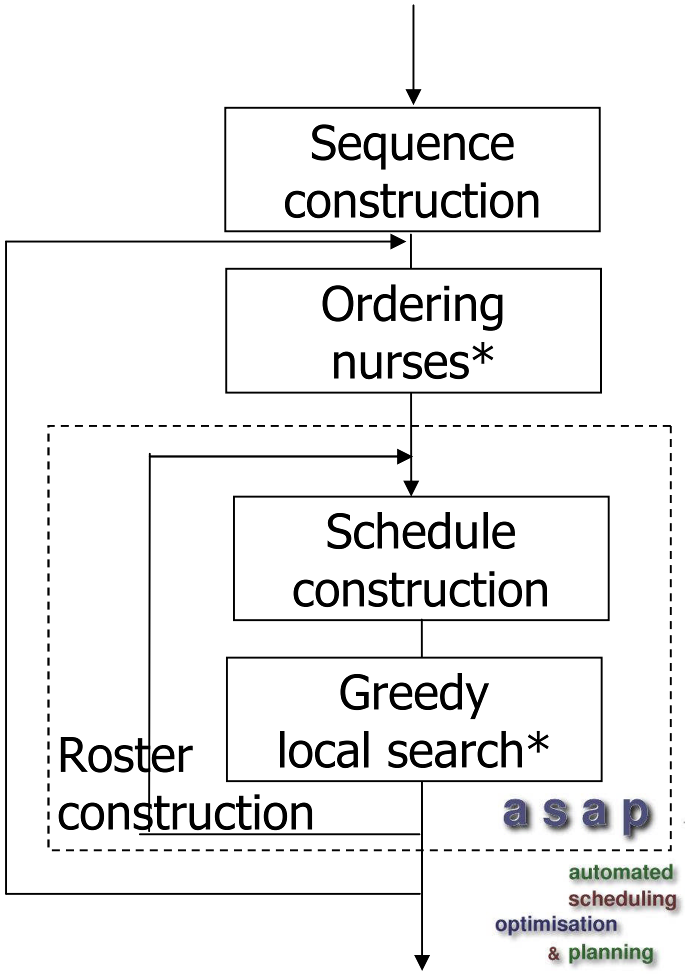
Sequence Based Adaptive Approach

- Stage II: schedule and roster construction
  - Build schedules based on sequences for each nurse considering only *schedule* related constraints
  - Iteratively combine schedules of nurses to construct rosters considering only *roster* related constraints



Sequence Based Adaptive Approach

- Stage II: schedule and roster construction
  - Hybridisations of different techniques are possible with this simple and fast approach
    - Greedy local search: improvement during and after roster construction
    - Adaptive ordering: nurses with worse schedules are scheduled first in the next iteration



Sequence Based Adaptive Approach

- Experiment results
  - Without adaptive ordering
    - Greedy local search does not make much improvement
  - With adaptive ordering
    - Improvement by greedy local search around 4%

Sequence Based Adaptive Approach

- Conclusions
  - Problem formulation to decompose the constraints of different types → smaller search space
  - Simple and fast technique, usually take a few seconds to 2 minutes for problems up to 46 nurses and more than four weeks
  - Easily hybridised with other techniques for further improvement; Relatively straightforward and highly effective
  - Superior to the existing algorithm in a commercial software

Other Recent Work - VDS

- Variable depth search (VDS)
  - Basic VNS
    - Move single shift to another nurse
    - Swap two shifts between nurses
  - Extend basic VNS
    - include neighbour solutions which differ by an exchange of a **block** of shifts between two nurses

E.K. Burke, T. Curtois, R. Qu and G. Vanden Berghe. A Time Pre-defined Variable Depth Search for Nurse Rostering. Technical Report NOTTCS-TR-2007-6, School of Computer Science, University of Nottingham. Under review at Journal of Heuristics, 2007.

Other Recent Work - VDS

- Basic VNS

a

December	1				
	4	5	6	7	8
	M	T	W	T	F
G		L			
H		N		D	D
A	DH	DH	DH	DH	

December	1				
	4	5	6	7	8
	M	T	W	T	F
G		N			
H		L		D	D
A	DH	DH	DH	DH	

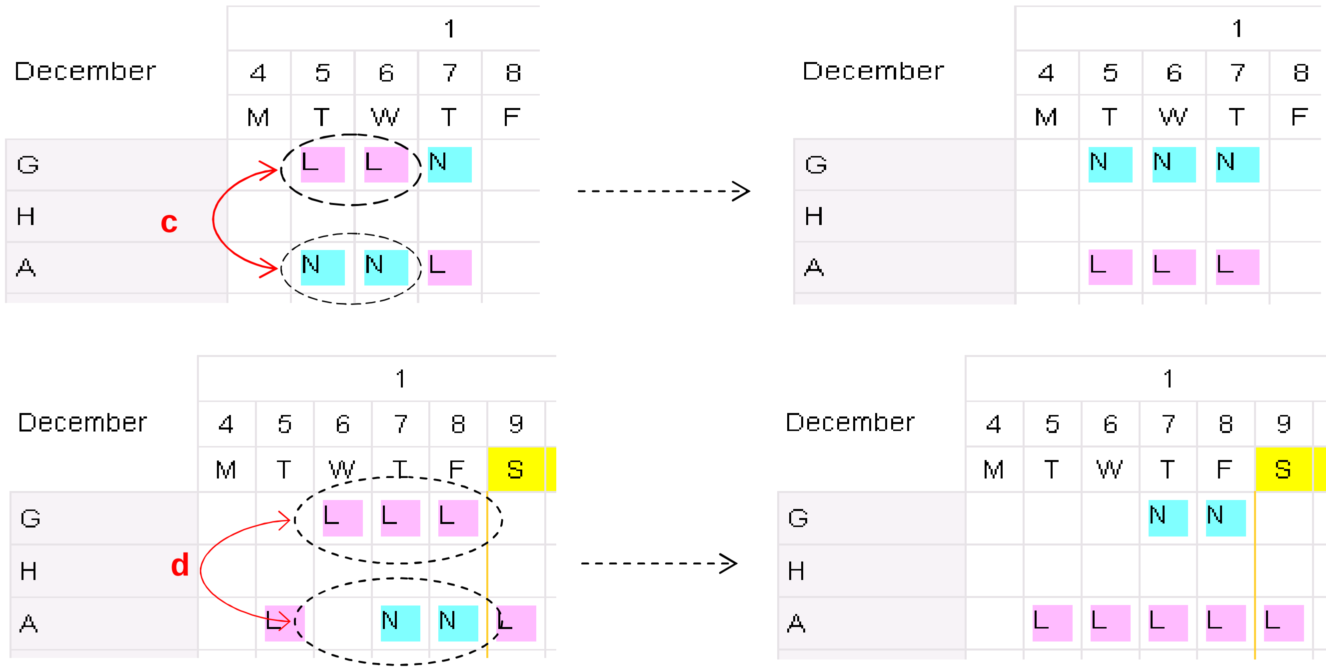
b

December	1				
	4	5	6	7	8
	M	T	W	T	F
G					
H		L		D	D
A	DH	DH	DH	DH	

December	1				
	4	5	6	7	8
	M	T	W	T	F
G		L			
H				D	D
A	DH	DH	DH	DH	

Other Recent Work - VDS

- Extend basic VNS



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Other Recent Work - VDS

- Form chains of moves/swaps
- Each neighbour in the neighbourhood for the best solution found so far is a possible starting point for the chain of moves
- If at any point a new best solution is found, set it as the current solution and look for another set of moves
- Algorithm terminates when no untried starting points in the current best solution

