# Automated Heuristic Design

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LANCS INITIATIVE
Foundational Operational Research: Building Theory for Practice

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# *Agenda*

- First Section: Introduction
  - General introduction and motivation
  - What is a hyper-heuristic?
  - Classification of hyper-heuristic approaches
- Second Section: Heuristic Selection Methodologies
  - A Constructive hyper-heuristic: Graph-based hyper-heuristic
  - A Perturbative hyper-heuristic: Tabu-search hyper-heuristic
  - *HyFlex* and the Cross-domain Heuristic Search Challenge
  - Conclusion and Future Work
- Third Section: Heuristic Generation Methodologies
  - Introduction
    - Hyper-heuristic Definition
    - What's the Point?
  - Case Study 1: Max-SAT
  - Case Study 2: Bin Packing
  - Conclusion

# *Automated Heuristic Design*

- Search and optimisation problems are everywhere, and search algorithms are getting increasingly powerful
- They are also getting increasingly complex
- Only autonomous self-managed systems that provide high-level abstractions can turn search algorithms into widely used methodologies
- **Research Goals**:
  - Reduce the role of the human expert in the process of designing optimisation algorithms and search heuristics
  - Software systems able to automatically tune, configure, generate and design optimisation algorithms and search heuristics.
  - Self-tuning, self-configuring and self-generating search heuristics

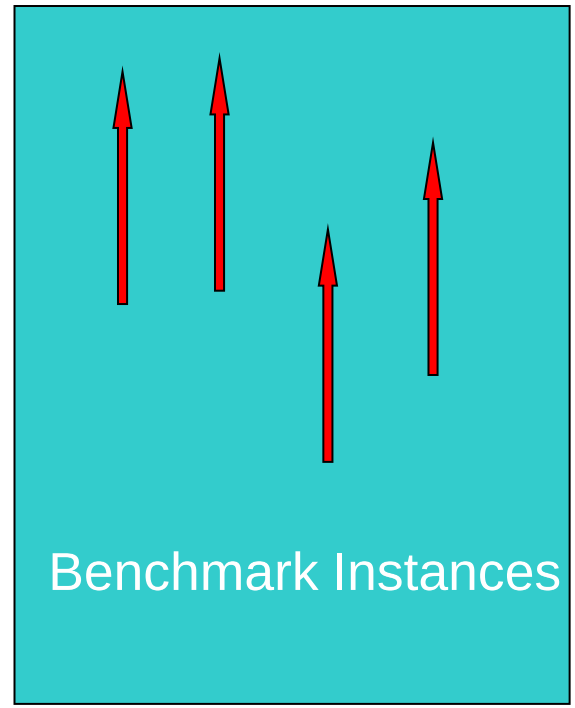
# *Automated Heuristic Design: Several Approaches*

- ***Online* approaches**
  - Self-tuning and self-adapting heuristics on the fly, effectively learning by doing until a solution is found
  - **Examples:** adaptive memetic algorithms, adaptive operator selection, parameter control in evolutionary algorithms, adaptive and self-adaptive search algorithms, reactive search
- ***Offline* approaches**
  - Learn, from a set of training instances, a method that would generalise to unseen instances
  - **Examples:** automated algorithm configuration, meta-learning, performance prediction, experimental methods, SPO
- **Hyper-heuristics (offline and online)**

# *Motivation*

## The "Up the Wall" game

- We have a problem (e.g. exam timetabling) and a set of benchmark instances
- We develop new methodologies (ever more sophisticated)
- Apply methodologies to benchmarks
- Compare with other "players"
- The goal is to "get further up the wall" than the other players
- Consequence: Made to measure (handcrafted) *Rolls-Royce* systems

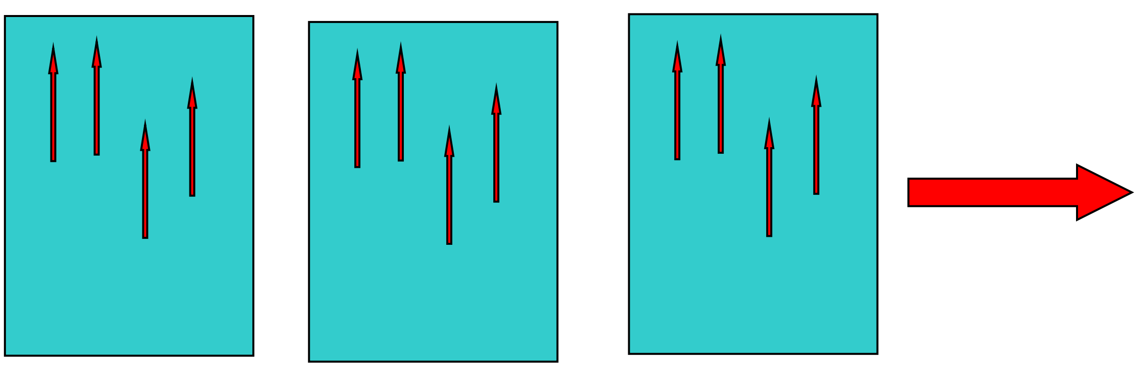



e.g. Exam Timetabling

# Motivation

## The "Many Walls" game

- Can we develop the ability to automatically work well on different problems?
- Raising the level of generality
- Still want to get as high up the wall as possible … BUT…
- We want to be able to operate on as many different walls as possible
- Consequence: Off the peg, *Ford* model



