School of Computer Science, University of Nottingham

G52MAL Machines and their Languages, Spring 2012

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Exercises, Set 1

Friday, 10th February 2012

Deadline: Wednesday 22nd February 2011 in your tutorial

1. Let L_1 and L_2 be two languages over the alphabet $\Sigma = \{a, b, c\}$, defined as follows:

$$\begin{aligned} L_1 &= \{a, ab\} \\ L_2 &= \{\varepsilon, bb, bbc\} \end{aligned}$$

   Enumerate the words in the following languages:

   (a) $L_3 = L_1 \cup L_2$

   (b) $L_4 = L_2\, L_1$

   (c) $L_5 = L_4\, \emptyset\, L_3$

   (d) $L_6 = L_1^* \cap L_2^*$

2. Given $\Sigma = \{a, b, c\}$ which of the following equations for $L_1, L_2 \in \mathcal{P}(\Sigma^*)$ are universally true:

   (a) $L_1 L_2 = L_2 L_1$

   (b) $L_1 \Sigma^* = L_1$

   (c) $L_1 L_1 = L_1$

   (d) $L_1^* L_1^* = L_1^*$

   (e) $(L_1 L_2)^* = L_1^* L_2^*$

   Either give a counterexample or give a short explanation why you think the equation is true.

3. Consider the following DFA A over the alphabet $\Sigma = \{b, c\}$:

$$\begin{aligned} A &= (\{0, 1, 2\}, \Sigma, \delta, 0, \{2\}) \\ &\quad \textbf{where} \\ &\qquad \delta\,(0, b) = 1 \\ &\qquad \delta\,(0, c) = 0 \\ &\qquad \delta\,(1, b) = 2 \\ &\qquad \delta\,(1, c) = 0 \\ &\qquad \delta\,(2, b) = 2 \\ &\qquad \delta\,(2, c) = 2 \end{aligned}$$

(a) Draw a transition table for A.

(b) Draw a transition diagram for A.

(c) Which of the following words belong to $L(A)$:

  i. ε

  ii. cb

  iii. $cbbcb$

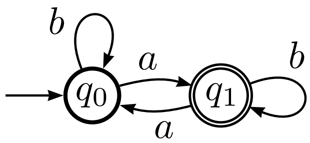
  iv. $bccbbbccb$

(d) Explicitly calculate $\hat{\delta}\,(0, bcbb)$. Clearly show each step of the calculation.

(e) Describe the language $L(A)$ in plain English.

4. **Bonus Exercise**

Recall the DFA D_1 from Lecture 3:



Encode D_1 in Haskell by giving definitions for the following data types and functions:

$$
\begin{array}{lcl}
\textbf{data}\ Q & = & \ldots \\
\textbf{data}\ \Sigma & = & \ldots \\
\textbf{type}\ \mathit{Word} & = & \ldots \\
q_0 & :: & Q \\
\mathit{final} & :: & Q \rightarrow \mathit{Bool} \\
\delta & :: & (Q, \Sigma) \rightarrow Q \\
\hat{\delta} & :: & (Q, \mathit{Word}) \rightarrow Q \\
\mathit{accept} & :: & \mathit{Word} \rightarrow \mathit{Bool}
\end{array}
$$

Note that for syntactical reasons, Σ^* has been renamed Word.