Other Recent Work

- Hybrid algorithm where integer programming is integrated with a variable neighbourhood search
- Investigations on multi-objective nurse rostering problems
- A scatter search on nurse rostering

* all papers can be downloaded from
  http://www.cs.nott.ac.uk/~rxq/publications.htm

References

- [AIC04] Aickelin U. and Dowsland K. A. An Indirect Genetic Algorithm for a Nurse Scheduling Problem. Journal of Computers & Operations Research, 31(5):761-778, 2004
- [AIC07] Aickelin U. and Li J. An Estimation of Distribution Algorithm for Nurse Scheduling. Annals of Operations Research, 155(1):289-309, 2007
- [AIC07a] Aickelin U., Burke E. K., and Li J. An Estimation of Distribution Algorithm with Intelligent Local Search for Rule-based Nurse Rostering. Journal of the Operational Research Society, 2007.
- [BUR06] Burke E. K., De Causmaecker P., Petrovic S., and Vanden Berghe G. Metaheuristics for Handling Time Interval Coverage Constraints in Nurse Scheduling. Applied Artificial Intelligence, 20(9):743-766, October 2006
- [BUR03] Burke E. K., Kendall G., and Soubeiga E. A Tabu-Search Hyperheuristic for Timetabling and Rostering. Journal of Heuristics, 9(6):451-470, Dec 2003

References

- [BUR07] Burke E. K., Curtois T. E., Post G., **Qu R.**, and Veltman B. A Hybrid Heuristic Ordering and Variable Neighbourhood Search for the Nurse Rostering Problem. European Journal of Operational Research, 2007.
- [BBCQ08] P. Brucker, E.K. Burke, T. Curtois, **R. Qu**. Adaptive Construction of Nurse Schedules: A Shift Sequence Based Approach. accepted by EJOR, under 2nd review.
- [BUR04] Burke E. K., De Causmaecker P., Vanden Berghe G., and Van Landeghem H. The State of the Art of Nurse Rostering. Journal of Scheduling, 7(6):441-499, Nov-Dec 2004
- [BUR07] Burke E.K., Li J. and **Qu R.** (2007): A Hybrid Model of Integer Programming and Variable Neighbourhood Search for Highly-constrainted Nurses Rostering Problems. (under review) European Journal of Operational Research.
- [BUR07a] Burke E.K., Li J. and Qu R. Pareto-Based Optimization for Multi-objective Nurse Scheduling, Technical Report, University of Nottingham, 2007

References

- [BUR07b] Burke E.K., Curtois T., Qu R. and Vanden Berghe G. A Time Pre-defined Variable Depth Search for Nurse Rostering, Technical Report, University of Nottingham, 2007
- [BUR07c] Burke E.K., Curtois T., Qu R. and Vanden Berghe G. A Scatter Search for the Nurse Rostering Problem. Technical Report, University of Nottingham, 2007
- [BRU05] Brucker P., Qu R, Burke E.K. and Post G. A Decomposition, Construction and Post-processing Approach for a Specific Nurse Rostering Problem. MISTA'05, 397-406. New York, USA, Jul 2005
- [DOW98] Dowsland K. Nurse Scheduling with Tabu Search and Strategic Oscillation. European Journal of Operational Research, 106: 393-407, 1998
- [OUS07] Oussedik S. (ILOG) . ODMS: A System for Developing Interactive. Optimization-Based Planning and Scheduling. Applications. MISTA’07, plenary talk, Paris, France, August, 2007.
- [OZC07] Ozcan E. Memes, Self-Generation and Nurse Rostering. PATAT’06 selected volume, LNCS 3687, 87-106

PART II: Other Ongoing and Previous Work

- Timetabling problems
  - Description & formulation
  - Brief literature review
  - Benchmarks
  - Approaches

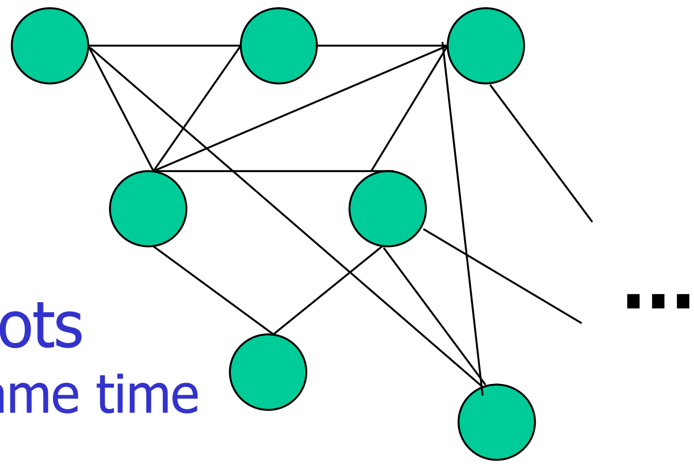
- Other ongoing work

Timetabling Problems

- Assigning a set of exams into limited timeslots satisfying a set of constraints
  - Hard constraints: cannot be violated
  - Soft constraints: desired
  - Quality of solutions: objective function

Timetabling Problems

- Important activities in all universities
- A general timetabling problem
  - A set of events
  - A set of timeslots
  - A set of rooms
  - Schedule the events to timeslots
    - No events for students at the same time
    - Spread students' events
    - …



Timetabling Algorithms

- Graph heuristics, constraint based techniques
- Meta-heuristics, multi-criteria techniques
- New trends
  - hybrid techniques, hyper-heuristics, VNS, ILS, GRASP, adaptive techniques, etc

R. Qu, E. K. Burke, B. McCollum, L.T.G. Merlot, and S.Y. Lee. A Survey of Search Methodologies and Automated System Development for Examination Timetabling. To appear at ***Journal of Scheduling***, 2008. DOI: 10.1007 / s10951-008-0060-1

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Benchmark Timetabling Problems

- Carter, Laporte & Lee (1996): 11 real world exam timetabling problems
    - Hard constraints: conflicts between exams
    - Soft constraints: spread out exams over slots
    - Objective function: C (t) = ($\sum_{s=0}^{4} w_s \times N_s$) / S

