School for Computer Science and Information Technology

Machines and their languages (G51MAL) Spring 2004

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Solutions to 4th Coursework

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1. We assume that L_1 would be regular. Hence, by the pumping lemma there is a number n such that we can split all words longer than n. Consider $w = \mathrm{a}^n\mathrm{b}\mathrm{c}^{n+1}$, we have $|w| \geq n$ and $w \in L_1$. By the pumping lemma there is a splitting of the word $w = xyz$ s.t. $|xy| \leq n$. Hence y may only contain as and it is not empty. Hence $xz = \mathrm{a}^m\mathrm{b}\mathrm{c}^{n+1} \in L_1$ with $m < n$. However, this is clearly false since $m + 1 \neq n + 1$ and hence our assumption that L_1 is regular must have been wrong.

2. We assume that L_2 would be regular. Hence, by the pumping lemma there is a number n such that we can split all words longer than n. Consider $w = 10^n10^n$, we have $|w| \geq n$ and $w \in L_2$. By the pumping lemma there is a splitting of the word $w = xyz$ s.t. $|xy| \leq n$ and hence xy is a prefix of 10^n where y is of the form 10^m or 0^m with $m < n$. In any case all the 1s in xz are now in the first half of the word and hence xz cannot be of the form $w = vv$ which contradicts the conclusion of the pumping lemma that $xz \in L_2$. Hence L_2 cannot be regular.

   If Σ contains only one letter than L_2 is the language of words with even length which is regular.