Towards a High Level Quantum Programming Language

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based on joint work with Jonathan Grattage

and discussions with V.P. Belavkin

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Background

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no Nature is classical after all!

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- Nielsen and Chuang, p.7, *Coming up with good quantum algorithms is hard.*
- Richard Josza, QPL 2004: *We need to develop quantum thinking!*

QML

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- Compiler under construction (Jonathan)

Example: Hadamard operation

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Matrix

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

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QML

$$\begin{aligned} &had : Q_2 \multimap Q_2 \\ &had\ x = \mathbf{if}^{\circ}\ x \\ &\qquad\qquad \mathbf{then}\ \{\mathrm{qfalse} \mid (-1)\ \mathrm{qtrue}\} \\ &\qquad\qquad \mathbf{else}\ \ \{\mathrm{qfalse} \mid \mathrm{qtrue}\} \end{aligned}$$

Deutsch algorithm

$$eq : Q_2 \multimap Q_2 \multimap Q_2$$

$$
\begin{array}{l}
eq\ a\ b = \\
\quad \mathbf{let}\ (x, y, (a', b')) = \\
\qquad \mathbf{if}^{\circ} \{\mathrm{qfalse} \mid \mathrm{qtrue}\} \\
\qquad \mathbf{then}\ (\mathrm{qtrue}, \mathbf{if}^{\circ}\ a \\
\qquad\qquad\qquad \mathbf{then}\ (\{\mathrm{qfalse} \mid (-1)\ \mathrm{qtrue}\}, (\mathrm{qtrue}, b)) \\
\qquad\qquad\qquad \mathbf{else}\ (\{(-1)\ \mathrm{qfalse} \mid \mathrm{qtrue}\}, (\mathrm{qfalse}, b))) \\
\qquad \mathbf{else}\ (\mathrm{qfalse}, \ \mathbf{if}^{\circ}\ b \\
\qquad\qquad\qquad \mathbf{then}\ (\{(-1)\ \mathrm{qfalse} \mid \mathrm{qtrue}\}, (a, \mathrm{qtrue})) \\
\qquad\qquad\qquad \mathbf{else}\ (\{\mathrm{qfalse} \mid (-1)\ \mathrm{qtrue}\}, (a, \mathrm{qfalse}))) \\
\quad \mathbf{in}\ had\ x
\end{array}
$$

Overview

1. Finite classical computation
2. Finite quantum computation
3. QML
4. Conclusions and further work

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Classical computations on finite types

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- **However:** Newtonian mechanics, Maxwellian electrodynamics are also time-reversible. . .
- . . . hence classical computation **should be** based on reversible operations.

Classical computation (FCC)

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Given finite sets A (input) and B (output):

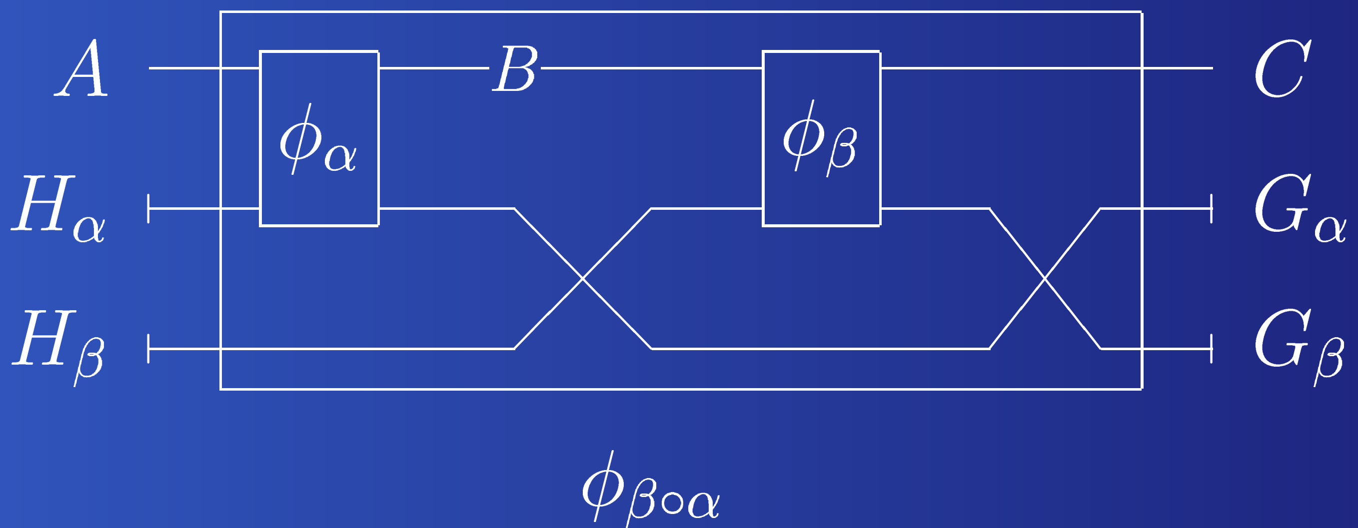
Classical computation (FCC)