- State-of-the-art approaches employing different "fine-tuned" techniques
    - Carter, Laporte and Lee (1996), Di Gaspero and Schaerf (2000), Caramia et al (2001), Merlot et al (2002), Casey and Thompson (2002), Burke and Newall (2002), etc

Benchmark Timetabling Problems

- Benchmark Course timetabling
  - Metaheuristics network: 11 benchmark course timetabling problems
  - The same problem format/structure as the International Competition on Timetabling
- The 2nd International Competition on Timetabling
  - http://www.cs.qub.ac.uk/itc2007/
  - Exam, course timetabling problems

A Graph Based Hyper-heuristic

- Hyper-heuristics
  - Heuristics that choose heuristics
    - High level heuristics: Meta-heuristics, Choice function, Ant Algorithm, CBR, Fuzzy ES, etc
    - Low level heuristics: different moving strategies, constructive heuristics, etc

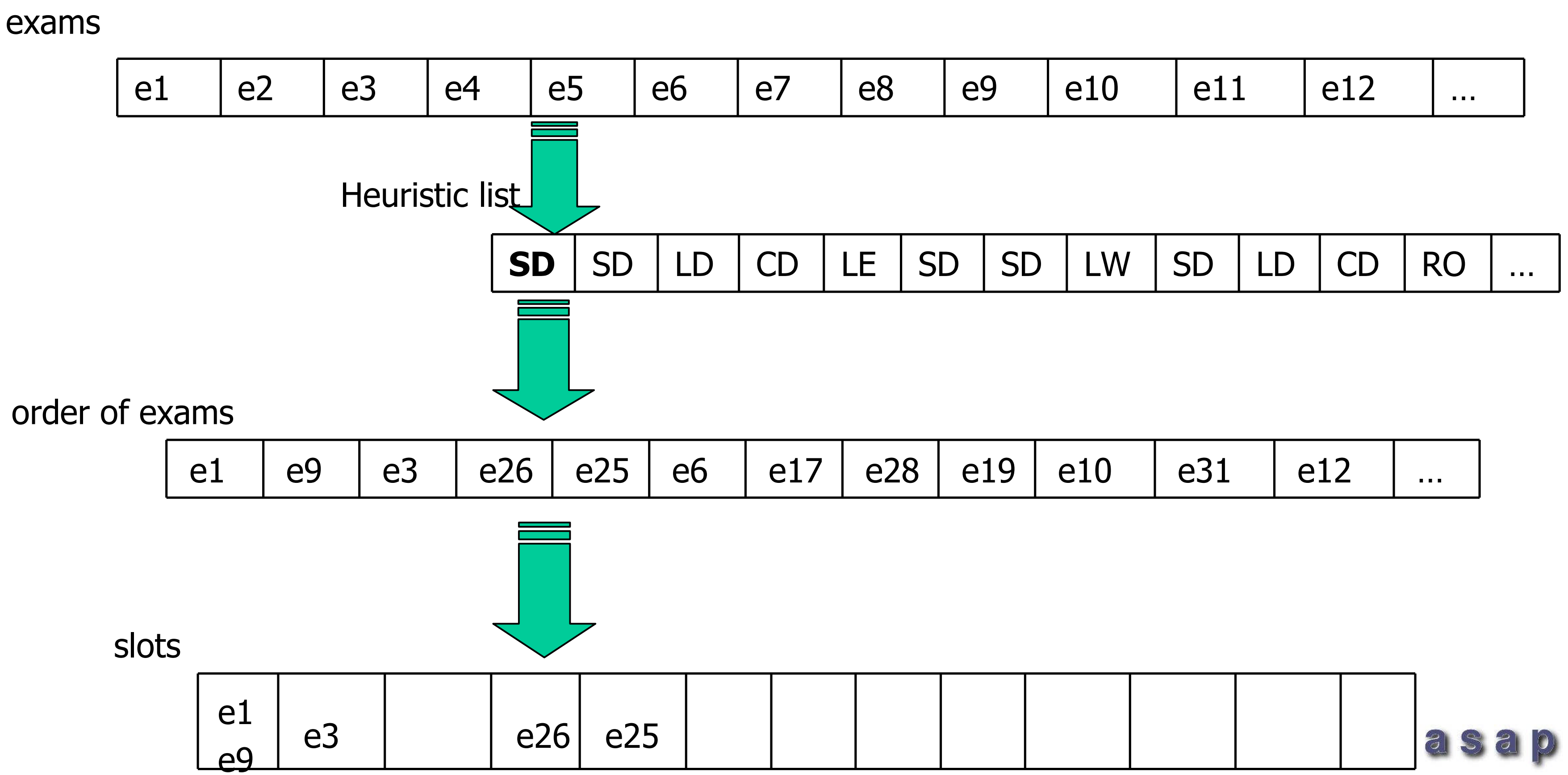
- Aim of hyper-heuristic
  - Exploring general techniques for wider problems
  - Searching techniques not look into domain knowledge

E. K. Burke, B. McCollum, A. Meisels, S. Petrovic and R. Qu. A Graph-Based Hyper Heuristic for Timetabling Problems. ***European Journal of Operational Research*** (EJOR), 176: 177-192, 2007

A Graph Based Hyper-heuristic

- High level heuristics that search for lists of graph heuristics to construct solutions
  - Low level graph heuristics: order events by how difficult to assign them
    - Saturation Degree: least available slots
    - Colour Degree: most conflicted with those scheduled
    - Largest Degree: most conflicted with the others
    - Largest Weighted Degree: students involved
    - Largest Enrolment: students enrolled
  - Random Ordering: brings randomness