One method that operates on several problems

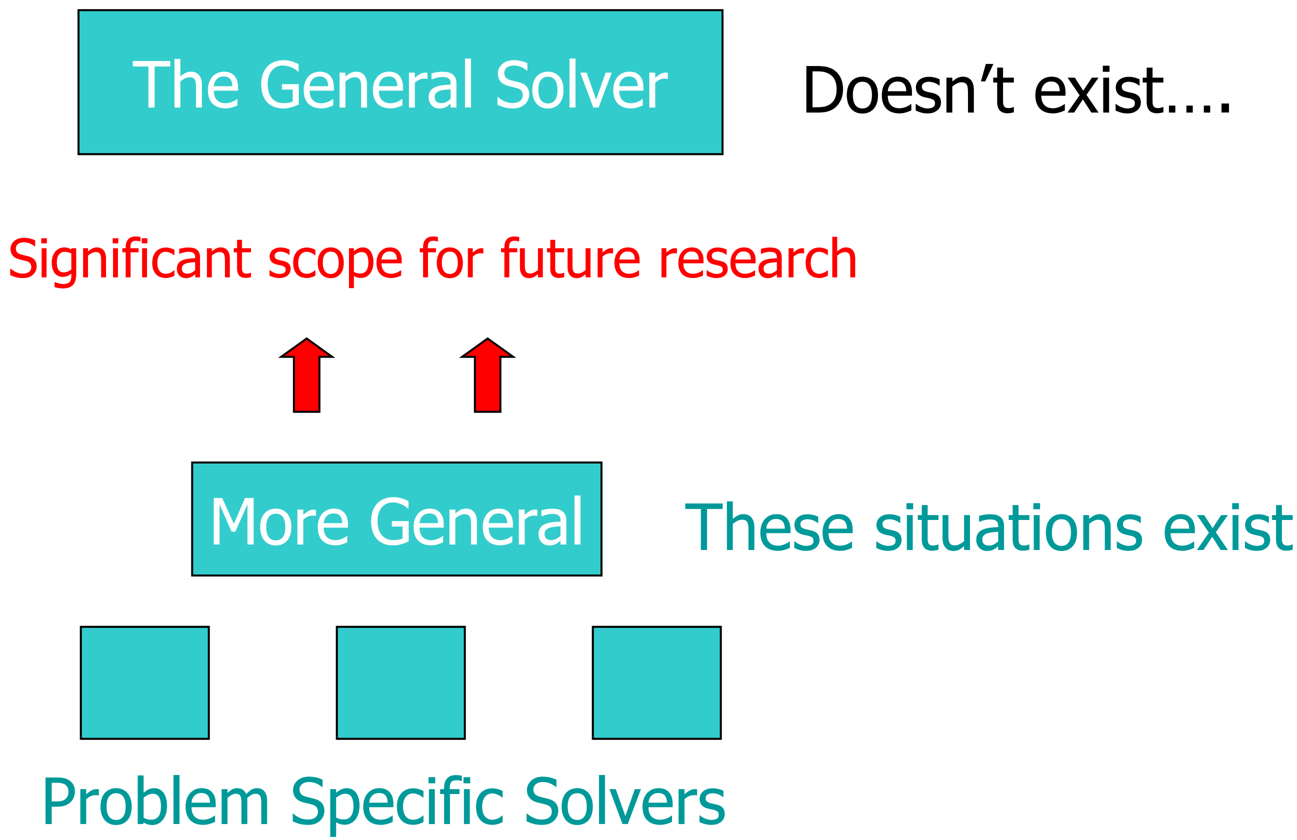
# Motivation

- Develop decision support systems that are *off the peg*
- Develop the ability to automatically work well on different problems

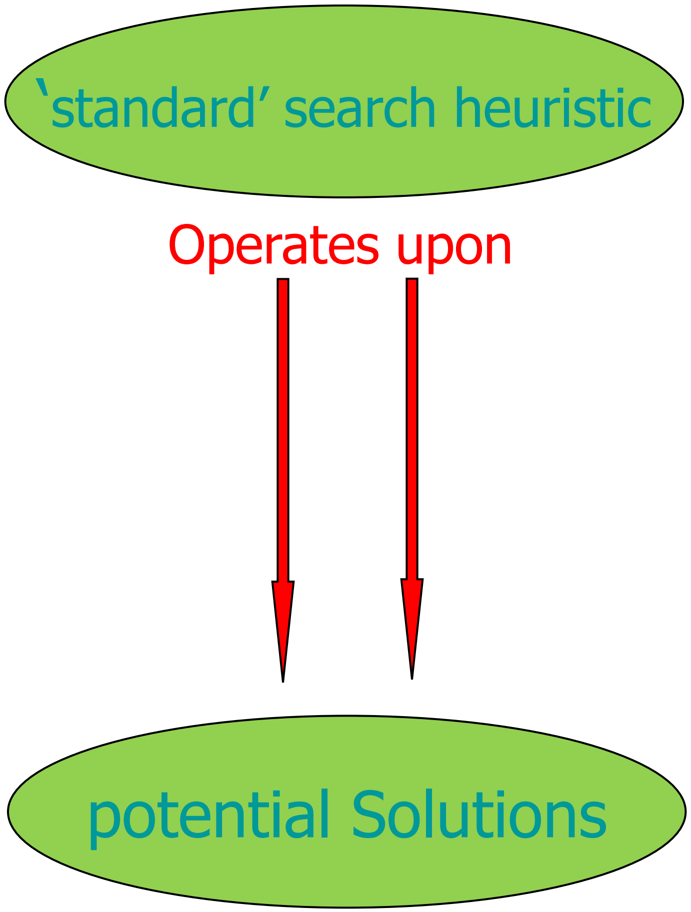
## Research challenges

- Automate heuristic design
  - Now made by human experts
  - Not cheap!
- How general we could make hyper-heuristics
  - No free lunch theorem

# *Motivation*

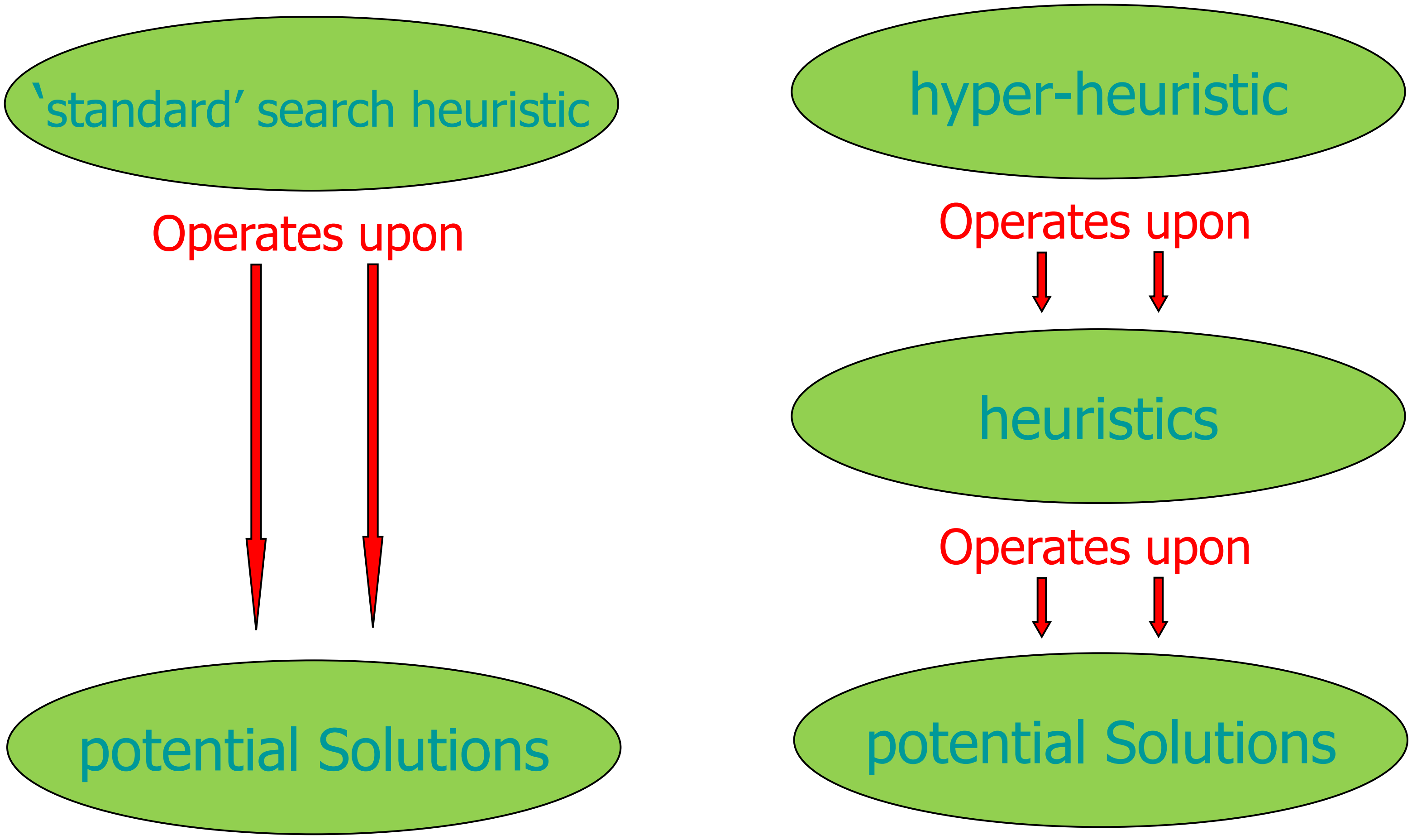

The General Solver
Doesn't exist….
Significant scope for future research
More General
These situations exist
Problem Specific Solvers

