School of Computer Science, University of Nottingham

G52MAL Machines and their Languages, Spring 2011

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Solutions to Exercises, Set 1

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1. (a) $L_3 \; = \; \{\, a, ab, \varepsilon, bb, bbc \,\}$
   (b) $L_4 \; = \; \{\, a, ab, bba, bbab, bbca, bbcab \,\}$
   (c) $L_5 \; = \; \emptyset$
   (d) $L_6 \; = \; \emptyset$

2. (a) Doesn't hold in general, i.e. let $L_1 = \{a\}, L_2 = \{b\}$ then $L_1L_2 = \{ab\} \neq L_2L_1 = ba$.

   (b) Doesn't hold in general, i.e. let $L_1 = \{\epsilon\}$, then $L_1\Sigma^* = \Sigma^* \neq \{\epsilon\}$.

   (c) Doesn't hold in general, i.e.let $L_1 = \{a\}$ then $L_1L_1 = \{aa\} \neq \{a\} = L_1$.

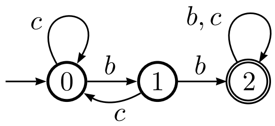
   (d) Is always true. We show first that $L_1* \subseteq L_1^*L_1*$ since given $w \in L_1^*$ we know $w = w\epsilon$ and $\epsilon \in L_1*$ hence $w = w\epsilon \in L_1^*L_1^*$. On the other hand $L_1^*L_1* \subseteq L_1^*$ since if $w \in L_1^*L_1^*$ we know that $w = uv$ with $u, v \in L_1^*$ but then $w = uv \in L_1^*$.

   (e) Doesn't hold in general, i.e. choose $L_1 = \{a\}, L_2 = \{b\}$ then $abab \in (L_1L_2)^*$ but $abab \notin L_1^*L_2^*$ (On the other hand $aab \in L_1^*L_2^*$ but $aab \notin (L_1L_2)^*$).

3. (a)

δ	b	c
$\to *0$	1	0
$*1$	2	0
$*2$	2	2

   (b)



   (c) i. $\varepsilon \notin L\,(A)$
   ii. $cb \notin L\,(A)$
   iii. $cbbcb \in L\,(A)$
   iv. $bccbbbccb \in L\,(A)$

   (d)

$\hat{\delta}\,(0, bcbb)$
$=$ $\{\text{def. } \hat{\delta}\}$
$\hat{\delta}\,(\delta\,(0, b), cbb)$
$=$ $\{\delta\,(0, b) \; = \; 1\}$
$\hat{\delta}\,(1, cbb)$
$=$ $\{\text{def. } \hat{\delta}\}$
$\hat{\delta}\,(\delta\,(1, c), bb)$
$=$ $\{\delta\,(1, c) \; = \; 0\}$
$\hat{\delta}\,(0, bb)$
$=$ $\{\text{def. } \hat{\delta}\}$

$\hat{\delta}\ (\delta\ (0, b), b)$
$=$ $\{\delta\ (0, b)\ =\ 1\}$
$\hat{\delta}\ (1, b)$
$=$ $\{\text{def. } \hat{\delta}\}$
$\hat{\delta}\ (\delta\ (1, b), \varepsilon)$
$=$ $\{\delta\ (1, b)\ =\ 2\}$
$\hat{\delta}\ (2, \varepsilon)$
$=$ $\{\text{def. } \hat{\delta}\}$
2

(e) $L\ (A)$ contains all words over $\{b, c\}$ which contain bb.

4. **data** $Q\ =\ Q_0\ \mid\ Q_1$
   data $\Sigma\ =\ A\ \mid\ B$
   type $Word\ =\ [\Sigma]$
   $q_0 :: Q$
   $q_0\ =\ Q_0$
   $final :: Q\ \rightarrow\ Bool$
   $final\ Q_0\ =\ False$
   $final\ Q_1\ =\ True$

$$
\begin{aligned}
&\delta :: (Q, \Sigma) \ \rightarrow \ Q \\
&\delta \ (Q_0, A) \ = \ Q_1 \\
&\delta \ (Q_0, B) \ = \ Q_0 \\
&\delta \ (Q_1, A) \ = \ Q_0 \\
&\delta \ (Q_1, B) \ = \ Q_1
\end{aligned}
$$

$$
\begin{aligned}
&\hat{\delta} :: (Q, \mathit{Word}) \ \rightarrow \ Q \\
&\hat{\delta} \ (q, [\,]) \qquad = \ q \\
&\hat{\delta} \ (q, x : w) \ = \ \hat{\delta} \ (\delta \ (q, x), w)
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{accept} :: \mathit{Word} \ \rightarrow \ \mathit{Bool} \\
&\mathit{accept} \ w \ = \ \mathit{final} \ (\hat{\delta} \ (q_0, w))
\end{aligned}
$$