A Graph Based Hyper-heuristic



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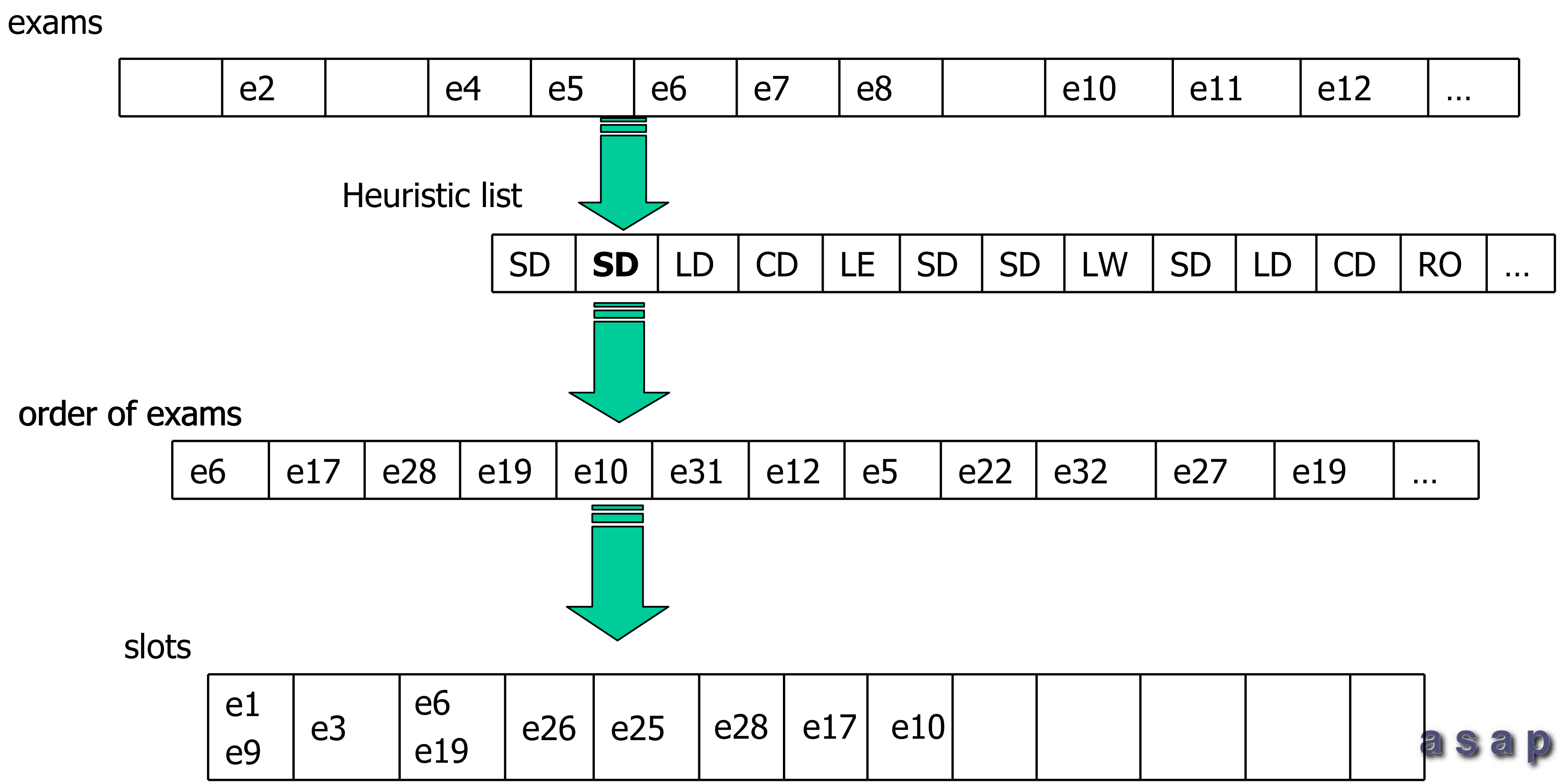
A Graph Based Hyper-heuristic



A Graph Based Hyper-heuristic

exams

	e2		e4	e5		e7	e8			e11	e12	...

Heuristic list

SD	SD	**LD**	CD	LE	SD	SD	LW	SD	LD	CD	RO	...

order of exams

e5	e32	e19	e22	e13	e31	e12	e7	e2	e15	e27	e12	...

slots

e1 e9	e3	e6 e19	e26	e25	e28	e17	e10	e5 e13	e32 e19	e13		

A Graph Based Hyper-heuristic

- Graph based Hyper-heuristics (GHH) Framework
  - Search space: permutations of graph heuristics, rather than actual solutions
  - Moving operator: randomly change two heuristics in the heuristic list
  - Objective function: map from heuristic lists to penalty of timetables constructed
  - "Walks" are allowed

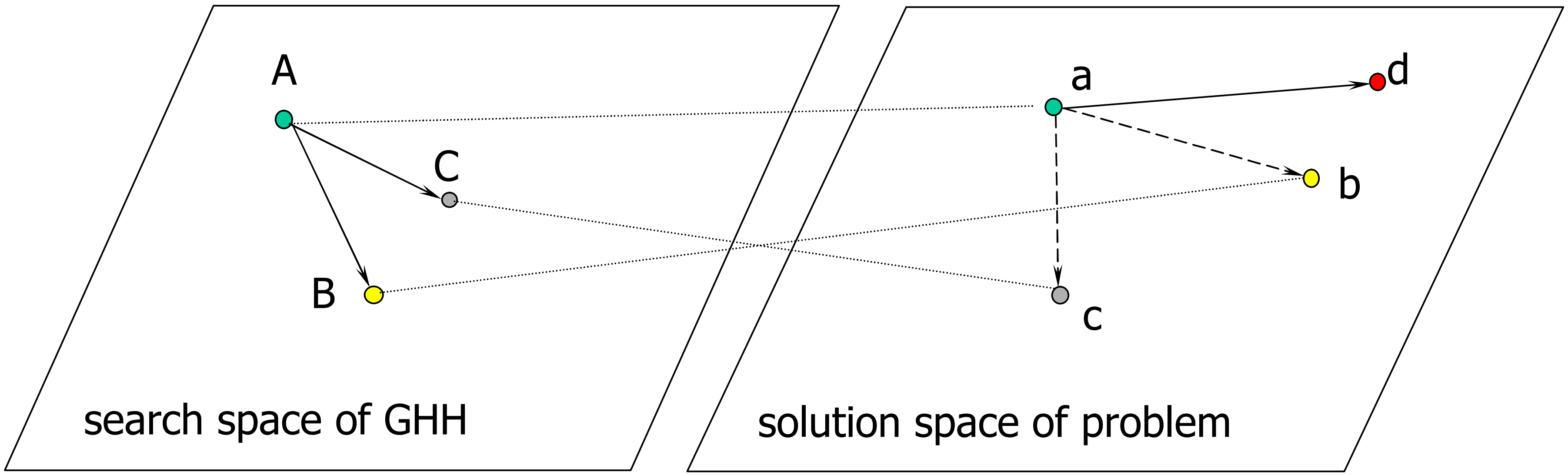
- Overall objective
  - Role of different high level heuristics (ILS, TS, SDM, VNS)
  - Characteristics of *heuristic* search space

R. Qu and E. K. Burke. Hybridisations within a Graph Based Hyper-heuristic Framework for University Timetabling Problems. Accepted by ***Journal of Operational Research Society***, 2008

A Graph Based Hyper-heuristic

- Observation A
  - Results are competitive to state-of-the-art approaches
- Observation B
  - Different high level heuristics (SD, TS, ILS, VNS)
  - Iterated techniques (ILS, VNS) are slightly better
  - ILS and VNS performed similar with same total number of evaluations

A Graph Based Hyper-heuristic



- Two search spaces
  - Search space of high level heuristics: permutations of low level heuristics
  - Solution space of problem: actual solutions