# *What is a Hyper-heuristic?*



# Hyper-heuristics:

*"Operate on a search space of heuristics"*



# What is a hyper-heuristic?

- Recent research trend in hyper-heuristics
  - Automatically *generate* new heuristics suited to a given problem or class of problems
  - Combining, i.e. by GP, *components* or *building-blocks* of human designed heuristics
- New definition:

A hyper-heuristic is an automated methodology for selecting or generating heuristics to solve hard computational search problems

E. K. Burke, M. Hyde, G. Kendall, G. Ochoa, E. Ozcan, and J. Woodward (2009). A Classification of Hyper-heuristics Approaches, *Handbook of Metaheuristics*, International Series in Operations Research & Management Science, M. Gendreau and J-Y Potvin (Eds.), Springer, pp.449-468.

# Origins and early approaches

- Term *hyper-heuristics*
  - First used 1997 (Dezinger et. al): a protocol for combining several AI methods in automated theorem proving
  - Independently used in 2000 (Colwing et. al): 'heuristic to choose heuristics' in combinatorial optimisation
  - First journal paper (Burke et. al, 2003)
- The ideas can be traced back to the 60s and 70s
  - Automated heuristic sequencing (early 60s and 90s)
  - Automated planning systems (90s)
  - Automated parameter control in evolutionary algorithms (70s)
  - Automated learning of heuristic methods (90s)
  - Automated prioritising: "Squeaky Wheel" optimisation (1999)

# Classification of hyper-heuristics

## Search paradigms

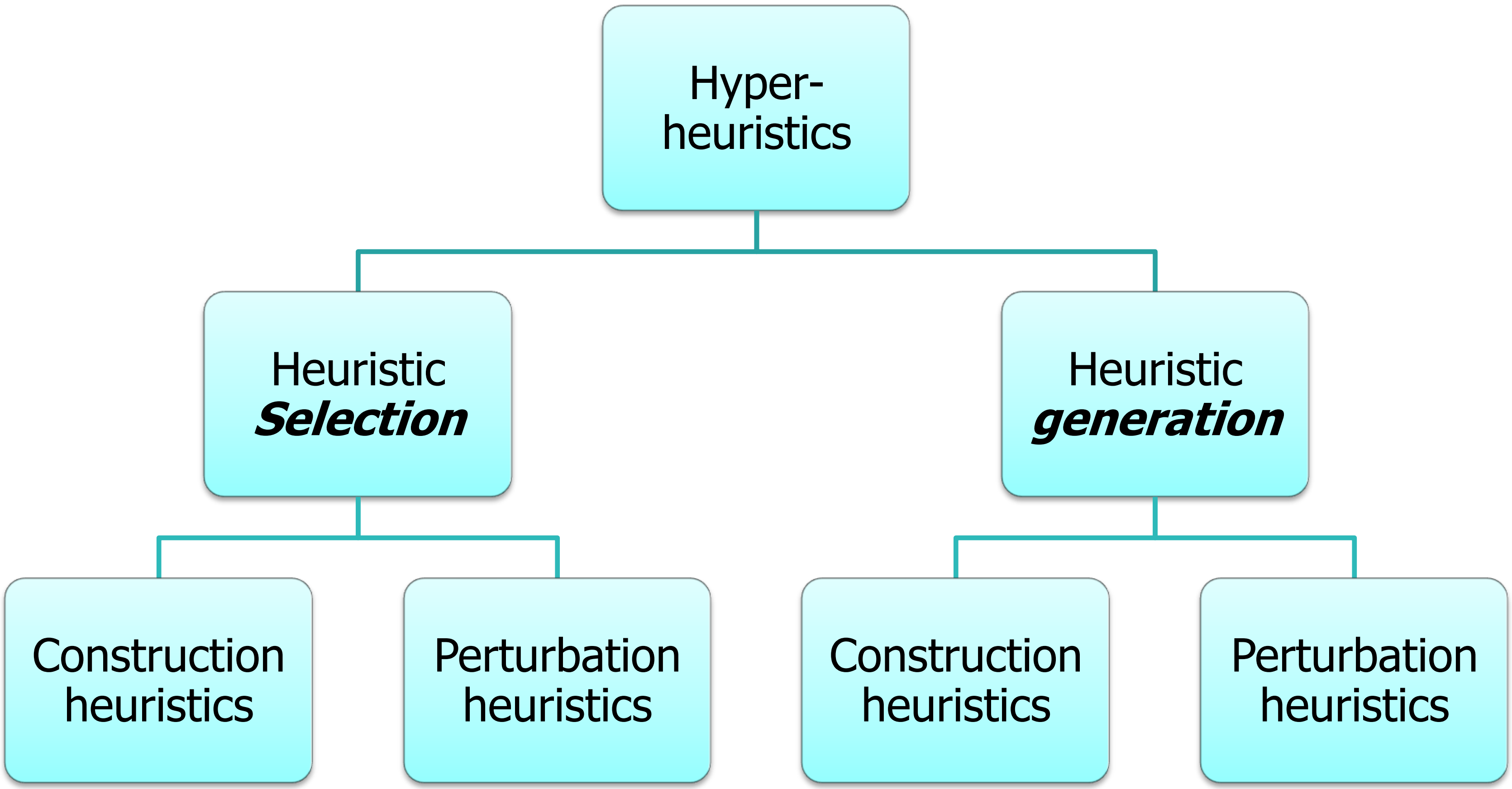
### Perturbation

- Search space: complete candidate solutions
- Search step: modification of one or more solution components
- TSP: 2-opt exchanges

### Construction

- Search space: partial candidate solutions
- Search step: extension with one or more solution components
- TSP: Next-neighbour

# *Classification of hyper-heuristics (nature of the search space)*



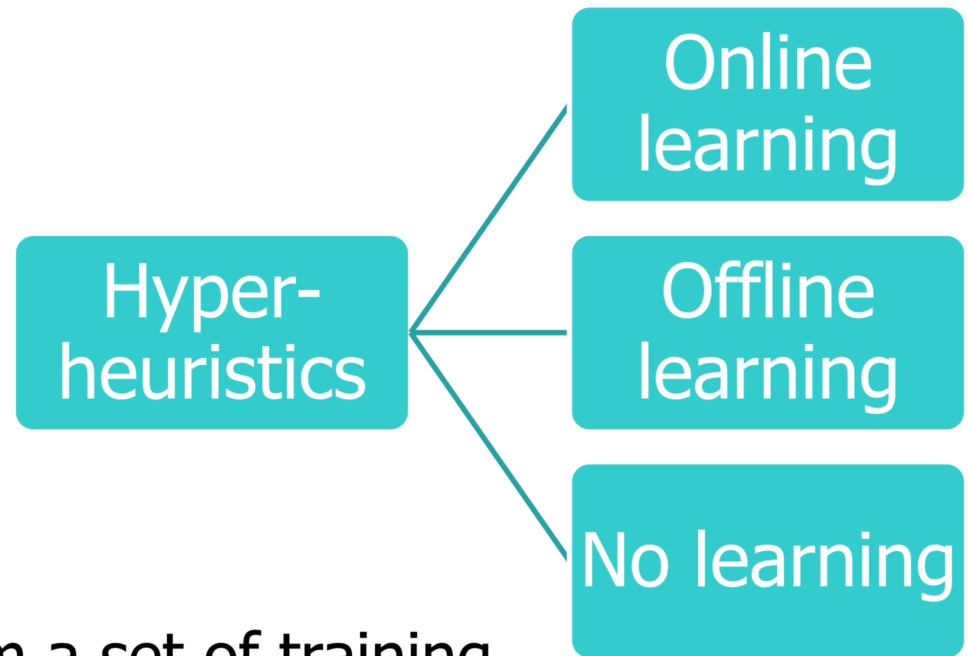

Fixed, human-designed low level heuristics

Heuristic *components*

# *Classification of hyper-heuristics (source of feedback during learning)*

## Online

- Learning while solving a single instance
- Adapt
- Examples: reinforcement learning, meta-heuristics




