School for Computer Science and Information Technology
A Taste of Proof Theory 2002
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1st Coursework

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See `http://www.cs.nott.ac.uk/~txa/pt/` for basic definitions and pointers.

1. Derive the following propositions of intuitionistic propositional logic in H,N,G.
   (a) $(A \Rightarrow B) \Rightarrow (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$
   (b) $(A \wedge B) \Rightarrow C \Leftrightarrow A \Rightarrow (B \Rightarrow C)$
   (c) $A \Rightarrow (B \vee C) \Leftrightarrow (A \Rightarrow B) \wedge (A \Rightarrow C)$
2. Extend the proof of equivalence of (N) and (G) to full intuitionistic propositional logic.
3. Derive
   (a) $\neg(A \wedge B) \Leftrightarrow \neg A \vee \neg B$
   (b) $\neg(A \vee B) \Leftrightarrow \neg A \wedge \neg B$

   in classical propositional logic using (N).

   For which parts of the derivations do wee need (RAA)?
4. (*) How can we show that Pierce's law $((A \Rightarrow B) \Rightarrow A) \Rightarrow A$ is not derivable in (N) without using (RAA)?