School of Computer Science, University of Nottingham

G52MAL Machines and their Languages, Spring 2012

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Solutions to Exercises, Set 5

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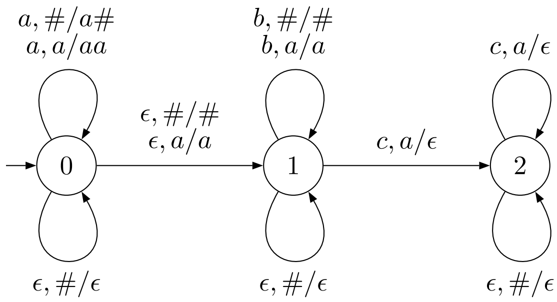
1. (a) When drawing PDA transition diagrams, you need to specify the initial stack symbol; in this case $Z_0 = \#$.



(b) i. $\varepsilon \in L(P)$
$(0, \varepsilon, \#) \vdash_P (0, \varepsilon, \varepsilon)$

ii. $a \notin L(P)$

iii. $ab \notin L(P)$

iv. $ac \in L(P)$
$(0, ac, \#) \vdash_P (0, c, a\#) \vdash_P (1, c, a\#) \vdash_P (2, \varepsilon, \#) \vdash_P (2, \varepsilon, \varepsilon)$

v. $bb \in L(P)$
$(0, bb, \#) \vdash_P (1, bb, \#) \vdash_P (1, b, \#) \vdash_P (1, \varepsilon, \#) \vdash_P (1, \varepsilon, \varepsilon)$

vi. $abc \in L(P)$
$(0, abc, \#) \vdash_P (0, bc, a\#) \vdash_P (1, bc, a\#) \vdash_P (1, c, a\#) \vdash_P (2, \varepsilon, \#) \vdash_P (2, \varepsilon, \varepsilon)$

vii. $aabbc \notin L(P)$

viii. $aabcc \in L(P)$
$(0, aabcc, \#) \vdash_P (0, abcc, a\#) \vdash_P (0, bcc, aa\#) \vdash_P (1, bcc, aa\#) \vdash_P (1, cc, aa\#)$
$\vdash_P (2, c, a\#) \vdash_P (2, \varepsilon, \#) \vdash_P (2, \varepsilon, \varepsilon)$

ix. $abbccc \notin L(P)$

(c) $L(P) = \{a^n b^m c^n \mid m, n \in \mathbb{N}\}$

2. Omitting the G subscript on the $\underset{G}{\Rightarrow}$ symbol is acceptable.

(a) i. $\varepsilon \in L(G)$
$S \underset{G}{\Rightarrow} X \underset{G}{\Rightarrow} \varepsilon$

ii. $a \notin L(G)$

iii. $c \in L(G)$
$S \underset{G}{\Rightarrow} cZ \underset{G}{\Rightarrow} c$

iv. $ab \in L(G)$
$S \underset{G}{\Rightarrow} X \underset{G}{\Rightarrow} aXb \underset{G}{\Rightarrow} ab$

v. $ba \notin L(G)$

vi. $bbc \in L(G)$

$S \underset{G}{\Rightarrow} Y \underset{G}{\Rightarrow} bbc$

vii. $cd \notin L(G)$

viii. $ccdd \in L(G)$

$S \underset{G}{\Rightarrow} cZ \underset{G}{\Rightarrow} ccZdd \underset{G}{\Rightarrow} ccdd$

ix. $cabba \notin L(G)$

x. $cccddd \notin L(G)$

xi. $aaabbb \in L(G)$

$S \underset{G}{\Rightarrow} X \underset{G}{\Rightarrow} aXb \underset{G}{\Rightarrow} aaXbb \underset{G}{\Rightarrow} aaaXbbb \underset{G}{\Rightarrow} aaabbb$

(b) $L(G) = \{a^n b^n \mid n \in \mathbb{N}\} \cup \{abba, bbc\} \cup \{c^{m+1} d^{2m} \mid m \in \mathbb{N} \wedge m \geqslant 1\}$

(c) No, it is not possible to construct a Regular Expression for this language because it is not a Regular Language. Intuitively, this can be seen in the productions $X \rightarrow aXb \mid \varepsilon$. These productions allow an arbitrarily long sequence of as followed by a sequence of bs. However, it is impossible to construct a finite automaton that counts up to an arbitrarily high number and then counts down from that number to ensure that the amount of as and bs is equal (the number of states limits how high an automaton can count, and there are only a finite number of states). As no such automaton can be constructed, the language cannot be regular.