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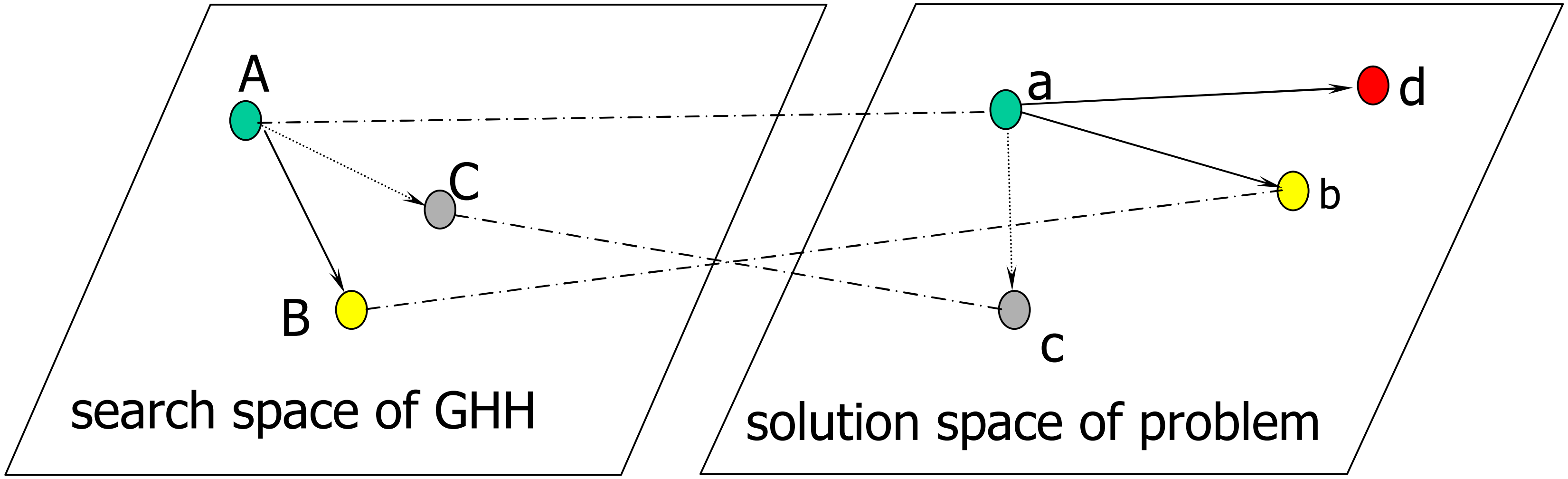
A Graph Based Hyper-heuristic



- With one move
  - Local search approaches — One bit different
  - Graph based hyper-heuristics — One part different (from the different part of the heuristic list)

A Graph Based Hyper-heuristic

- Local search based algorithms
  - Make moves within a limited search areas
  - Easily stuck to local optima: different mechanisms developed
  - Chaotic attractor: a limited portion of search space

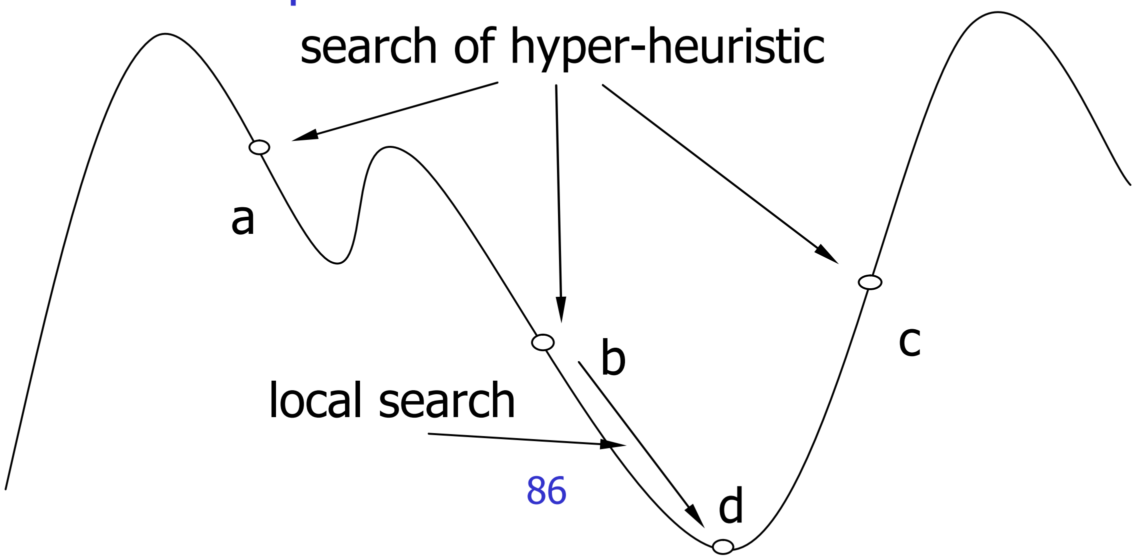
- GHH
  - Change the way of building the solutions at a high level
  - Local search move in search space of heuristic maps to solutions far from each other in solution space
  - Key feature: coverage of the solution space

A Graph Based Hyper-heuristic

- Landscape of high level heuristic space
  - More likely to have “walks” or plateau
    - Only a subset of the neighbourhoods can be evaluated before a move can be made
    - Not mapped to all solutions in solution space (hypothesis)
    - Size of neighbourhoods is very large
    - Computational time: limited number of evaluations within a limited time

A Graph Based Hyper-heuristic

- Hypothesis: search is upon heuristics, not solutions – not all the solutions in solution space are reachable
- Hybridisation with greedy local search
  - Diversification vs. intensification
  - Coverage of solution space



