School of Computer Science, University of Nottingham

G52MAL Machines and their Languages, Spring 2011

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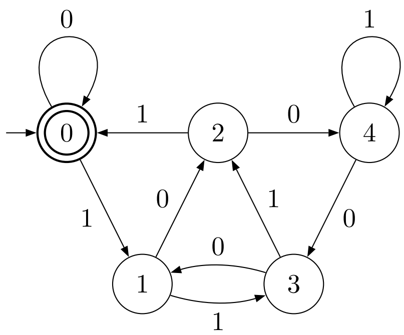
Solutions to Exercises, Set 2

Friday 2nd March 2012

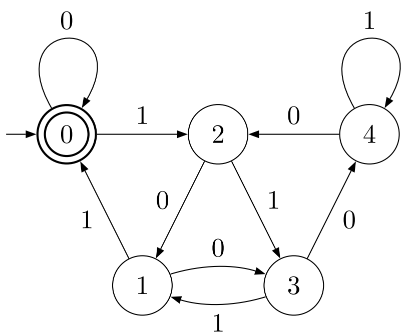
1. (a)



(b) Each state i represents the current remainder. If the current remainder is i and we read a digit x then we have to shift the remainder, i.e. multiply it by 2 and add x modulo 5. E.g. if the current remainder is 2 and we read a 1 then the remainder becomes 0 since $2 \times 2 + 1 \equiv 0 \mod 5$.

(c)

The reverse of an NFA can be constructed by reversing all arrows and swapping initial and terminal states. This doesn't necessarily give rise to a DFA but in the present case it does - so that's all we need to do.



2. (a) Transition table for N:

δ	a	b	c	d
$\to 0$	$\{0,2\}$	$\emptyset$	$\emptyset$	$\emptyset$
$\to 1$	$\emptyset$	$\{2\}$	$\emptyset$	$\emptyset$
2	$\emptyset$	$\{2\}$	$\{2,3\}$	$\{2\}$
$* 3$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

(b)

$$\begin{aligned}
&\hat{\delta}\ (\{0,1\}, acb) \\
&\qquad\qquad \{\text{def. } \hat{\delta}\} \\
&\hat{\delta}\ (\textstyle\bigcup\{\delta\ (0,a), \delta\ (1,a)\}, cb) \\
&\qquad\qquad \{\delta\ (0,a) = \{0,2\} \\
&\qquad\qquad \ \ \delta\ (1,a) = \emptyset\} \\
&\hat{\delta}\ (\textstyle\bigcup\{\{0,2\},\emptyset\}, cb) \\
&\qquad\qquad \{\textstyle\bigcup\{\{0,2\},\emptyset\} = \{0,2\}\} \\
&\hat{\delta}\ (\{0,2\}, cb) \\
&\qquad\qquad \{\text{def. } \hat{\delta}\} \\
&\hat{\delta}\ (\textstyle\bigcup\{\delta\ (0,c), \delta\ (2,c)\}, b) \\
&\qquad\qquad \{\delta\ (0,c) = \emptyset \\
&\qquad\qquad \ \ \delta\ (2,c) = \{2,3\}\} \\
&\hat{\delta}\ (\textstyle\bigcup\{\emptyset, \{2,3\}\}, b) \\
&\qquad\qquad \{\textstyle\bigcup\{\emptyset,\{2,3\}\} = \{2,3\}\} \\
&\hat{\delta}\ (\{2,3\}, b) \\
&\qquad\qquad \{\text{def. } \hat{\delta}\} \\
&\hat{\delta}\ (\textstyle\bigcup\{\delta\ (2,b), \delta\ (3,b)\}, \varepsilon) \\
&\qquad\qquad \{\delta\ (2,b) = \{2\} \\
&\qquad\qquad \ \ \delta\ (3,b) = \emptyset\} \\
&\hat{\delta}\ (\textstyle\bigcup\{\{2\},\emptyset\}, \varepsilon) \\
&\qquad\qquad \{\textstyle\bigcup\{\{2\},\emptyset\} = \{2\}\} \\
&\hat{\delta}\ (\{2\}, \varepsilon) \\
&\qquad\qquad \{\text{def. } \hat{\delta}\} \\
&\{2\}
\end{aligned}$$

(c)
i. $\varepsilon \notin L\,(N)$
ii. $a \notin L\,(N)$
iii. $bc \in L\,(N)$
iv. $dac \notin L\,(N)$
v. $bbcc \in L\,(N)$
vi. $acdac \notin L\,(N)$
vii. $aadbdc \in L\,(N)$

(d) $L\ (N)$ is the language of all words over $\{a, b, c, d\}$ that start with either the symbol b or a sequence of one or more as, and are then followed by a sequence of any length made up of bs, cs and ds, and then finish with the symbol c.

3. (a) $runDFA :: Eq\ q \Rightarrow DFA\ q\ \sigma \rightarrow [\sigma] \rightarrow Bool$
$runDFA\ (DFA\ \delta\ q_0\ fs)\ w = \hat{\delta}\ q_0\ w\ \text{`}elem\text{`}\ fs$
where
$\hat{\delta}\ q\ [\,] = q$
$\hat{\delta}\ q\ (x : w) = \hat{\delta}\ (\delta\ q\ x)\ w$

(b) $runNFA :: Eq\ q \Rightarrow NFA\ q\ \sigma \rightarrow [\sigma] \rightarrow Bool$
$runNFA\ (NFA\ \delta\ ss\ fs)\ w = (\hat{\delta}\ ss\ w\ \text{`}intersect\text{`}\ fs) \not\equiv [\,]$
where
$\hat{\delta}\ qs\ [\,] = qs$
$\hat{\delta}\ qs\ (x : w) = \hat{\delta}\ (unions\ [\delta\ q\ x \mid q \leftarrow qs])\ w$

(c) $dfa2nfa :: DFA\ q\ \sigma \rightarrow NFA\ q\ \sigma$
$dfa2nfa\ (DFA\ \delta\ q_0\ fs) = NFA\ \delta'\ [q_0]\ fs$
where
$\delta'\ q\ x = [\delta\ q\ x]$

(d) $\mathit{nfa2dfa} :: (\mathit{Eq}\ q, \mathit{Enum}\ q, \mathit{Bounded}\ q) \Rightarrow \mathit{NFA}\ q\ \sigma \rightarrow \mathit{DFA}\ [q]\ \sigma$

$\mathit{nfa2dfa}\ (\mathit{NFA}\ \delta\ \mathit{ss}\ \mathit{fs}) = \mathit{DFA}\ \delta'\ \mathit{ss}\ \mathit{fs}'$

where

$\delta'\ p\ x = \mathit{unions}\ [\delta\ q\ x \mid q \leftarrow p]$

$\mathit{fs}' = [p \mid p \leftarrow \mathit{powerset}\ \mathit{enumerate}, (p\ \text{`}\mathit{intersect}\text{`}\ \mathit{fs}) \not\equiv [\,]]$