G51MCS - Assignment 2

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To be handed in by Thursday, 1 November 2012 at 16:00. The work must be stamped and put in the mailbox at the School Office. [Maximum number of points for this assignment: 25.]

Problem 1 Make the truth tables for the following formulas and say which ones are tautologies [1 point each]:

$$\begin{array}{l} \neg A \Rightarrow \neg B \\ A \wedge B \Rightarrow A \vee B \\ A \vee B \Rightarrow A \wedge B \\ A \vee (B \wedge C) \Rightarrow (\neg B \Rightarrow A) \\ (A \Rightarrow B) \wedge (C \Rightarrow B) \Rightarrow \neg B \end{array}$$

Problem 2 Here is the proof of an equality of propositional formulas in Boolean algebra. The justification of each step has been omitted and replaced with dots. Complete the derivation by writing in place of the dots in each step the name of the Boolean law that was used [1 point for each step]:

$$\begin{aligned} A \vee \neg(A \wedge \neg B) &= A \vee (\neg A \vee \neg\neg B) && \cdots \\ &= (A \vee \neg A) \vee \neg\neg B && \cdots \\ &= \mathsf{true} \vee \neg\neg B && \cdots \\ &= \mathsf{true} \vee B && \cdots \\ &= B \vee \mathsf{true} && \cdots \\ &= \mathsf{true} && \cdots \end{aligned}$$

Problem 3 This exercise is complementary to the previous one: the law to be used at each step is given, but you have to apply it and write the right formula [1 point for each step]:

$$\begin{aligned} B \wedge (A \Rightarrow \neg B) &= \cdots && \text{definition of implication} \\ &= \cdots && \text{distributivity of conjunction over disjunction} \\ &= \cdots && \text{contradiction} \\ &= \cdots && \text{unit of disjunction} \end{aligned}$$

Problem 4 Prove the following equalities using Boolean algebra [2 points each]:

$$
\begin{aligned}
A \vee (A \wedge A) &= A \\
A \vee (B \wedge \neg A) &= A \vee B \\
(A \vee B) \wedge (A \vee \neg B) &= A \\
\neg(A \Rightarrow \neg B) &= A \wedge B \\
A \Leftrightarrow B &= (A \wedge B) \vee (\neg A \wedge \neg B)
\end{aligned}
$$