Given finite sets A (input) and B (output):

- a finite set of initial heaps H,
- an initial heap $h \in H$,
- a finite set of garbage states G,
- a bijection $\phi \in A \times H \simeq B \times G$,

Composing computations

Composing computations



Extensional equality

Extensional equality

- A classical computation $\alpha = (H, h, G, \phi)$ induces a function $\mathsf{U}\alpha \in A \to B$ by

$$
\begin{array}{ccc}
A \times H & \xrightarrow[\phi]{} & B \times G \\
\Big\uparrow{\scriptstyle (-,h)} & & \Big\downarrow{\scriptstyle \pi_1} \\
A & \xrightarrow[\mathsf{U}\,\alpha]{} & B
\end{array}
$$

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- We say that two computations are **extensionally equivalent**, if they give rise to the same function.

Extensional equality . . .

- **Theorem:**

$$\mathsf{U}\,(\beta \circ \alpha) = (\mathsf{U}\,\beta) \circ (\mathsf{U}\,\alpha)$$

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- **Hence, classical computations upto extensional equality give rise to the category FCC.**
- **Theorem: Any function $f \in A \to B$ on finite sets A, B can be realized by a computation.**
- ***Translation for Category Theoreticians:***
  U is full and faithful.

Example π_1 :

function

$$\pi_1 \in (2, 2) \to 2$$
$$\pi_1\ (x, y) = x$$

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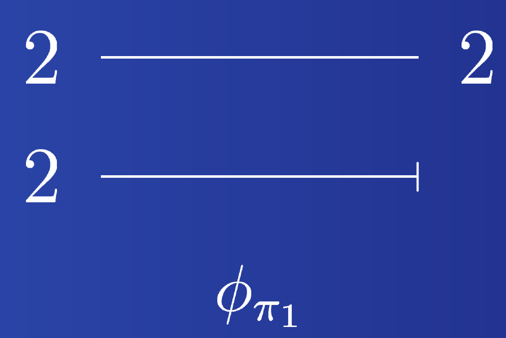

Example δ :

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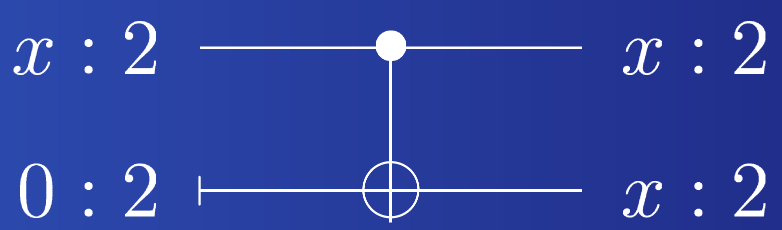

ϕ_δ

$$\phi_\delta \in (2,2) \to (2,2)$$
$$\phi_\delta\ (0, x) = (0, x)$$
$$\phi_\delta\ (1, x) = (1, \neg\ x)$$

2. Finite quantum computation

1. Finite classical computation
2. Finite quantum computation
3. QML basics
4. Compiling QML
5. Conclusions and further work

Pure quantum values

Pure quantum values

- A pure quantum value over a finite set A is given by $\vec{v} \in A \to \mathbb{C}$ with unit norm:

$$||\vec{v}|| = \Sigma a \in A. |\vec{v} a|^2 = 1$$

Pure quantum values

- A pure quantum value over a finite set A is given by $\vec{v} \in A \to \mathbb{C}$ with unit norm:

$$||\vec{v}|| = \Sigma a \in A.|\vec{v}a|^2 = 1$$

- $A \to \mathbb{C}$ is monadic, giving rise to the category of (finite dimensional) vector spaces.

Vector spaces as a monad

$$\textbf{type Vec } a = a \to \mathbb{C}$$

$$\mathit{return} \in \textbf{Eq } a \Rightarrow a \to \textbf{Vec } a$$

$$\mathit{return}\ a\ b = \textbf{if } a \equiv b \textbf{ then } 1 \textbf{ else } 0$$

$$(\gg\!=) \in \textbf{Finite } a \Rightarrow \textbf{Vec } a \to (a \to \textbf{Vec } b) \to \textbf{Vec } b$$

$$\mathit{as} \gg\!= f = \lambda b \to \mathit{sum}\ [(\mathit{as}\ a) * (f\ a\ b) \mid a \leftarrow \mathit{enumerate}]$$

Reversible quantum operations

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- On finite dimensional vector spaces: unitary = norm preserving linear iso.
- The inverse is given by the adjoint:

$$adj \in (a \to \mathbf{Vec}\ b) \to b \to \mathbf{Vec}\ a$$

$$adj\ f\ \ b\ \ a = conjugate\ (f\ \ a\ \ b)$$

Quantum computations (FQC)