## Offline

- Gather knowledge from a set of training instances
- Generalise
- Examples: classifier systems, case-based, GP

# Section2: Heuristic Selection Methodologies

## *A constructive Hyper-heuristic*

# Graph-based hyper-heuristics

- A general framework (GHH) employing a set of low level constructive graph colouring heuristics
- Low level heuristics: sequential methods that order events by the difficulties of assigning them
  - 5 graph colouring heuristics
  - Random ordering strategy
- Applied to exam and course timetabling problem

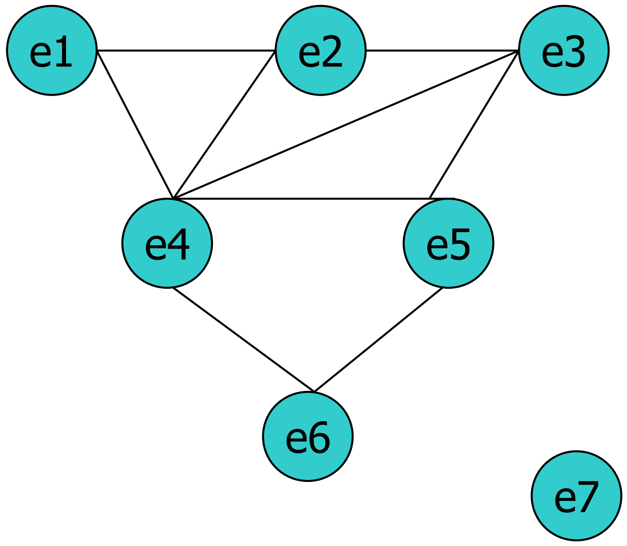
E.K.Burke, B.McCollum, A.Meisels, S.Petrovic & R.Qu. A Graph-Based Hyper Heuristic for Educational Timetabling Problems. EJOR, 176: 177-192, 2007.

# *Examination timetabling*

- A number of exams (*e1, e2, e3, …*), taken by different students (*s1, s2, s3, …*), need to be scheduled to a limited time periods (*t1, t2, t3, …*) and certain rooms (*r1, r2, r3, …*)
- Hard Constraints
  - Exams taken by common students can't be assigned to the same time period
  - Room capacity can't be exceeded
- Soft Constraints
  - Separation between exams
  - Large exams scheduled early

# *Examination timetabling*

- How can we model this problem?
  - There are 7 exams, e1 ~ e7
  - 5 students taking different exams
    - s1: e1, e2, e4
    - s2: e2, e3, e4
    - s3: e3, e4, e5
    - s4: e4, e5, e6
    - s5: e7

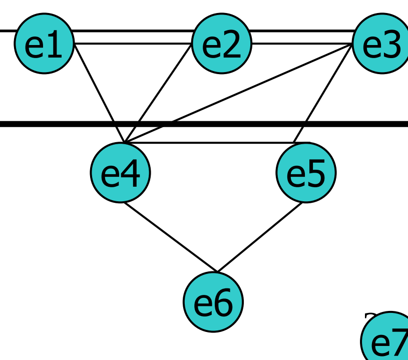



Objective: assign colours (time periods) to nodes (exams), adjacent nodes with different colour, minimising time periods used

