G52MAL
Machines and their Languages
Lecture 5: Equivalence between NFAs and DFAs

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DFAs are NFAs

- A DFA is just a special case of an NFA, where there is:
  - exactly one initial state;
  - exactly one transition from each state per symbol.
- Thus all DFA transition diagrams also define an NFA that accepts the same language.

Converting DFAs to NFAs

Given a DFA

$$A = (Q, \Sigma, \delta_A, q_0, F)$$

an equivalent NFA $N(A)$ can be constructed as follows:

$$N(A) = (Q, \Sigma, \delta_{N(A)}, \{q_0\}, F)$$

where

$$\delta_{N(A)}(q, x) = \{\delta_A(q, x)\}$$

NFA Observations

- An NFA is always in a set of states.
- When reading an input symbol, the machine enters a new set of states.
- How many possible sets of states are there?
  - For each state, the machine is either in that state or not — i.e. 2 possibilities per state.
  - Thus $2^{|Q|}$ possibilities.
- This could be a lot, but it is finite.
- So we could convert an NFA to a DFA by taking each set of NFA states to be a single DFA state!

Converting NFAs to DFAs: The Subset Construction

Given an NFA

$$A = (Q, \Sigma, \delta_A, S, F_A)$$

an equivalent DFA $D(A)$ can be constructed as follows:

$$D\left(A\right) = \left(\mathcal{P}\left(Q\right), \Sigma, \delta_{D(A)}, S, F_{D(A)}\right)$$

where

$$\delta_{D(A)}\left(P, x\right) = \bigcup\{\delta_A\left(q, x\right) \mid q \in P\}$$

$$F_{D(A)} = \{P \mid P \in \mathcal{P}\left(Q\right) \wedge \left(P \cap F_A \neq \emptyset\right)\}$$

Summary

- A DFA is a special case of an NFA.
- An NFA can be converted to a DFA using the Subset Construction.
- Thus DFAs and NFAa are interconvertible, and therefore equivalent in the sense that they characterise exactly the same class of languages: the Regular Languages.

Recommended Reading

- Introduction to Automata Theory, Languages, and Computation (3rd edition), pages 60–71
- G52MAL Lecture Notes, pages 11–13