Note that this argument is only valid because the other productions only allow a finite amount of other words that contain as and bs. If it were possible to derive all words consisting of as and bs (and likewise cs and ds) through other productions, then the language would be regular (if all words are being accepted, there is no need to count the number of as).

3. (a) i. $\varepsilon \notin L(Exp)$

ε is not a sentential form of Exp.

ii. $g\ (13 + f\ ()) \in L(Exp)$

$g\ (13 + f\ ())$ is a left-sentential form of Exp.

$T \underset{lm}{\Rightarrow} F \underset{lm}{\Rightarrow} P \underset{lm}{\Rightarrow} N\ (A) \underset{lm}{\Rightarrow} g\ (A) \underset{lm}{\Rightarrow} g\ (T) \underset{lm}{\Rightarrow} g\ (T + T) \underset{lm}{\Rightarrow} g\ (F + T) \underset{lm}{\Rightarrow} g\ (P + T) \underset{lm}{\Rightarrow} g\ (I + T) \underset{lm}{\Rightarrow} g\ (DI + T) \underset{lm}{\Rightarrow} g\ (1\ I + T) \underset{lm}{\Rightarrow} g\ (1\ D + T) \underset{lm}{\Rightarrow} g\ (13 + T) \underset{lm}{\Rightarrow} g\ (13 + F) \underset{lm}{\Rightarrow} g\ (13 + P) \underset{lm}{\Rightarrow} g\ (13 + N\ (A)) \underset{lm}{\Rightarrow} g\ (13 + f\ (A)) \underset{lm}{\Rightarrow} g\ (13 + f\ ())$

iii. $I * g\ (D + f + g\ ()) \notin L(Exp)$

$I * g\ (D + f + g\ ())$ is not a sentential form of Exp.

iv. $2\ (f\ (17)) + g\ (4) \notin L(Exp)$

$2\ (f\ (17)) + g\ (4)$ is not a sentential form of Exp.

v. $N\ (3 * F) * h\ () \notin L(Exp)$

$N\ (3 * F) * h\ ()$ is a sentential form of Exp, but not a left-sentential form.

vi. $33 + 7) * h\ (21 + 6 * f\ (13 * 542)) \notin L(Exp)$

$33 + 7) * h\ (21 + 6 * f\ (13 * 542))$ is not a sentential form of Exp.

(b) i. Delete all old productions for A and add the following productions:

$A \rightarrow B \mid \varepsilon$
$B \rightarrow T, B \mid T$

B is a new nonterminal symbol, and "," is a new terminal symbol.

ii. The productions for B generate non-empty lists of Ts, separated by commas. The production $A \to B$ thus generates argument lists of one or more arguments, while the production $A \to \varepsilon$ takes care of the case of zero arguments.

iii. $T \underset{rm}{\Rightarrow} F \underset{rm}{\Rightarrow} P \underset{rm}{\Rightarrow} N\ (A) \underset{rm}{\Rightarrow} N\ (B) \underset{rm}{\Rightarrow} N\ (T, B) \underset{rm}{\Rightarrow} N\ (T, T, B) \underset{rm}{\Rightarrow} N\ (T, T, T) \underset{rm}{\Rightarrow} N\ (T, T, F) \underset{rm}{\Rightarrow} N\ (T, T, P) \underset{rm}{\Rightarrow} N\ (T, T, I) \underset{rm}{\Rightarrow} N\ (T, T, D) \underset{rm}{\Rightarrow} N\ (T, T, 3) \underset{rm}{\Rightarrow} N\ (T, F, 3) \underset{rm}{\Rightarrow} N\ (T, P, 3) \underset{rm}{\Rightarrow} N\ (T, I, 3) \underset{rm}{\Rightarrow} N\ (T, D, 3) \underset{rm}{\Rightarrow} N\ (T, 2, 3) \underset{rm}{\Rightarrow} N\ (F, 2, 3) \underset{rm}{\Rightarrow} N\ (P, 2, 3) \underset{rm}{\Rightarrow} N\ (I, 2, 3) \underset{rm}{\Rightarrow} N\ (D, 2, 3) \underset{rm}{\Rightarrow} N\ (1, 2, 3) \underset{rm}{\Rightarrow} f\ (1, 2, 3)$