G51MCS - Assignment 3

Venanzio Capretta

To be handed in by Thursday, 15 November 2012 at 16:00. The work must be stamped and put in the mailbox at the School Office. [Maximum number of points for this assignment: 25.]

Problem 1 For each of the following statements, say if it is true or false. The variable names x, y, z, h, k refer to generic **real** numbers: [1 point each]

1. $x \leq y < z \Rightarrow x < z$
2. $x < y \wedge h \geq 0 \Rightarrow h \cdot x < h \cdot y$
3. $x \leq y \wedge h < k \Rightarrow x + h < y + k$
4. $x \cdot h \leq y \cdot h \Rightarrow x \leq y$
5. $x \leq y \Rightarrow x^2 \leq y^2$
6. $\lfloor x \rfloor \leq \lfloor y \rfloor \Rightarrow x \leq y$
7. $\lceil x \rceil > \lceil y \rceil \Rightarrow x > y$
8. $x < y \Rightarrow \lfloor x \rfloor < \lfloor y \rfloor$
9. $(\lfloor x \rfloor)^2 = \lfloor x^2 \rfloor$
10. $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$

Problem 2 Consider the following recursive definition of a function myLuc on natural numbers:

$$\begin{aligned} &\mathsf{myLuc}(0) = 0 \\ &\mathsf{myLuc}(1) = 1 \\ &\mathsf{myLuc}(n) = 3 \times \mathsf{myLuc}(n-1) - \mathsf{myLuc}(n-2) \qquad \text{if } n > 1. \end{aligned}$$

(a) Compute the values of myLuc up to $\mathsf{myLuc}(10)$. [2 points]

(b) Give a recursive definition for a function yourLuc such that it generates the following values: [5 points]

$$\begin{aligned} &\mathsf{yourLuc}(0) = 0 \\ &\mathsf{yourLuc}(1) = 1 \qquad \mathsf{yourLuc}(6) = 21 \\ &\mathsf{yourLuc}(2) = 1 \qquad \mathsf{yourLuc}(7) = 43 \\ &\mathsf{yourLuc}(3) = 3 \qquad \mathsf{yourLuc}(8) = 85 \\ &\mathsf{yourLuc}(4) = 5 \qquad \mathsf{yourLuc}(9) = 171 \\ &\mathsf{yourLuc}(5) = 11 \qquad \mathsf{yourLuc}(10) = 341. \end{aligned}$$

Hint: The recursive definition will be similar to that of the Fibonacci numbers, given in the lecture notes, and that of myLuc.

Problem 3. [8 points]

Prove by induction that the following property,

$$P(n): \quad \sum_{i=0}^{n}(2 \cdot i + 5) = (n + 5) \cdot (n + 1)$$

is true for every natural number n.