G52MAL Machines and their Languages Lecture 4: Nondeterministic Finite Automata (NFAs)

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DFAs

- DFA D $= (Q, \Sigma, \delta, q_0, F)$, where:
  - Q: Finite set of states
  - Σ: Alphabet
  - $\delta \in (Q \times \Sigma \to Q)$: Transition function
  - $q_0 \in Q$: Initial state
  - $F \subseteq Q$: Set of accepting states

- Extended Transition Function:

$$\begin{aligned} &\hat{\delta} \in (Q \times \Sigma^* \to Q) \\ &\hat{\delta}\,(q, \varepsilon) = q \\ &\hat{\delta}\,(q, xw) = \hat{\delta}\,(\delta\,(q, x), w) \end{aligned}$$

- Language of a DFA:

$$L\,(D) = \{w \mid \hat{\delta}\,(q_0, w) \in F\}$$

(Informal) NFA Rules

NFAs are mostly the same as DFAs, except:

- An NFA can "make choices".
- A word is accepted if choices can be made such that the machine ends in a final state.

Another way of looking at this is that:

- An NFA can be in multiple states at the same time.
- A word is accepted if any of the ending states are final.

Formal Definition of an NFA

A NFA N is a 5-tuple, $N = (Q, \Sigma, \delta, S, F)$, where:

- Q is a finite set of states
- Σ is an alphabet
- $\delta \in (Q \times \Sigma \to \mathcal{P}(Q))$ is a transition function
- $S \subseteq Q$ is a set of initial states
- $F \subseteq Q$ is a set of accepting (or final) states

The Extended Transition Function for NFAs

$$\hat{\delta} \in (\mathcal{P}(Q) \times \Sigma^* \to \mathcal{P}(Q))$$
$$\hat{\delta}\,(P, \varepsilon) = P$$
$$\hat{\delta}\,(P, xw) = \hat{\delta}\,(\bigcup\{\delta\,(q, x) \mid q \in P\}, w)$$

where

$$P \subseteq Q$$
$$x \in \Sigma$$
$$w \in \Sigma^*$$

The Language of an NFA

Given an NFA $N = (Q, \Sigma, \delta, S, F)$, the language of N is defined as:

$$L(N) = \{w \mid (\hat{\delta}(S, w) \cap F) \neq \emptyset\}$$

Recommended Reading

- Introduction to Automata Theory, Languages, and Computation (3rd edition), pages 55–59
- G52MAL Lecture Notes, pages 9–11