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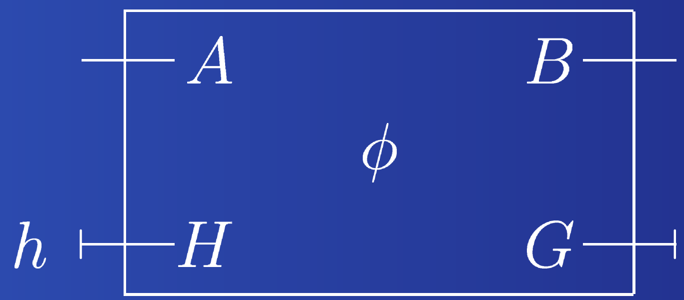

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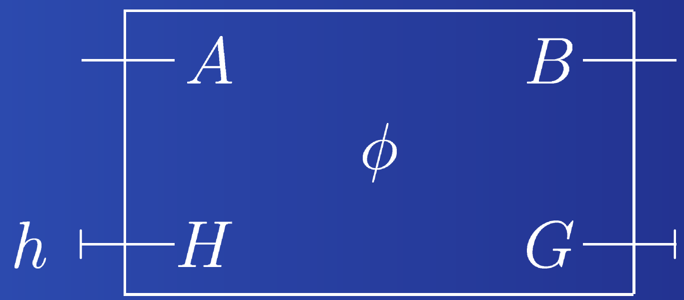

- a finite set H, the base of the space of initial heaps,
- a heap initialisation vector $\vec{h} \in H \to \mathbb{C}$,
- a finite set G, the base of the space of garbage states,
- a unitary operator $\phi \in A \otimes H \multimap_{\text{unitary}} B \otimes G$.

Composing quantum computations

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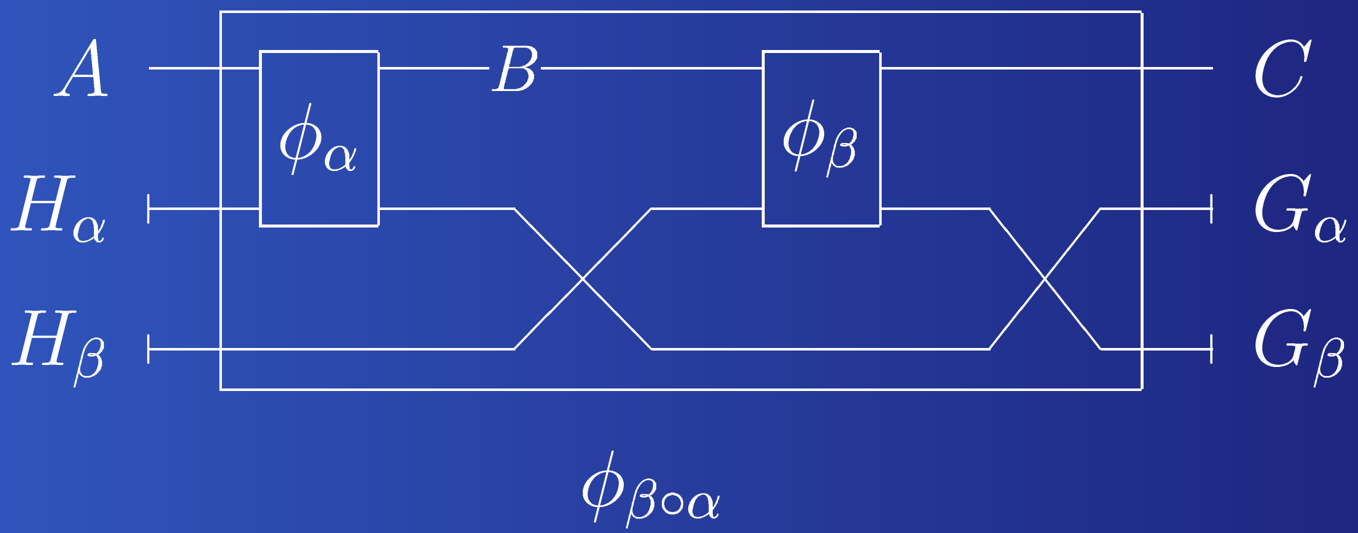

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- . . . is a bit more subtle.
- There is no (sensible) operator on vector spaces replacing $\pi_1 \in B \times G \to B$.
- **Indeed:** Forgetting part of a **pure state** results in a **mixed state**.

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- $\rho\vec{v} = \lambda\vec{v}$ is interpreted as the system is in the pure state $\vec{v}$ with probability λ.

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- Every unitary operator ϕ gives rise to a superoperator $\widehat{\phi}$.
- There is an operator

$$\mathrm{tr}_{B,G} \in B \otimes G \multimap_{\text{super}} B$$

called *partial trace*.

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- A quantum computation $\alpha \in \mathbf{FQC}\,A\,B$ gives rise to a superoperator $\mathsf{U}\,\alpha \in A \multimap_{\text{super}} B$

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  (U is full and faithful).