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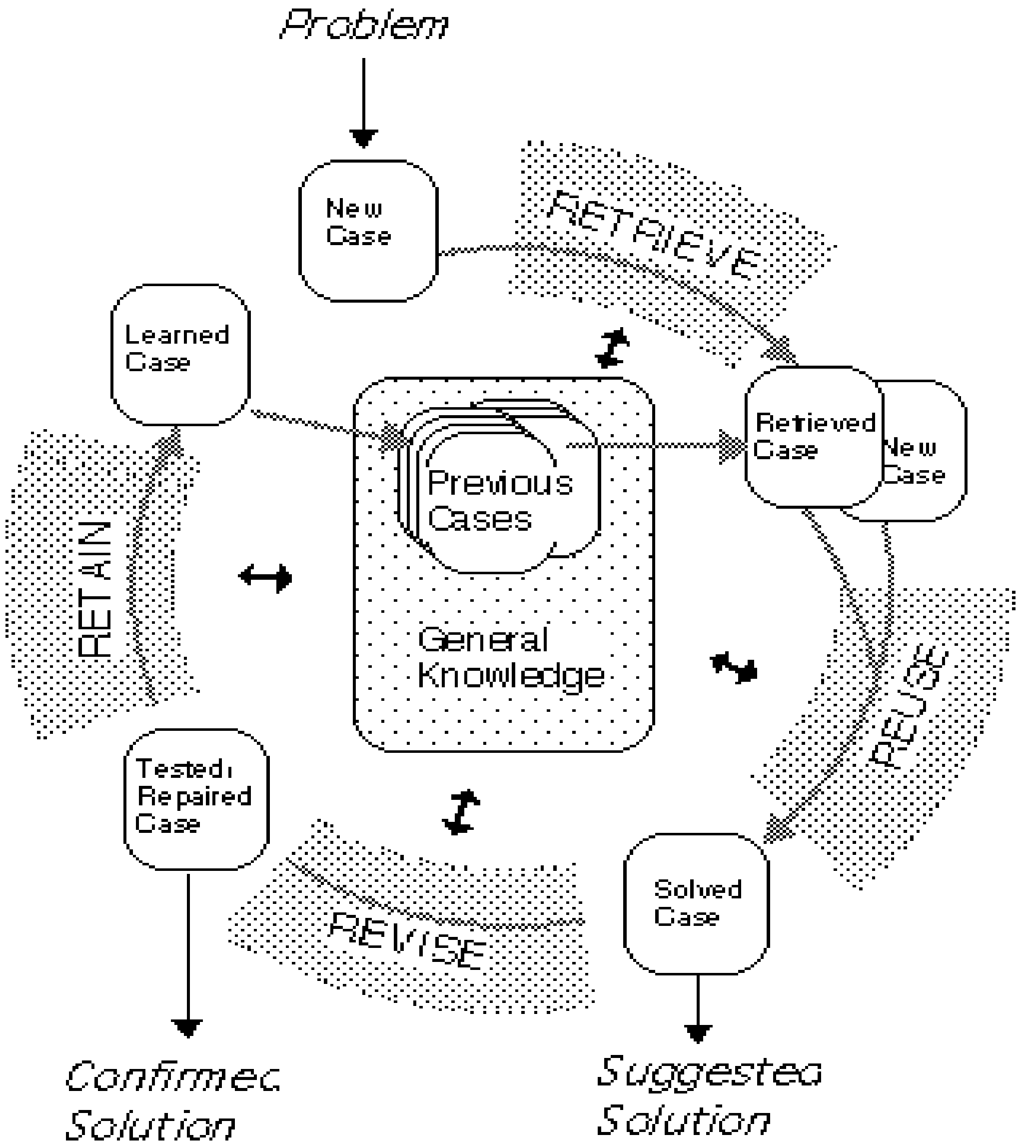
A Case Based Heuristic Selection

- Extract/record knowledge of heuristic selection during problem solving
- Learn to select good heuristics for particular situations
- Suggesting good heuristics in different situations
- Obtained good results on simulated problems, test on real-world problems

E. Burke, S. Petrovic, R. Qu, Case Based Heuristic Selection for Timetabling Problems. ***Journal of Scheduling***, 9: 115-132, 2006

A Case Based Heuristic Selection



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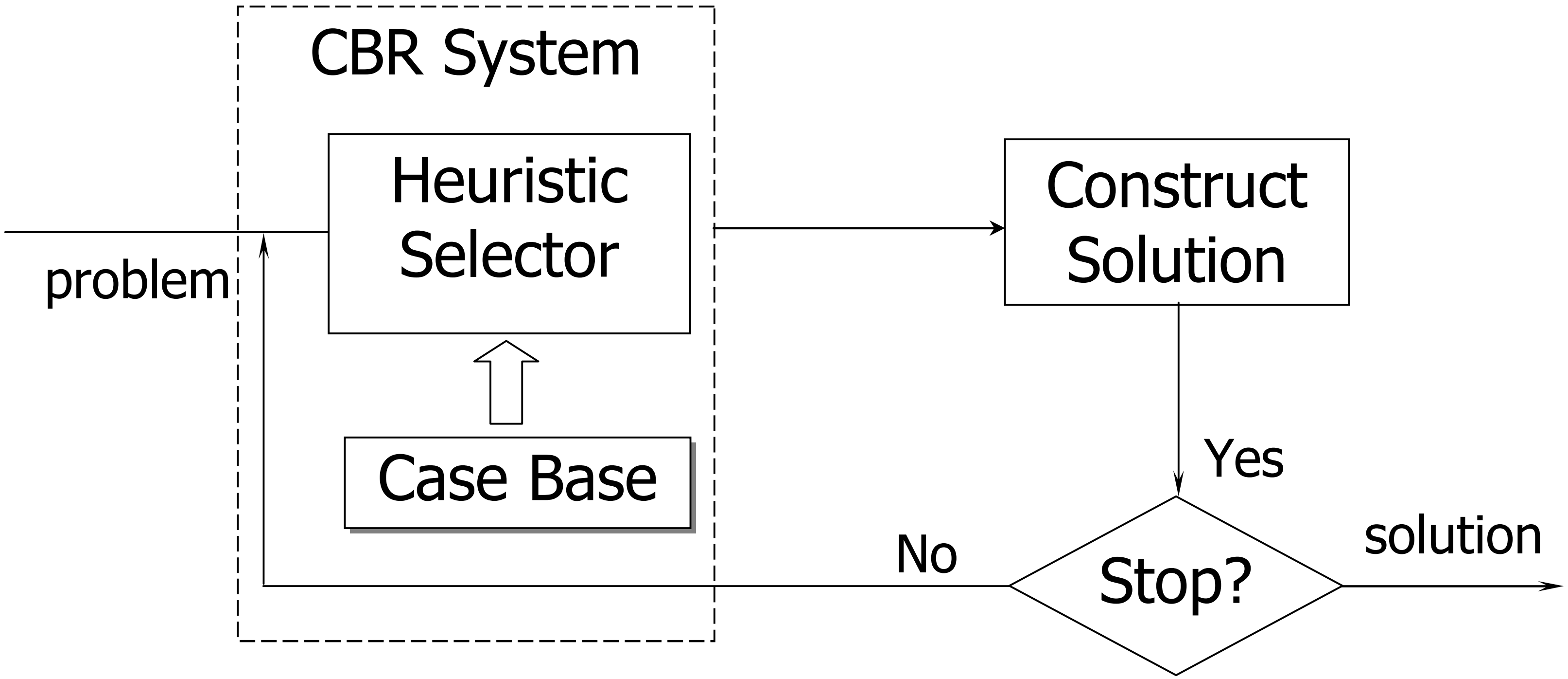
A Case Based Heuristic Selection

- In CBR system
  - Cases: problem description and solutions
  - Case base: collection of previously solved problems
  - Similarity measure: calculate how similar two cases are
  - Retrieval: find from the case base the most similar case
  - Adaptation: utilise the retrieved solution for new problem

A Case Based Heuristic Selection



A Case Based Heuristic Selection

- Using knowledge/experience to solve similar problems
  - Reuse previous good solutions for similar problems
  - Reuse methodology/heuristics in similar situations

