G52MAL
Machines and their Languages
Lecture 3: Deterministic Finite Automata (DFAs)

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The Problem

- An important question:

  Given a language $L \subseteq \Sigma^*$ and a word $w \in \Sigma^*$, can we determine if $w \in L$?

- A possible solution:

  'Construct' a machine that accepts a word w iff $w \in L$.

- Such a machine would also be a formal description of L.

(Informal) DFA Rules

- A machine has a finite number of states.
- A machine is in exactly one state at a time.
- Input is read in one symbol at a time.
- State changes according to current state and current symbol.

Formal Definition of a DFA

A DFA D is a 5-tuple, $D = (Q, \Sigma, \delta, q_0, F)$, where:

- Q is a finite set of states
- Σ is an alphabet
- $\delta \in (Q \times \Sigma \to Q)$ is a transition function
- $q_0 \in Q$ is the initial state
- $F \subseteq Q$ is a set of accepting (or final) states

The Extended Transition Function ($\hat{\delta}$)

The transition function can be extended to operate on words (rather than symbols):

$$\hat{\delta} \in (Q \times \Sigma^* \to Q)$$

$$\hat{\delta}\,(q, \varepsilon) = q \tag{1}$$

$$\hat{\delta}\,(q, xw) = \hat{\delta}\,(\delta\,(q, x), w) \tag{2}$$

where

$$q \in Q$$
$$x \in \Sigma$$
$$w \in \Sigma^*$$

Intuitively, if we start in state q, read an input word w, and end up in state q', then $\hat{\delta}\,(q, w) = q'$

The Language of a DFA

Given a DFA $D = (Q, \Sigma, \delta, q_0, F)$, the language of D is defined as:

$$L(D) = \{w \mid \hat{\delta}(q_0, w) \in F\}$$

Recommended Reading

- Introduction to Automata Theory, Languages, and Computation (3rd edition), pages 45–52
- G52MAL Lecture Notes, pages 7–8