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finite sets	

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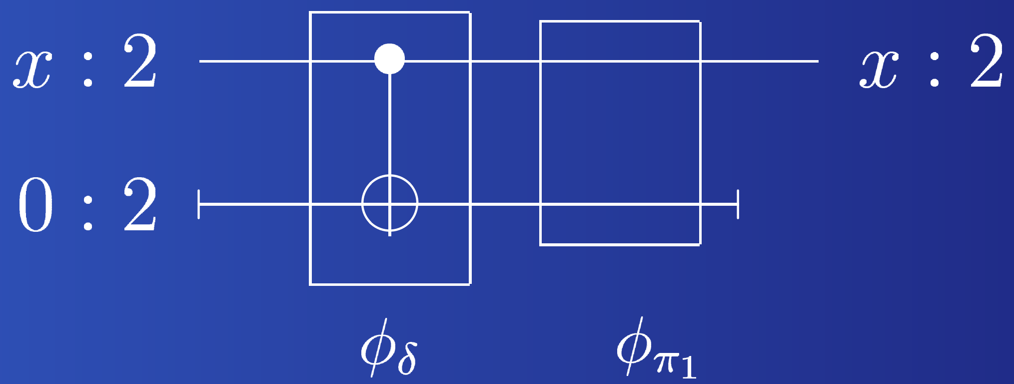

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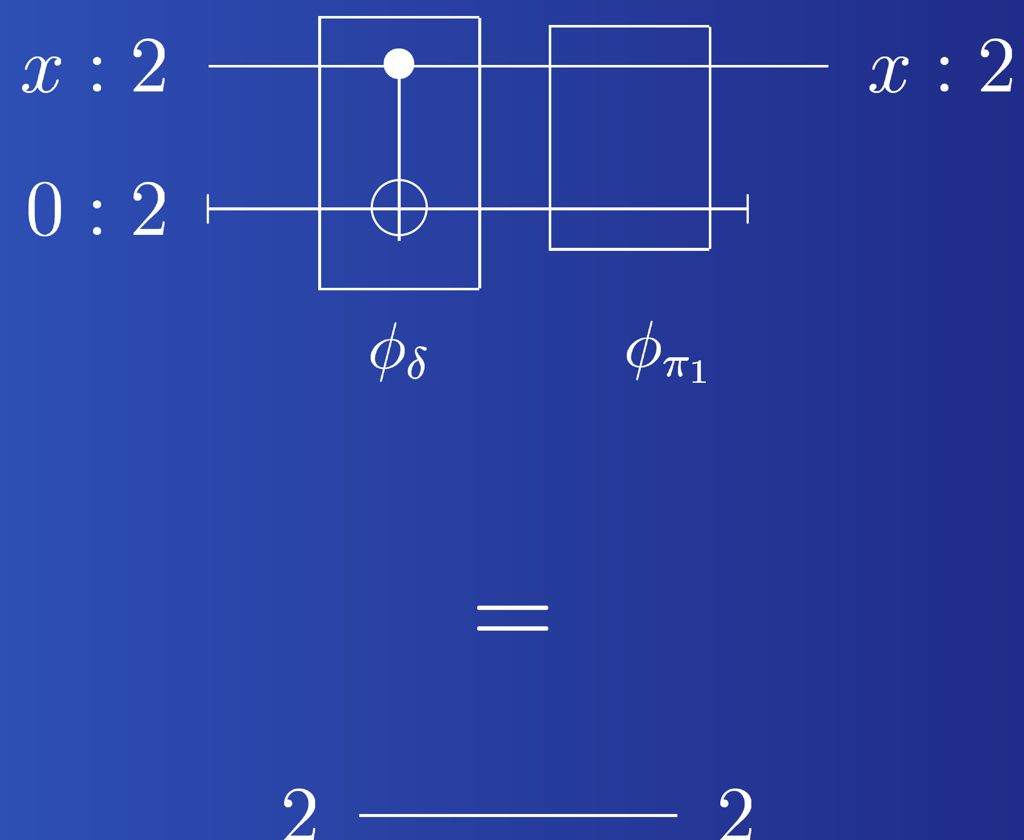

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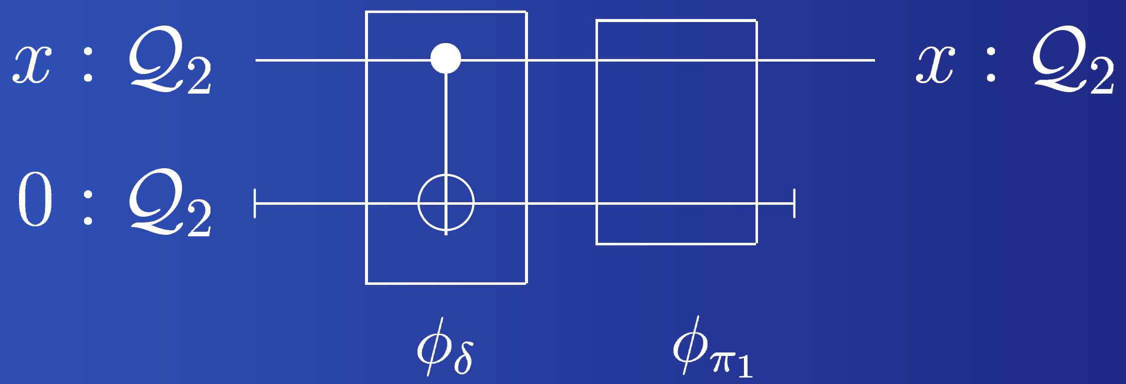