- Assumption: similar problems, similar solutions

A Case Based Heuristic Selection

exams

	e2		e4	e5	e6	e7	e8		e10	e11	e12	...

heuristic list

SD	**SD**	LD	CD	LE	SD	SD	LW	SD	LD	CD	RO	...

order of exams

e6	e17	e28	e19	e10	e31	e12	e5	e22	e32	e27	e19	...

slots

e1 e9	e3	e6 e19	e26	e25	e28	e17	e10					

Case Based Heuristic Selection

- Basic idea
  - CBR suggests good constructive heuristics that worked well in previous similar situations during problem solving employing the knowledge stored in system
- Case base
  - Timetabling problems and their partial solutions during problem solving
  - best heuristics for that situations

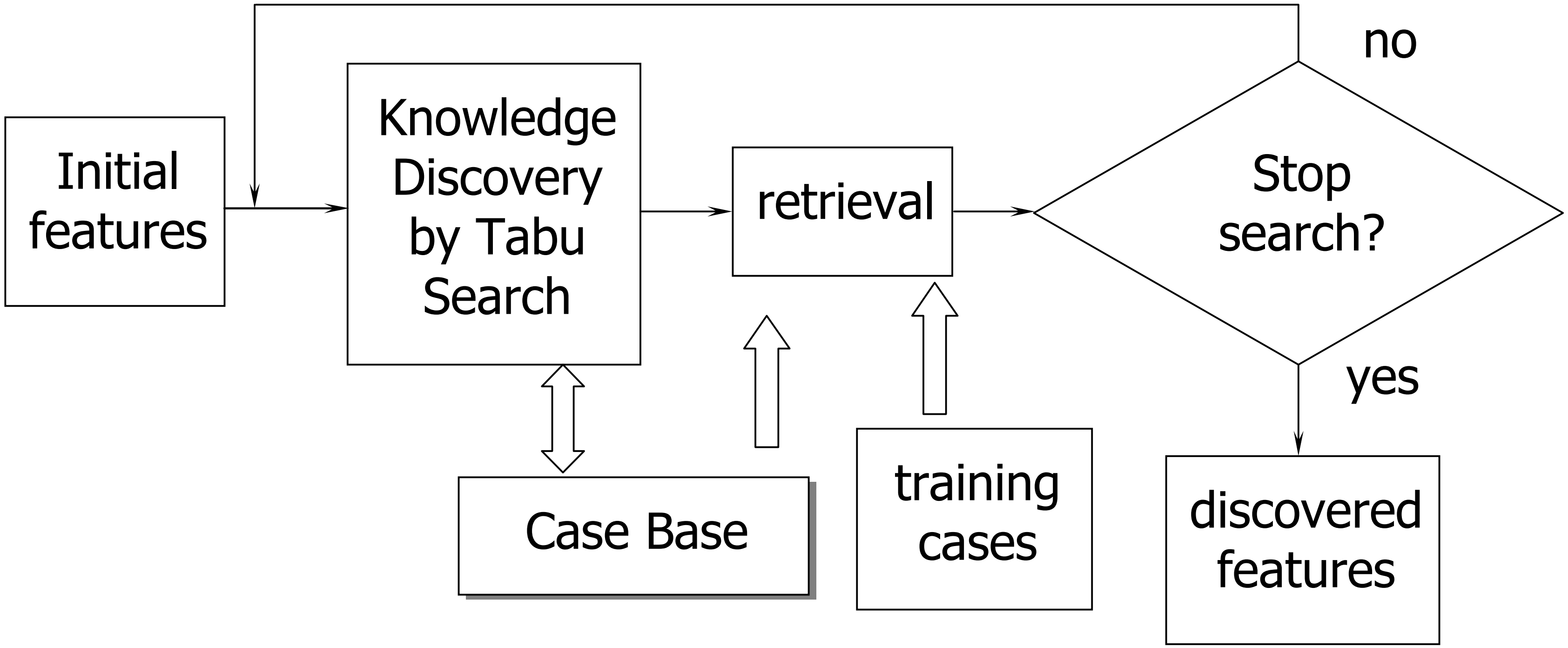
Case Based Heuristic Selection

- Similarity measure
  - nearest neighbourhood approach
- Key issue of meaningful comparison between two problem solving situations
  - features describe the characteristics of problem and partial solution (cases)

Case Based Heuristic Selection

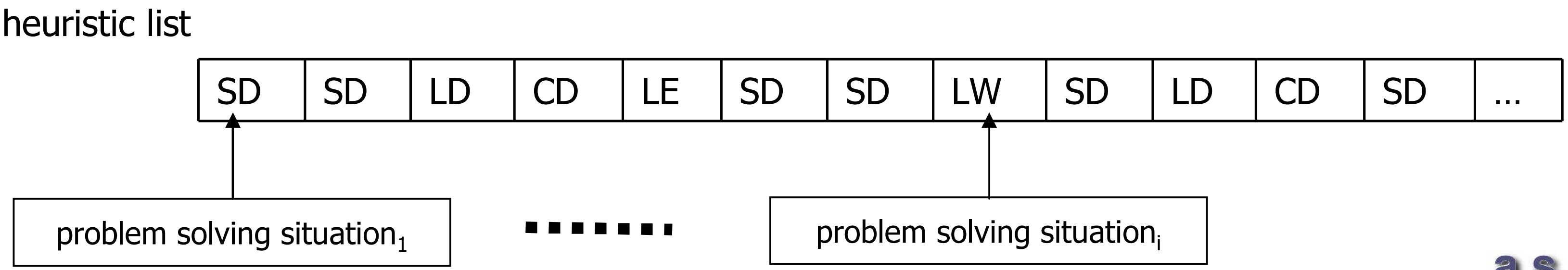
- A Tabu Search algorithm has been used to do the training on the feature list
  - Search for most relevant features by which cases (problems and problem solving situations) can be compared concerning the most appropriate heuristics used
- Training process on cases in case base
  - Refine the cases stored in case base
  - Only cases that may make contribution to problem solving are retained

Case Based Heuristic Selection



Case Based Heuristic Selection

- Training cases
  - Run a hyper heuristic on a set of timetabling problems
  - Get the heuristic lists that generate the best solutions
  - Keep record of problem solving situations + heuristics employed at particular situations



Case Based Heuristic Selection

- Data set: real world benchmark problems (Carter et al 1996)

- 2 case bases built
  - $CBRhs_{random}$ – from random generated problems
  - $CBRhs_{real}$ – from 4 out of 11 real problems

Case Based Heuristic Selection

- CBRhsrandom (from random generated problems)
  - Incapable of solving most of real problems
- CBRhsreal
  - Capable of getting results close/within state-of-the-art/fine-tuned approaches
  - Need more knowledge of real problem solving

Adaptive Decomposition

- Squeaky Wheel Optimisation (Joslin and Clements, 1999)