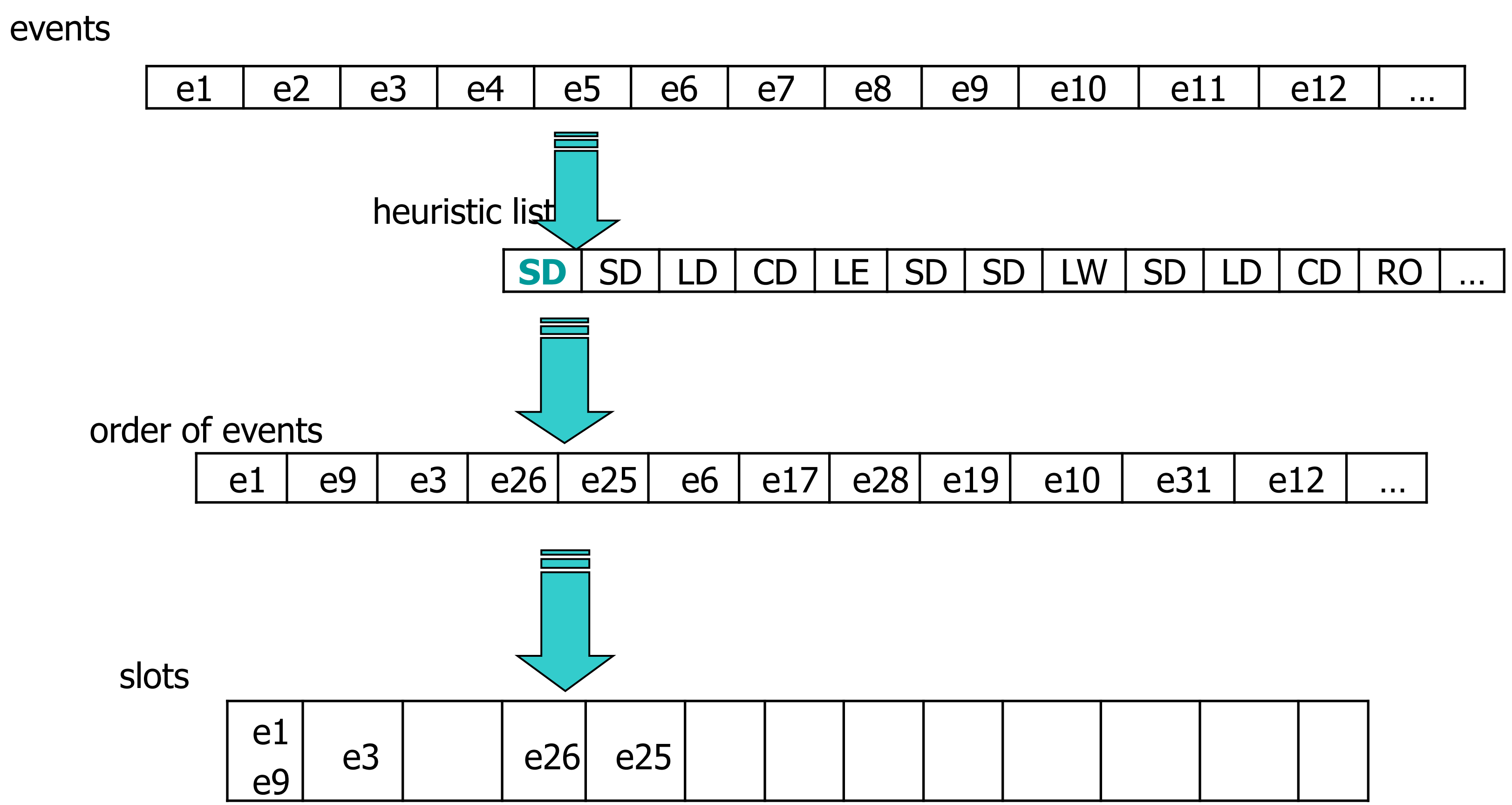
# *Low-level heuristics*

Order events by how *difficult* to schedule them

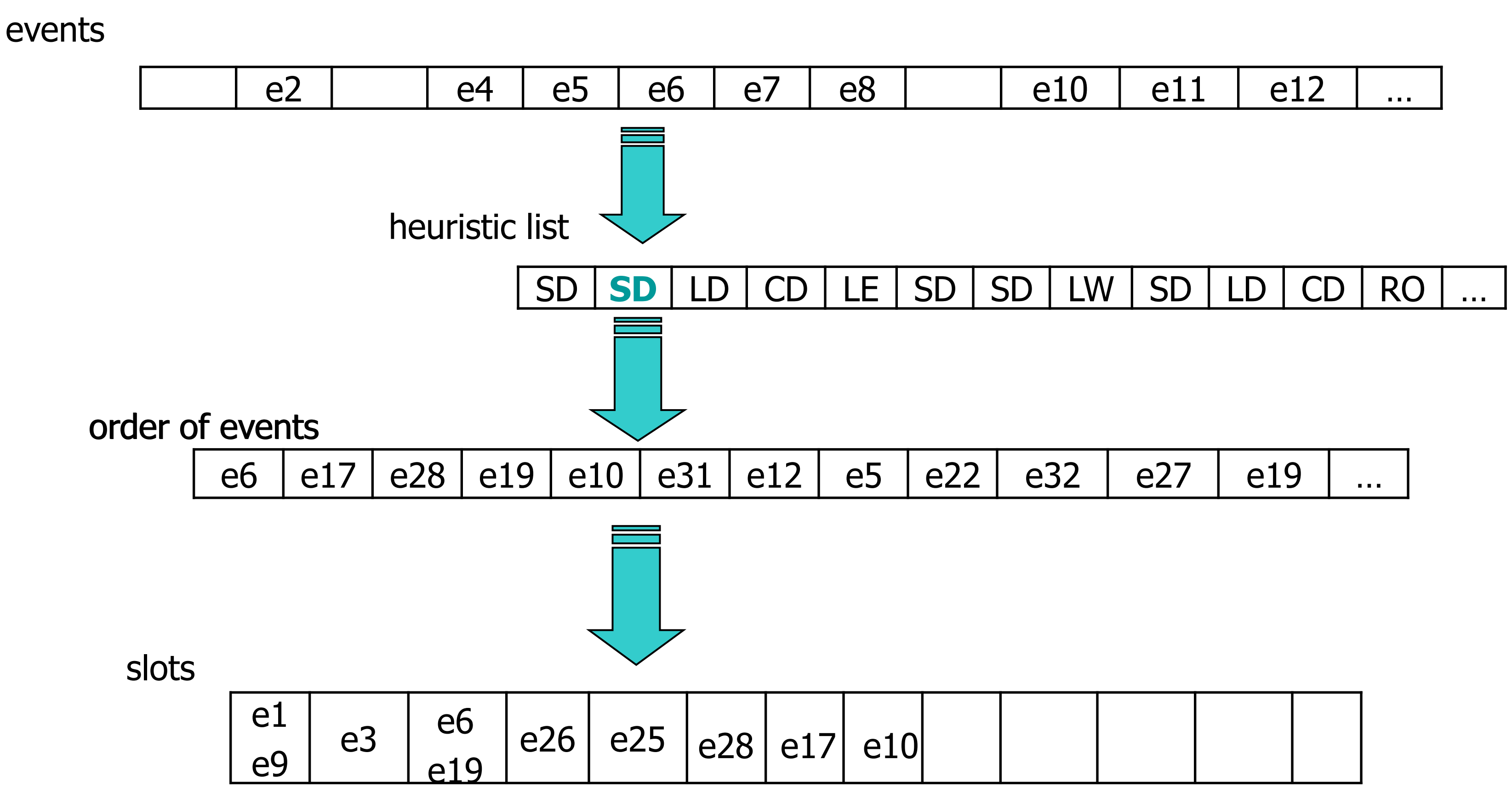
| **Graph Heuristics** | **Ordering strategies** |
| --- | --- |
| Largest degree (LD) | Number of clashed events |
| Largest weighted degree (LW) | LD with number of common students |
| Saturation degree (SD) | Number of valid remaining time periods |
| Largest enrolment (LE) | Number of students |
| Colour degree (CD) | Number of clashed event that are scheduled |
| + | |
| Random ordering (RO) | Randomly |



