School of Computer Science, University of Nottingham

G52MAL Machines and their Languages, Spring 2012

Thorsten Altenkirch

Exercises, Set 5

Friday 23rd March 2012

Deadline: Wednesday 3rd May 2012, in your tutorial

1. Consider the following Pushdown Automaton P:

$$P = (Q = \{0,1,2\}, \Sigma = \{a,b,c\}, \Gamma = \{a,\#\}, \delta, q_0 = 0, Z_0 = \#)$$

where the transition function δ is given by:

$$\begin{array}{lcl}
\delta\,(0,\varepsilon,\#) & = & \{(0,\varepsilon),(1,\#)\} \\
\delta\,(0,a,\#) & = & \{(0,a\#)\} \\
\delta\,(0,a,a) & = & \{(0,aa)\} \\
\delta\,(0,\varepsilon,a) & = & \{(1,a)\} \\
\delta\,(1,\varepsilon,\#) & = & \{(1,\varepsilon)\} \\
\delta\,(1,b,\#) & = & \{(1,\#)\} \\
\delta\,(1,b,a) & = & \{(1,a)\} \\
\delta\,(1,c,a) & = & \{(2,\varepsilon)\} \\
\delta\,(2,\varepsilon,\#) & = & \{(2,\varepsilon)\} \\
\delta\,(2,c,a) & = & \{(2,\varepsilon)\} \\
\delta\,(_,_,_) & = & \emptyset
\end{array}$$

Acceptance is by *empty stack*.

(a) Draw a transition diagram for P.

(b) For each of the following words, state whether they are accepted by P, and, for those that are, give a sequence of *Instantaneous Descriptions* leading to an accepting configuration.

i. ε
ii. a
iii. ab
iv. ac
v. bb
vi. abc
vii. $aabbc$
viii. $aabcc$
ix. $abbccc$

Note: Just give the sequence of IDs separated by $\vdash$'s, a formal calculation with hints is not required.

(c) Give a set comprehension defining the language accepted by P.

2. Consider the Context-Free Grammar

$$G = (\{S,X,Y,Z\}, \{a,b,c,d\}, P, S)$$

where P is given by:

$$\begin{array}{lcl}
S & \to & X \mid Y \mid cZ \\
X & \to & aXb \mid \varepsilon \\
Y & \to & bbc \mid abba \\
Z & \to & cZdd \mid \varepsilon
\end{array}$$

(a) For each of the following words, state whether they are in the language generated by G, and, for those that are, give a complete derivation sequence from the start symbol S.

i. ε
ii. a
iii. c
iv. ab
v. ba
vi. bbc
vii. cd
viii. $ccdd$
ix. $cabba$
x. $cccddd$
xi. $aaabbb$

(b) Give a set expression (using set comprehensions and operations on sets like union) denoting the language $L(G)$.

(c) Is it possible to construct a Regular Expression R such that $L(R) = L(G)$? If so, do so. If not, give a brief justification of why not.

3. Consider the following Context-Free Grammar *Exp*:

$$
\begin{aligned}
T &\rightarrow T + T \mid F \\
F &\rightarrow F * F \mid P \\
P &\rightarrow N\ (A) \mid (T) \mid I \\
N &\rightarrow f \mid g \mid h \\
A &\rightarrow T \mid \varepsilon \\
I &\rightarrow DI \mid D \\
D &\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9
\end{aligned}
$$

T, F, P, N, A, I, D are nonterminals;
$+, *, f, g, h,), (, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ are terminals;
T is the start symbol.

(a) For each of the following strings of terminals and nonterminals:

- state whether it is in the language generated by *Exp*;
- state whether it is a sentential form of *Exp*;
- if it is a sentential form, state whether it is a left-sentential form;
- if it is a left-sentential form, give a complete *leftmost* derivation sequence from the start symbol T.

i. ε
ii. $g\ (13 + f\ ())$
iii. $I * g\ (D + f + g\ ())$
iv. $2\ (f\ (17)) + g\ (4)$
v. $N\ (3 * F) * h\ ()$
vi. $33 + 7) * h\ (21 + 6 * f\ (13 * 542))$

(b) **Bonus Exercise**

i. Modify the relevant productions of the grammar *Exp* so that a function symbol (one of f, g, h) can be applied to zero, one, or more arguments, instead of just zero or one arguments. When there are two or more arguments, they should each be separated by a single comma. For example, it should be possible to derive words such as

$$f\ (7, g\ (), h\ (3 + 4))$$

ii. Explain your construction.

iii. Give a complete rightmost derivation sequence for the word $f(1, 2, 3)$ using your modified grammar.