School for Computer Science and Information Technology

Machines and their languages (G51MAL) Spring 2004

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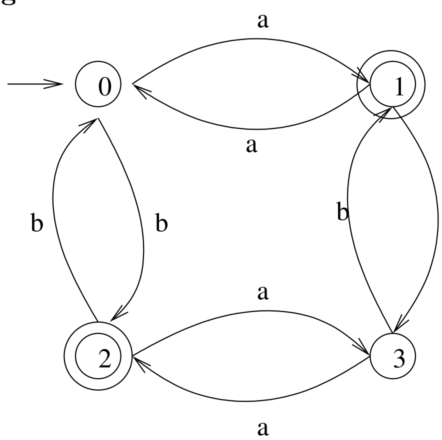
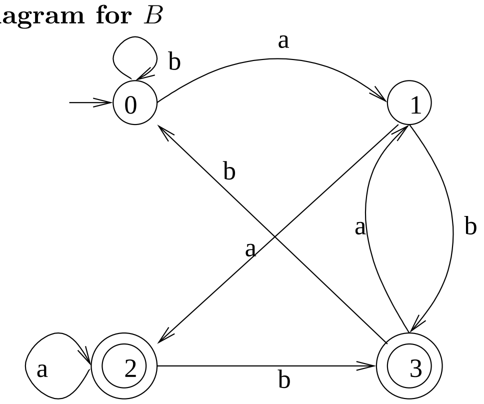
Solutions to 1st Coursework

10. 2. 2004

1. (a) **Transition diagram for** A



Transition diagram for B



(b) Determine which of the following words belong to $L(A), L(B)$:

i. $\epsilon \in L(A)$ $\epsilon \notin L(B)$

ii. $\mathrm{aabb} \in L(A)$ $\mathrm{aabb} \notin L(B)$

iii. $\mathrm{aaab} \notin L(A)$ $\mathrm{aaab} \in L(B)$

iv. $\mathrm{bbb} \in L(A)$ $\mathrm{bbb} \notin L(B)$

(c)

$$\begin{array}{rcll}
\hat{\delta}_A(0, \mathrm{bab}) & = & \hat{\delta}_A(\delta_A(0, \mathrm{b}), \mathrm{ab}) & \text{definition of } \hat{\delta} \\
& = & \hat{\delta}_A(2, \mathrm{ab}) & \text{because } \delta_A(0, \mathrm{b}) = 2 \\
& = & \hat{\delta}_A(\delta_A(2, \mathrm{a}), \mathrm{b}) & \text{definition of } \hat{\delta} \\
& = & \hat{\delta}_A(3, \mathrm{b}) & \text{because } \delta_A(2, \mathrm{a}) = 3 \\
& = & \hat{\delta}_A(\delta_A(3, \mathrm{b}), \epsilon) & \text{definition of } \hat{\delta} \\
& = & \hat{\delta}_A(1, \epsilon) & \text{because } \delta_A(3, \mathrm{b}) = 1 \\
& = & 1 & \text{definition of } \hat{\delta}
\end{array}$$

$$
\begin{array}{rcll}
\hat{\delta}_B(0, \mathrm{bab}) & = & \hat{\delta}_B(\delta_B(0, \mathrm{b}), \mathrm{ab}) & \text{definition of } \hat{\delta} \\
& = & \hat{\delta}_B(0, \mathrm{ab}) & \text{because } \delta_B(0, \mathrm{b}) = 0 \\
& = & \hat{\delta}_B(\delta_B(0, \mathrm{a}), \mathrm{b}) & \text{definition of } \hat{\delta} \\
& = & \hat{\delta}_B(1, \mathrm{b}) & \text{because } \delta_B(0, \mathrm{a}) = 1 \\
& = & \hat{\delta}_B(\delta_B(1, \mathrm{b}), \epsilon) & \text{definition of } \hat{\delta} \\
& = & \hat{\delta}_B(3, \epsilon) & \text{because } \delta_B(1, \mathrm{b}) = 3 \\
& = & 3 & \text{definition of } \hat{\delta}
\end{array}
$$

(d) $L(A)$ contains all the words such that the number of a's and b's have a different remainder when divided by 2. Writing $\#(x, w)$ for the number of x's in w we can express this by:

$$L(A) = \{w \mid \#(a, w) \not\equiv \#(b, w) \mathrm{mod} 2\}$$

$L(B)$ contains all words such that the letter before the last one is a.

$$L(B) = \{wax \mid w \in \{a, b\}^*, x \in \{a, b\}\}$$

2.