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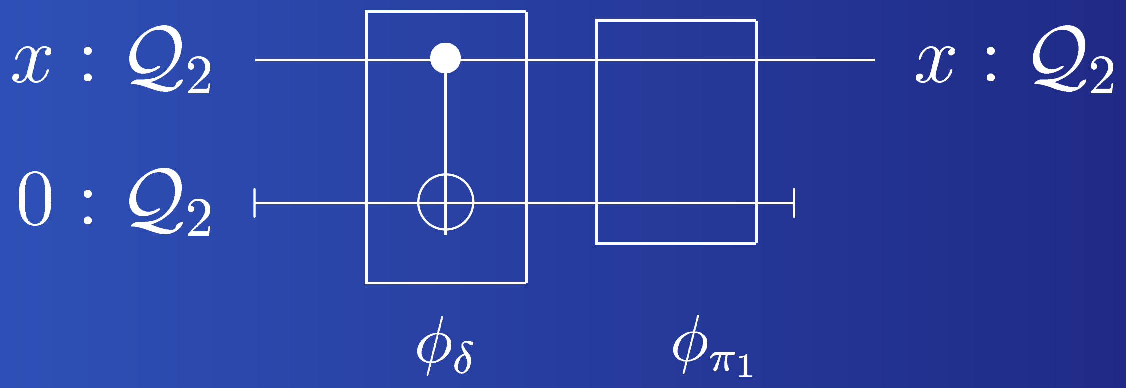
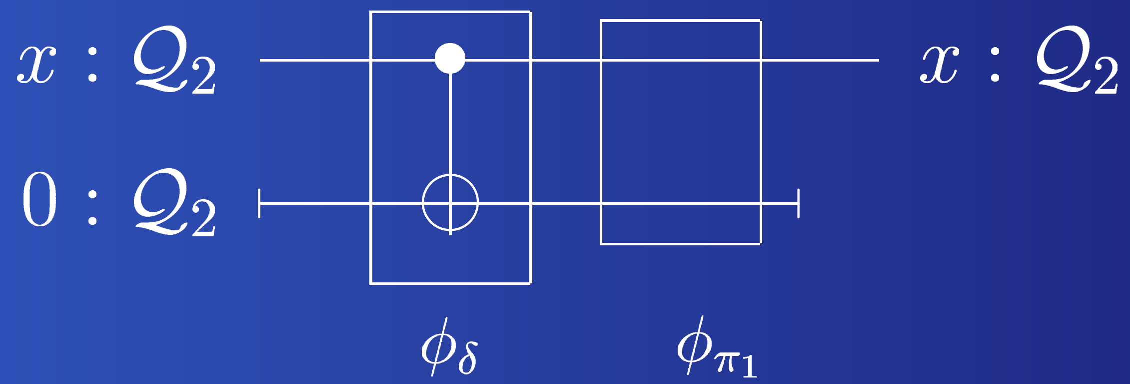


input: $\left\{ \frac{1}{\sqrt{2}} \left|0\right\rangle + \frac{1}{\sqrt{2}} \left|1\right\rangle \right\}$

$\pi_1 \circ \delta$, quantum



input: $\{\frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\}$

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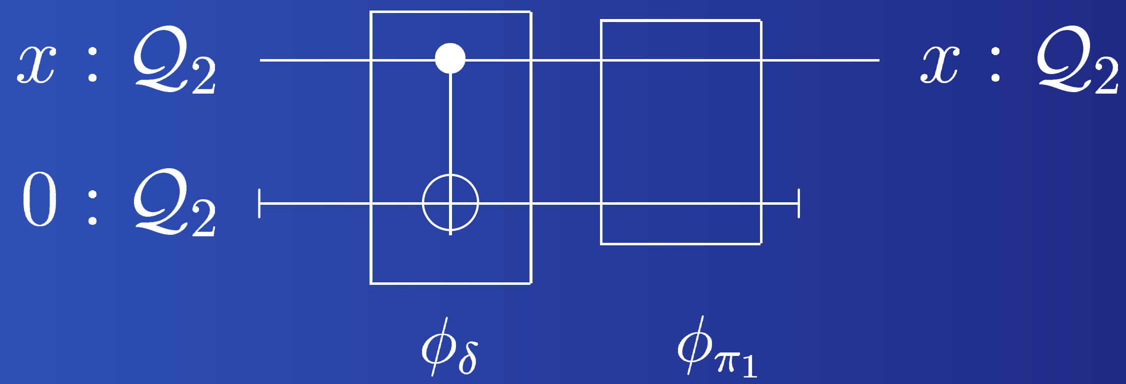

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Decoherence!

