Exercises for Naive Type Theory
Part 2

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1. Define

   $\text{swap}_{A,B} : A \times B \to B \times A$

   using only

   $\text{uncurry} : (X \to Y \to Z) \to (X \times Y \to Z)$

2. Using if-then-else

   $$\begin{aligned} \text{ifThenElse} &: \text{Bool} \to A \to A \\ \text{ifThenElse}\,\text{true}\,a\,b &:\equiv a \\ \text{ifThenElse}\,\text{false}\,a\,b &:\equiv b \end{aligned}$$

   define

   $f : (\text{Bool} \to A) \to A \times A$
   $g : A \times A \to (\text{Bool} \to A)$

   which are inverse to each other (you don't need to prove this).

3. Using the propositions as types translation, try to verify the de Morgan-laws:

   $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
   $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

   Which one(s) are provable? And which one(s) are partially provable (one direction of $\Leftrightarrow$)?

4. Show that the principle of exclude middle, that is for all propositions P

   $P \vee \neg P$

   is equivalent to the principle of indirect proof, that is for all Q

   $\neg\neg Q \implies Q$