G51MCS - Assignment 4

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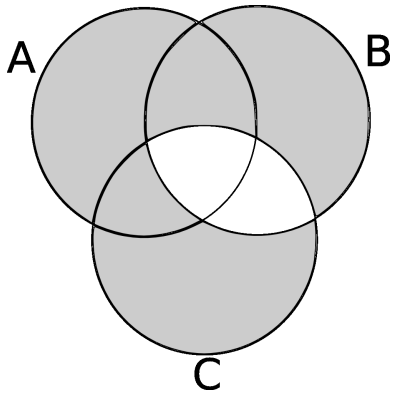
To be handed in by Thursday, 29 November 2012 at 16:00. The work must be stamped and put in the mailbox at the School Office. [Maximum number of points for this assignment: 25.]

Problem 1 Answer these questions. (2 points each)

(a) Write an expression in the variables A, B and C, combined using intersection, union, difference and complement, that corresponds to the shaded area in the following Venn diagram:



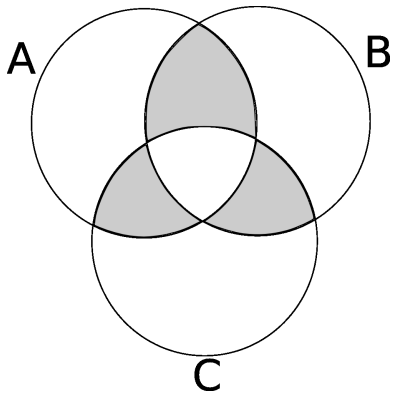
(b) Now take the sets A, B and C to be these:

$$A = \{\mathsf{Sirius}, \mathsf{Vega}, \mathsf{Altair}\}$$
$$B = \{\mathsf{Betelgeuse}, \mathsf{Vega}, \mathsf{Mizar}\}$$
$$C = \{\mathsf{Sirius}, \mathsf{Mizar}, \mathsf{Vega}\}$$

Which of the following stars belong to the set you wrote in part (a)?

$$\mathsf{Sirius}, \mathsf{Vega}, \mathsf{Altair}, \mathsf{Betelgeuse}, \mathsf{Mizar}$$

(c) Write an expression in the variables A, B and C, combined using intersection, union, difference and complement, that corresponds to the shaded area in the following Venn diagram:



(d) Now take the sets A, B and C to be these:

$$A = \{n \in \mathbb{N} \mid n \text{ is even}\}$$
$$B = \{n \in \mathbb{N} \mid 3 \text{ divides } n\}$$
$$C = \{n \in \mathbb{N} \mid n \text{ is a multiple of } 5\}$$

Which of the following numbers belong to the set you wrote in part (c)?

$$6, 7, 15, 25, 30$$

Problem 2 Consider the following function:

$f : \{\mathsf{Sirius}, \mathsf{Vega}, \mathsf{Altair}, \mathsf{Betelgeuse}, \mathsf{Mizar}\} \to \{\mathsf{Sirius}, \mathsf{Vega}, \mathsf{Altair}, \mathsf{Betelgeuse}, \mathsf{Mizar}\}$
$f(\mathsf{Sirius}) = \mathsf{Mizar}$
$f(\mathsf{Vega}) = \mathsf{Betelgeuse}$
$f(\mathsf{Altair}) = \mathsf{Sirius}$
$f(\mathsf{Betelgeuse}) = \mathsf{Altair}$
$f(\mathsf{Mizar}) = \mathsf{Vega}$

Answer these questions. (2 point each)

(a) Is it injective? If it isn't, give two elements on which it has the same value.

(b) Is it surjective? If it isn't, give an element that is not a result.

(c) Is it bijective? If it is, write down its inverse.

Problem 3 Consider the following function:

$$f : \mathbb{N} \to \mathbb{N}$$
$$f(n) = \mathsf{rem}(n, 7)$$

Answer these questions. (2 point each)

(a) Is it injective? If it isn't, give two elements on which it has the same value.

(b) Is it surjective? If it isn't, give an element that is not a result.

(c) Is it bijective? If it is, write down its inverse.

Problem 4 Let X be a set of people, with at least two elements. Some of them are friends and some aren't. If a person A is friend of another person B, then also B is a friend of A. Prove that then there must be two people in X that have exactly the same number of friends. [5 points]

[Hint: use the pigeonhole principle in a similar way as was done for the handshakes puzzle.]