Control of decoherence

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Control of decoherence

- QML is based on strict linear logic
- Contraction is implicit and realized by ϕ_δ.
- Weakening is explicit and leads to decoherence.

3. QML

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QML overview

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Types

$$\sigma = 1 \mid \sigma \otimes \tau \mid \sigma \oplus \tau$$

QML overview

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$$\sigma = 1 \mid \sigma \otimes \tau \mid \sigma \oplus \tau$$

Terms

$$\begin{aligned} t = {} & x \mid \mathbf{let}\ x = t\ \mathbf{in}\ u \mid x \uparrow \vec{y} \\ \mid\ & () \mid (t, u) \mid \mathbf{let}\ (x, y) = t\ \mathbf{in}\ u \\ \mid\ & \mathrm{qinl}\ t \mid \mathrm{qinr}\ u \\ \mid\ & \mathbf{case}\ t\ \mathbf{of}\ \{\mathrm{qinl}\ x \Rightarrow u \mid \mathrm{qinr}\ y \Rightarrow u'\} \\ \mid\ & \mathbf{case}^{\circ}\ t\ \mathbf{of}\ \{\mathrm{qinl}\ x \Rightarrow u \mid \mathrm{qinr}\ y \Rightarrow u'\} \\ \mid\ & \{(\kappa)\ t \mid (\iota)\ u\} \end{aligned}$$

Qbits

$$Q_2 = 1 \oplus 1$$

$$\mathrm{qtrue} = \mathrm{qinl}\ ()$$

$$\mathrm{qfalse} = \mathrm{qinr}\ ()$$

$$\textbf{if}\ t\ \textbf{then}\ u\ \textbf{else}\ u'$$

$$= \textbf{case}\ \{\mathrm{qinl}\ \_ \Rightarrow u \mid \mathrm{qinr}\ \_ \Rightarrow u'\}$$

$$\textbf{if}^\circ\ t\ \textbf{then}\ u\ \textbf{else}\ u'$$

$$= \textbf{case}^\circ\{\mathrm{qinl}\ \_ \Rightarrow u \mid \mathrm{qinr}\ \_ \Rightarrow u'\}$$

QML overview . . .

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Typing judgements

$\Gamma \vdash t : \sigma$	programs
$\Gamma \vdash^{\circ} t : \sigma$	strict programs

QML overview ...

Typing judgements

$\Gamma \vdash t : \sigma$ programs

$\Gamma \vdash^{\circ} t : \sigma$ strict programs

Semantics

$$\frac{\Gamma \vdash t : \sigma}{[\![t]\!] \in \mathbf{FQC}[\![\Gamma]\!][\![\sigma]\!]} \qquad \frac{\Gamma \vdash^{\circ} t : \sigma}{[\![t]\!] \in \mathbf{FQC}^{\circ}[\![\Gamma]\!][\![\sigma]\!]}$$

The let-rule

$$\frac{\Gamma \vdash t : \sigma \qquad \Delta, x : \sigma \vdash u : \tau}{\Gamma \otimes \Delta \vdash \mathtt{let}\ x = t\ \mathtt{in}\ u : \tau}\ \text{let}$$

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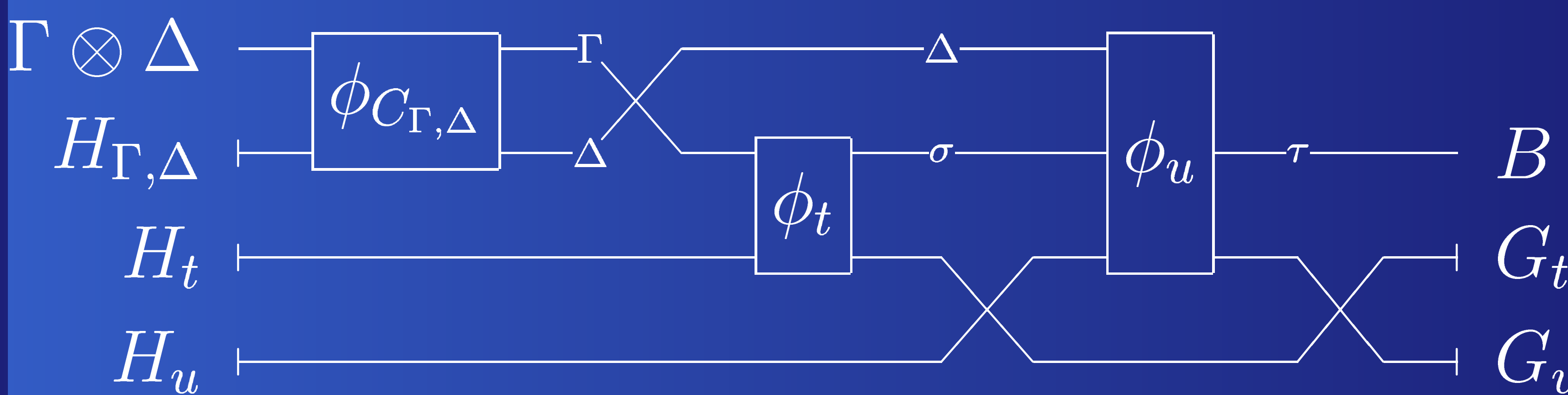