In parallel

Solution construction

- Greedy algorithm
- Analyse of trouble elements

Adjustment of problematic elements in previous problem solving

- Priorities of troublesome elements increased in the next iteration in greedy algorithm

Adaptive Decomposition

- Decomposition
  - Basic idea: divide and conquer
  - Benefits
    - Smaller search space
    - Problem complexity significantly reduced
    - (Near-)optimal solutions for sub-problems

Adaptive Decomposition

- Decomposition
  - Basic idea: divide and conquer
  - Problems
    - Problem specific
    - How to combine the sub-solutions?
      - Global constraints not considered in sub-problems
      - Combined solutions not even feasible
      - Lose of optimality

Adaptive Decomposition

- Adaptive ordering on timetabling [BN04]
  - Order exams by how difficult they were scheduled
  - Increase priorities of exams in ordering
  - Difficult exams
    - Contribute costs > threshold
    - Cannot be scheduled

- Efficient on benchmark exam timetabling problems

Adaptive Decomposition

- Adaptive ordering on timetabling [BN04]
- Parameters considered
  - Different initial ordering
    - LD, SD, random
  - Increment of priorities of exams
    - 1, exponential, random (N)
  - Threshold
    - If priorities of exams need to be adjusted
    - Gradually changed

Adaptive Decomposition

- Based on adaptive ordering
  - Reduce search space
    - Assignment: t^e (t: timeslots)
    - Ordering: e! (e: exams)
  - Reduce parameters

R. Qu and E.K. Burke. Adaptive Decomposition and Construction for Examination Timetabling Problems. ***MISTA'07***, 418-425, Aug, 2007, Paris, France.

Adaptive Decomposition

- Initial order by Saturation Degree
  - How many valid timeslots left in the timetable

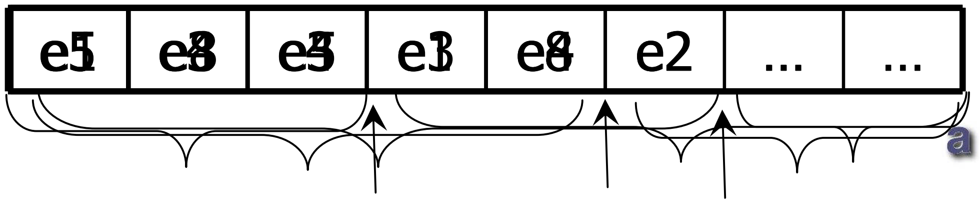
e1	e8	e5	e3	e4	e2	…	…

- Adaptively decompose the exams
  - *Difficult set*
    - Iteratively include difficult exams
    - Iteratively adjust size of *difficult set*
  - *Easy set*

e1	e8	e5	e3	e4	e2	…	…

Adaptive Decomposition

- *Difficult set*
  - Re-order exams in the *difficult set*, fix *easy set*
  - Construct timetable using the ordered *difficult set* & *easy set*
  - If feasible timetable generated
    - Expand *difficult set* to include more potential exams
  - Else
    - Move forward the exam causing infeasibility
    - Adjust set size



Adaptive Decomposition

- *Easy set*
  - Re-order exams in the *easy set,* fix *difficult set*
  - Construct timetable using the ordered *difficult set* & *easy set*

e5	e4	e3	e1	e8	e2	…	…

Adaptive Decomposition

	car91	car92	ear83	hec92	kfu93	lse91	sta83	tre92	ute92	uta93	yor83
distinct	**5.4**	4.53	36.8	11. 5	14.8	11.2	157.4	8.83	26.9	**3.5**	**42**
size %	32	23	38	61	23	14	15	46	37	22	44
cost %	66	62	60	71	83	58	12	79	57	66	47

overlap	5. 5	**4.5**	**36.15**	**11.4**	**14.7**	**10.9**	**157.2**	**8.79**	**26.7**	3.6	42.2
size %	32	23	38	61	23	14	15	46	37	22	44
cost %	65	62	59	70	78	58	23	79	54	66	46

adapt order	5-5.6	4.3-4.7	36.16-38.6	11.6-12.8	15-16.5	11-12.5	161.9-170.5	8.4-9	27.4-29.7	3.4-3.6	40.8-43

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Adaptive Decomposition

- A simple and general approach for exam timetabling problems
  - Could be applicable to other problems
- Adaptively detect *difficult* elements in the problem
- Adaptively decompose problems
- Quick and constructive

Finally …

- Ongoing projects
  - Search space study on hyper-heuristics
    - Fundamental study of heuristic space*
  - Modelling on complex real world staff scheduling problems
    - General staff scheduling problems in super market, call center, etc
    - Constraints vary depend on problem scenarios

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E. Burke, G. Ochoa, and R. Qu. Constructive Hyper-heuristic Landscapes: Definition and Analysis. Under review Annuals of OR, 2008.

Finally …

- Ongoing projects
  - Constraint programming on vehicle routing problems
    - Service a number of customers with a fleet of vehicles
    - Multi- objectives: minimise distance & number of vehicles
    - Special case of VRP: travelling salesman problems (TSP)
  - Stochastic network optimisation problems
    - Network routing optimisation
    - Quality of service (QoS)

References

- E. K. Burke, A. Meisels, S. Petrovic and R. Qu A Graph-based Hyper-Heuristic for Exam Timetabling Problems. European Journal of Operational Research, 176: 177-192, 2007.
- E. Burke, M. Dror, S. Petrovic, R. Qu, Hybrid Graph Heuristics within a Hyper-heuristic Approach to Exam Timetabling Problems. B.L. Golden, S. Raghavan and E.A. Wasil (eds.). The Next Wave in Computing, Optimization, and Decision Technologies. Kluwer Academic Publishers. Jan 2005.
- R. Qu and E. K. Burke. Hybridisations within a Graph Based Hyper-heuristic Framework for University Timetabling Problems. Accepted by Journal of Operational Research Society (JORS), 2008.
- E. Burke, G. Ochoa, and R. Qu. Constructive Hyper-heuristic Landscapes: Definition and Analysis. Under review at Annuals of OR, 2008.

References

- E. Burke, S. Petrovic, R. Qu, Case Based Heuristic Selection for Timetabling Problems. Journal of Scheduling, 9: 115-132, 2006 .
- Burke, E.K. and Newall, J.: Solving Examination Timetabling Problems through Adaptation of Heuristic Orderings. Annals of OR, 129:107-134, (2004).
- Joslin D.E. and Clements D.P.: “Squeaky Wheel" Optimization. Journal of Artificial Intelligence Research, 10: 353-373. 1999.
- Qu R., Burke E.K., McCollum B., Merlot L.T.G. and Lee S.Y.: A Survey of Search Methodologies and Automated Approaches for Examination Timetabling. To appear at Journal of Scheduling, 2008.

Questions and Discussions

- Nurse rostering research
- Related applications