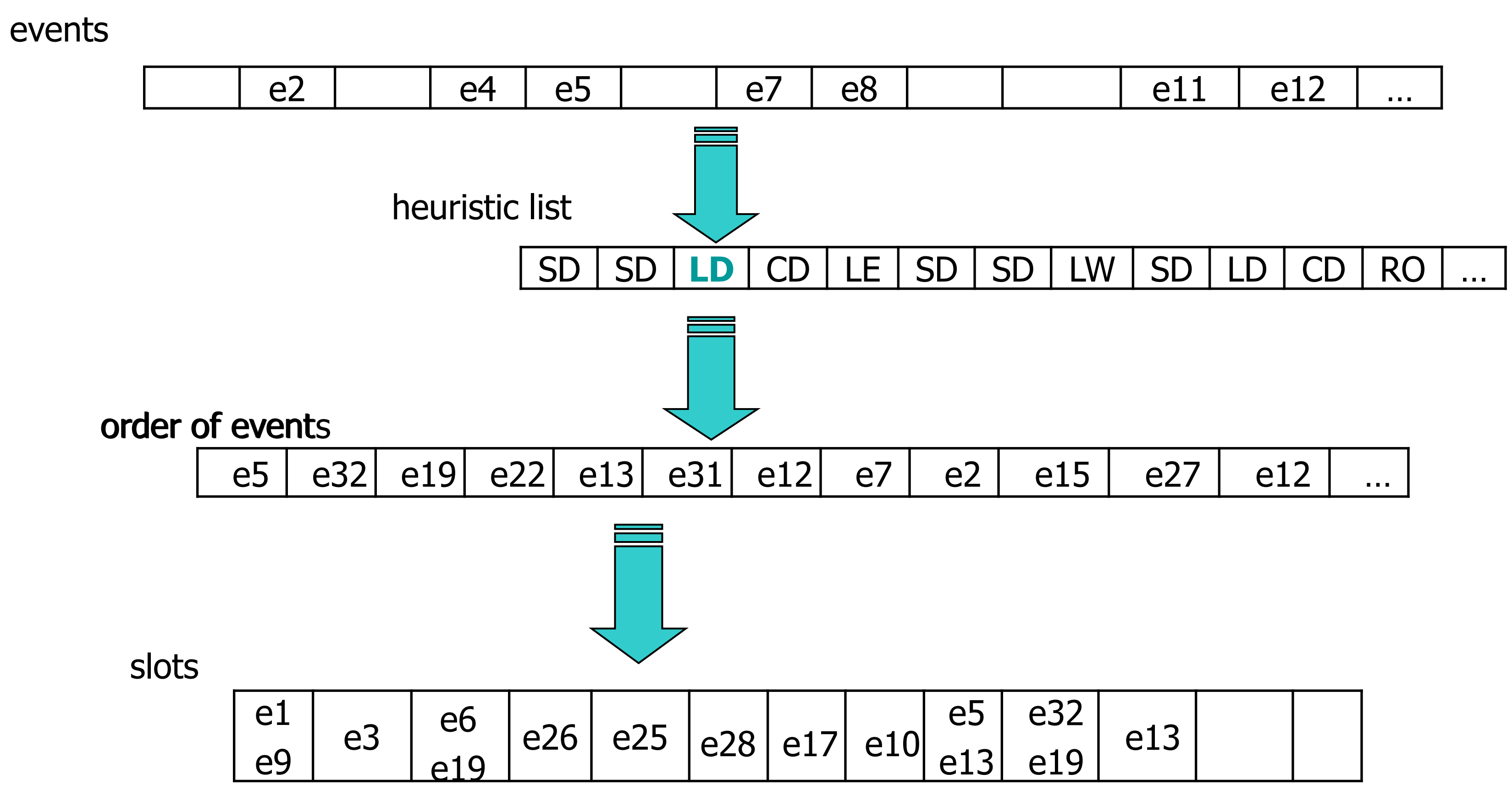
# *Graph-based hyper-heuristics*



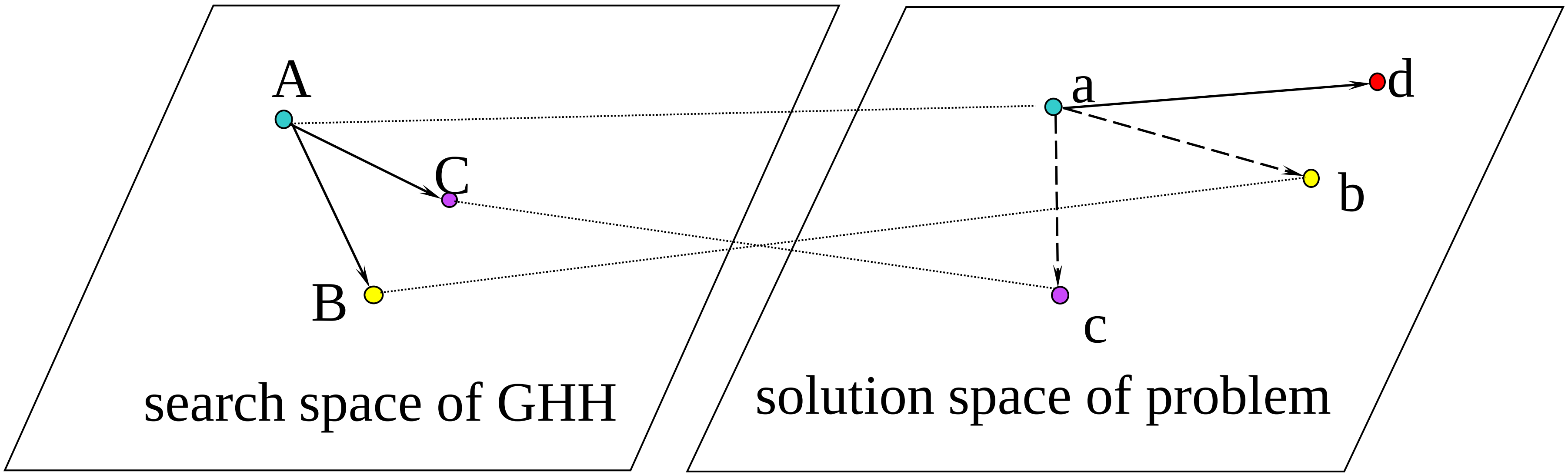
# *Graph-based hyper-heuristics*



# Graph-based hyper-heuristics



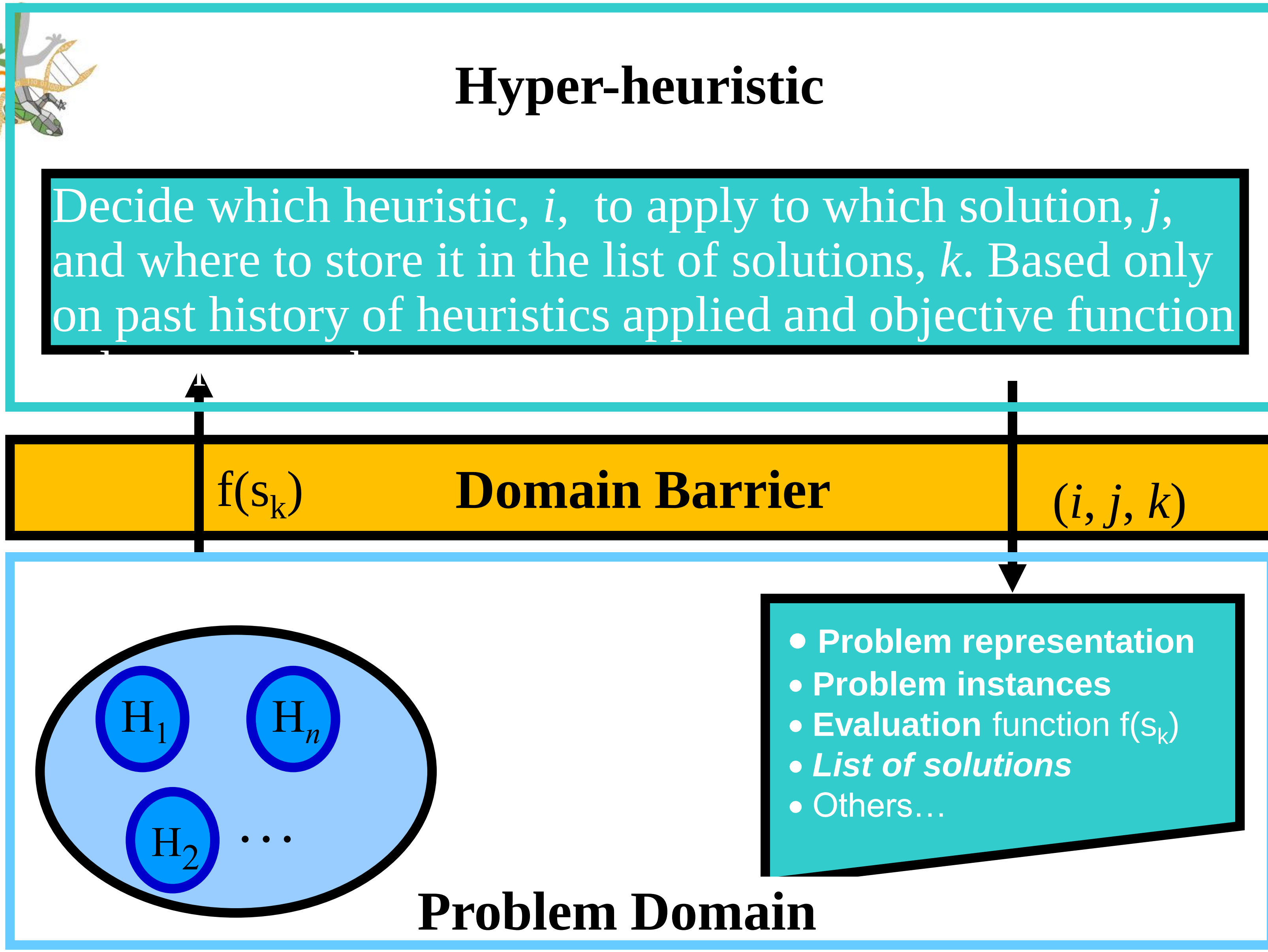
# *Graph-based hyper-heuristics*




- Tabu Search and other meta-heuristics (VNS, ILS) used to search the heuristic search space
- Objective function: quality of solutions (timetables) built by the corresponding heuristic list