⊗ on contexts

$\otimes$ on contexts

$$
\begin{array}{lcll}
\Gamma, x:\sigma \otimes \Delta, x:\sigma & = & (\Gamma \otimes \Delta), x:\sigma & \\
\Gamma, x:\sigma \otimes \Delta & = & (\Gamma \otimes \Delta), x:\sigma & \text{if } x \notin \mathsf{dom}\,\Delta \\
\bullet \otimes \Delta & = & \Delta &
\end{array}
$$

$\otimes$ on contexts

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\begin{aligned}
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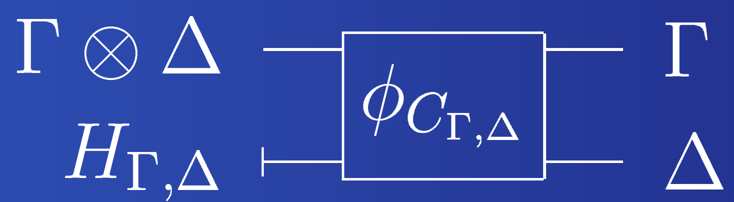
\bullet \otimes \Delta &= \Delta
\end{aligned}
$$



Another source of decoherence

Another source of decoherence

- forget mentions x

$$\mathit{forget} : 2 \multimap 2$$

$$\mathit{forget}\ x = \mathbf{if}\ x\ \mathbf{then}\ \mathrm{qtrue}\ \mathbf{else}\ \mathrm{qtrue}$$

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- *forget* mentions x

$$forget : 2 \multimap 2$$

$$forget\ x = \textbf{if}\ x\ \textbf{then}\ \mathrm{qtrue}\ \textbf{else}\ \mathrm{qtrue}$$

- but doesn't use it.

Another source of decoherence

- forget mentions x

$$\mathit{forget} : 2 \multimap 2$$

$$\mathit{forget}\ x = \mathbf{if}\ x\ \mathbf{then}\ \mathrm{qtrue}\ \mathbf{else}\ \mathrm{qtrue}$$

- but doesn't use it.
- Hence, it **has** to measure it!

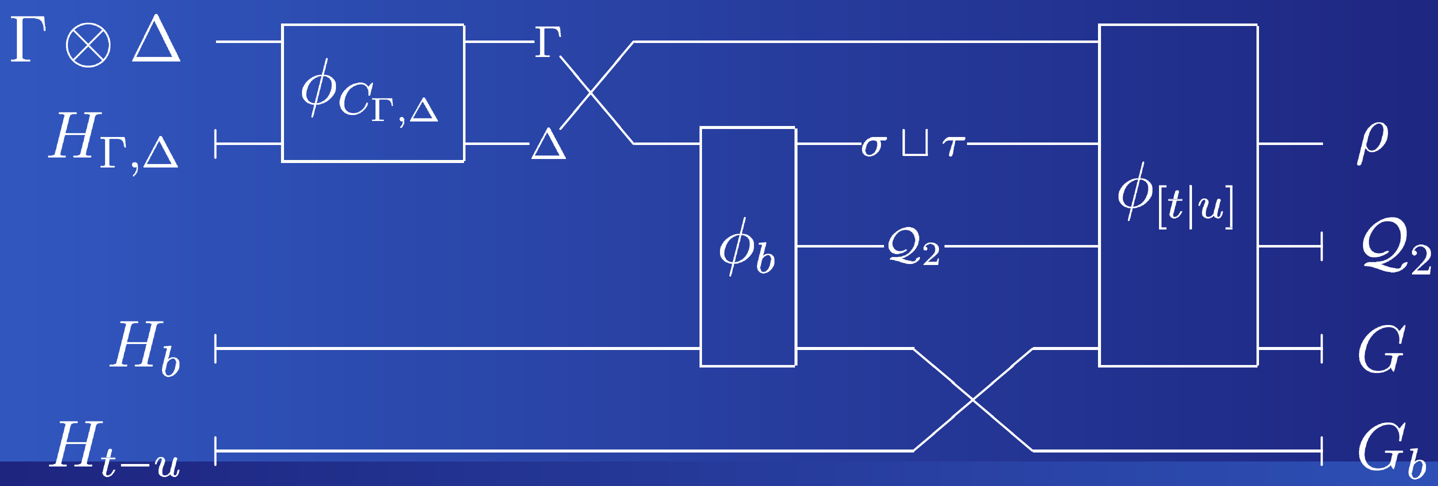
⊕-elim

$\bigoplus$-elim

$$\frac{\Gamma \vdash c : \sigma \oplus \tau \qquad \Delta, x : \sigma \vdash t : \rho \qquad \Delta, y : \tau \vdash u : \rho}{\Gamma \otimes \Delta \vdash \texttt{case}\ c\ \texttt{of}\ \{\texttt{inl}\ x \Rightarrow t \mid \texttt{inr}\ y \Rightarrow u\} : \rho} + \mathrm{elim}$$

$\oplus$-elim

$$\frac{\Gamma \vdash c : \sigma \oplus \tau \qquad \Delta, x : \sigma \vdash t : \rho \qquad \Delta, y : \tau \vdash u : \rho}{\Gamma \otimes \Delta \vdash \texttt{case}\ c\ \texttt{of}\ \{\texttt{inl}\ x \Rightarrow t \,|\, \texttt{inr}\ y \Rightarrow u\} : \rho} + \mathrm{elim}$$



$\oplus$-elim decoherence-free

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$$\frac{\Gamma \vdash^a c : \sigma \oplus \tau \qquad \Delta,\ x : \sigma \vdash^\circ t : \rho \qquad \Delta,\ y : \tau \vdash^\circ u : \rho \quad t \perp u}{\Gamma \otimes \Delta \vdash^a \texttt{case}^\circ\ \ c\ \texttt{of}\ \ \{\texttt{inl}\ x \Rightarrow t \mid \texttt{inr}\ y \Rightarrow u\} : \rho} \oplus - \mathrm{elim}^\circ$$

$\oplus$-elim decoherence-free

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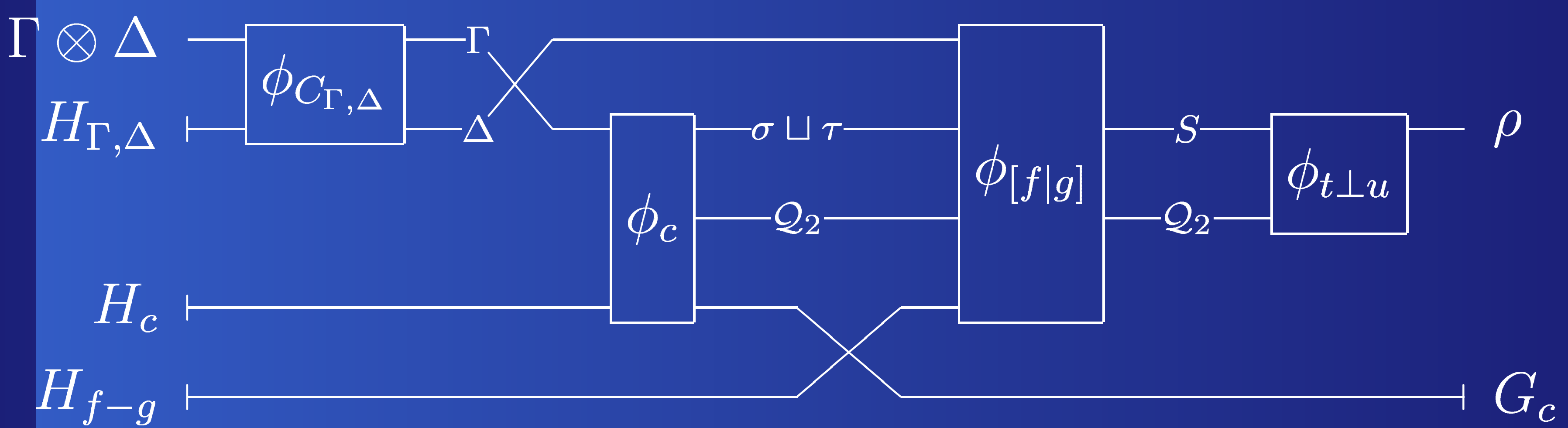

if°

$\mathbf{if}^\circ$

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- $$qnot : 2 \multimap 2$$
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- $$qnot : 2 \multimap 2$$
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- This program typechecks, because `qfalse` $\perp$ `qtrue`.

4. QML

1. Finite classical computation
2. Finite quantum computation
3. QML
4. Conclusions and further work

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- Our analysis also highlights the differences between classical and quantum programming.
- Quantum programming introduces the problem of *control of decoherence*, which we address by making forgetting variables explicit and by having different if-then-else constructs.

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- Are we able to come up with completely new algorithms using QML?
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- How to deal with infinite datatypes?
- Investigate the similarities/differences between FCC and FQC from a categorical point of view.

The end

Thank you for your attention.

Draft paper: quant-ph/0409065 from arxiv.org