# Heuristic Selection Methodologies

- The domain barrier
- A perturbative hyper-heuristic: Tabu-search hyper-heuristic
- HyFlex and the Cross-domain Heuristic Search Challenge

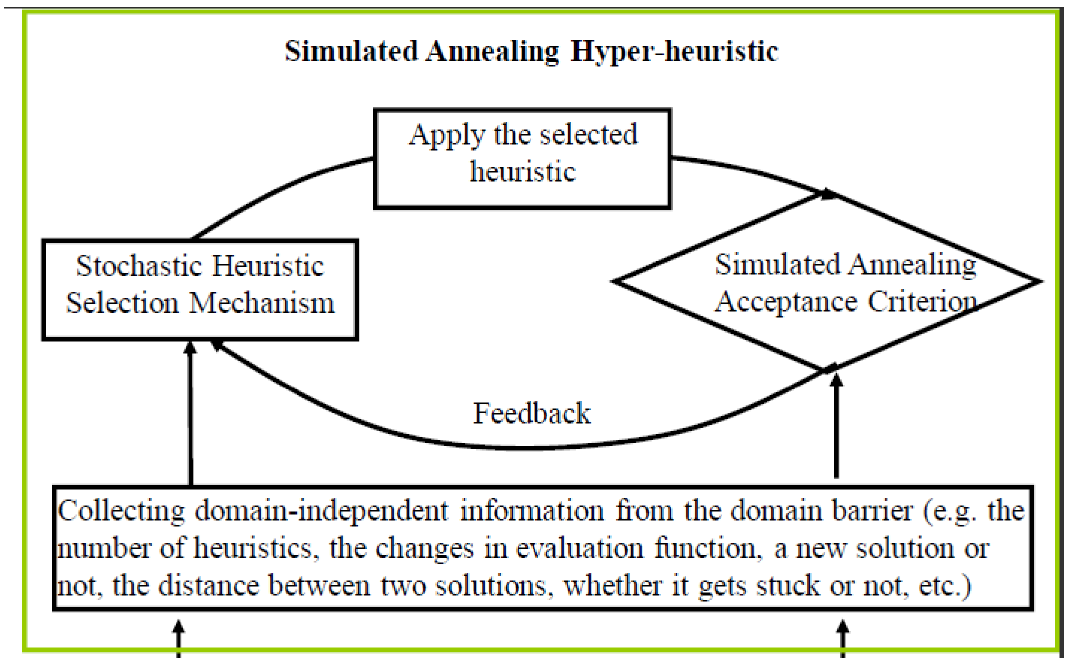



HH fremework:(Cowling P., Kendall G. and Soubeiga, 2000, 2001), (E. K. Burke et al., 2003)

Extension: J. Woodward, A. J. Parkes, G. Ochoa, A Mathematical Framework for Hyper-heuristics. PPSN Hyper-heuristics Workshop. 2008

# Tabu-search hyper-heuristics

- Heuristics selected according to learned ranks (using reinforcement learning)
- Dynamic tabu list of heuristics that are temporarily excluded from the selection pool

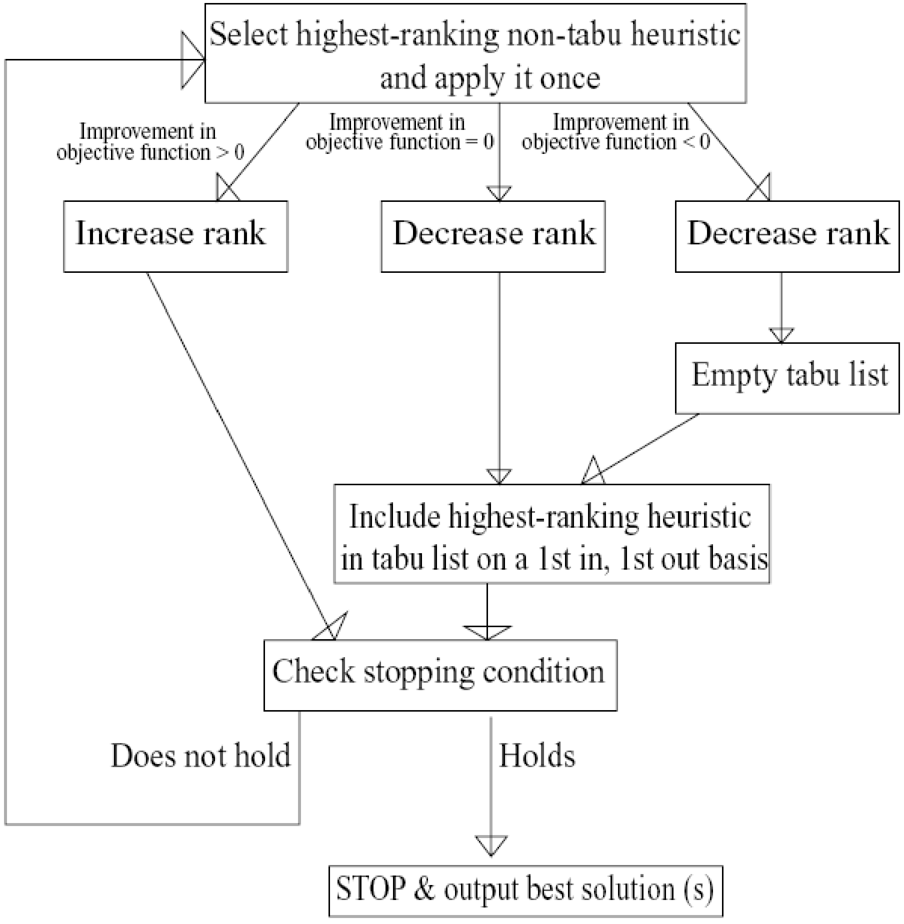



Later combined with SA acceptance

Burke, E.K., Kendall. G., Soubeiga. E. (2003) A Tabu-Search Hyperheuristic for Timetabling and Rostering, *Journal of Heuristics*,Vol 9

# Tabu search hyper-heuristics

Each heuristic $k$ is assigned a rank $r_k$ initialised to $0$ and allowed to increase and decrease within interval $[r_{min}, r_{max}]$



# *HyFlex (Hyper-heuristics Flexible framework)*

- **Question**: Can we produce a benchmark to test the generality of heuristic search algorithms?

- A software framework (problem library) for designing and evaluating general-purpose search algorithms
- Provides the *problem-specific* components
- Efforts focused on designing high-level strategies

E. K. Burke, T. Curtois, M. Hyde, G. Ochoa (2011) HyFlex: A Benchmark Framework for Cross-Domain Heuristic Search, *Evolutionary Computation*, (under review)

# *HyFlex: a benchmark for cross-domain heuristic search*

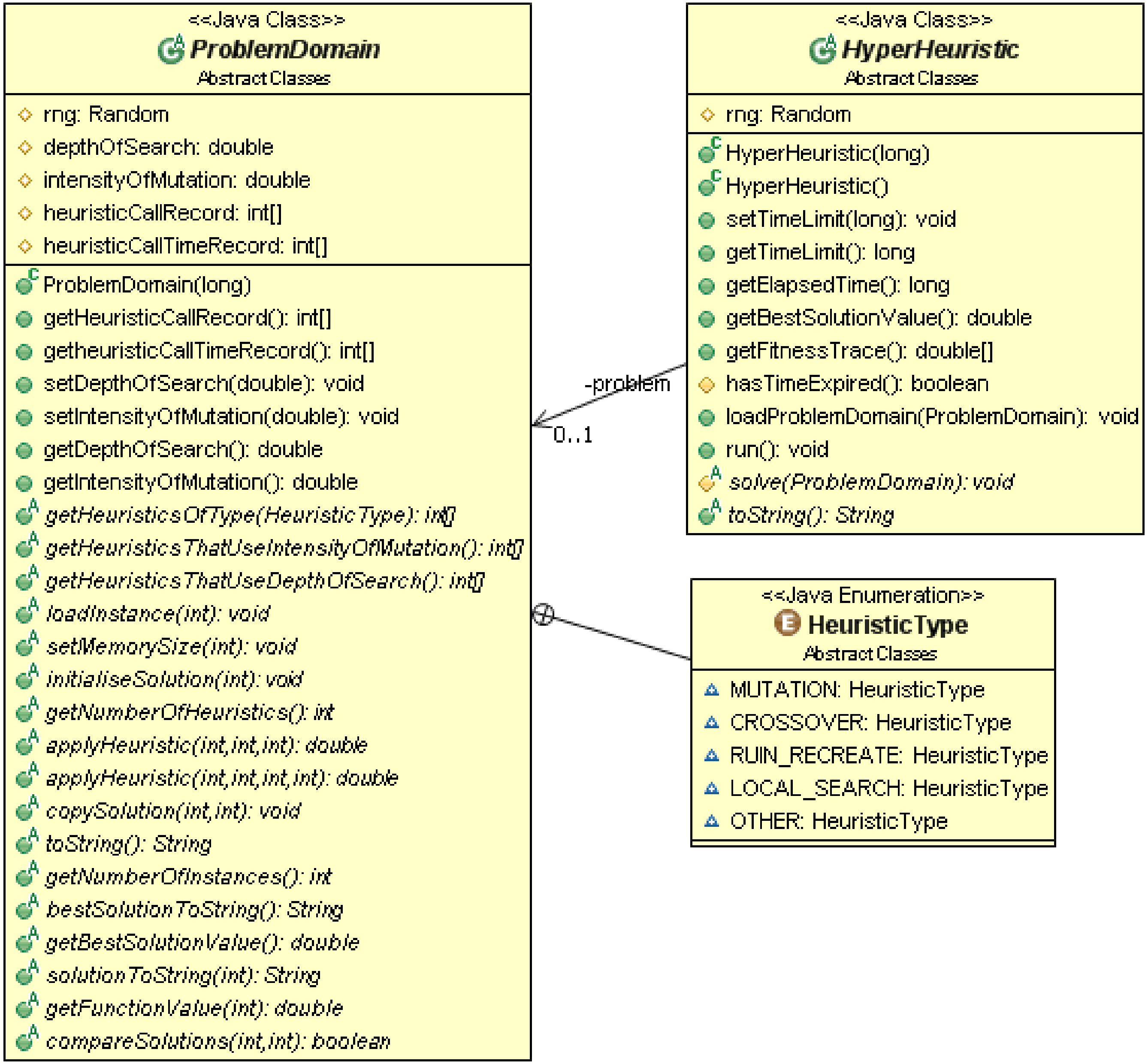

- Six different domains, hard combinatorial problems, interesting and varied set of operators and instances
- Implemented using the same software framework (common software interface)
- A single high-level strategy can operate and solve all the domains
- What are the principles and design strategies of successful cross-domain search heuristics?

CHeSC 2011
CROSS-DOMAIN HEURISTIC SEARCH CHALLENGE

http://www.asap.cs.nott.ac.uk/chesc2011/

# *Overview of the problem domain modules*

1. A routine to initialise (randomised) solutions
2. A population or list of solutions
3. A set of heuristics to modify solutions
   a. Mutational: makes a random modification
   b. Ruin-recreate: partially destroy a solution and rebuild it using a constructive procedure
   c. Local-search (hill-climbing): iterative procedures searching on the neighbourhood of solutions for non-worsening solutions
   d. Crossover: takes parent solutions and produce offspring solution
4. A set of interesting instances, that can be easily loaded


<<Java Class>>
ProblemDomain
Abstract Classes
rng: Random
depthOfSearch: double
intensityOfMutation: double
heuristicCallRecord: int[]
heuristicCallTimeRecord: int[]
ProblemDomain(long)
getHeuristicCallRecord(): int[]
getheuristicCallTimeRecord(): int[]
setDepthOfSearch(double): void
setIntensityOfMutation(double): void
getDepthOfSearch(): double
getIntensityOfMutation(): double
getHeuristicsOfType(HeuristicType): int[]
getHeuristicsThatUseIntensityOfMutation(): int[]
getHeuristicsThatUseDepthOfSearch(): int[]
loadInstance(int): void
setMemorySize(int): void
initialiseSolution(int): void
getNumberOfHeuristics(): int
applyHeuristic(int,int,int): double
applyHeuristic(int,int,int,int): double
copySolution(int,int): void
toString(): String
getNumberOfInstances(): int
bestSolutionToString(): String
getBestSolutionValue(): double
solutionToString(int): String
getFunctionValue(int): double
compareSolutions(int,int): boolean
-problem
0..1
<<Java Class>>
HyperHeuristic
Abstract Classes
rng: Random
HyperHeuristic(long)
HyperHeuristic()
setTimeLimit(long): void
getTimeLimit(): long
getElapsedTime(): long
getBestSolutionValue(): double
getFitnessTrace(): double[]
hasTimeExpired(): boolean
loadProblemDomain(ProblemDomain): void
run(): void
solve(ProblemDomain): void
toString(): String
<<Java Enumeration>>
HeuristicType
Abstract Classes
MUTATION: HeuristicType
CROSSOVER: HeuristicType
RUIN_RECREATE: HeuristicType
LOCAL_SEARCH: HeuristicType
OTHER: HeuristicType

# *HyFlex as a research tool*

"Civilization advances by extending the number of important operations which we can perform without thinking about them."

Alfred North Whitehead, *Introduction to Mathematics (1911)*

*Crowdsourcing*: "the act of taking a job traditionally performed by a designated agent (usually an employee) and outsourcing it to an undefined, generally large group of people in the form of an open call".

*Jeff Howe, Wired Magazine, 2006*

# Conclusions of 1st Section

A hyper-heuristic is an automated methodology for selecting or generating heuristics to solve hard computational search problems

- Main feature: search in a space of heuristics
- Term used for '*heuristics to choose heuristics*' in 2000
- Ideas can be traced back to the 60s and 70s
- Two main type of approaches
  - Heuristic selection
  - Heuristic generation
- Ideas from online and offline machine learning are relevant, as are ideas of meta-level search

# Future work

- Generalisation:  By far the biggest challenge is to develop methodologies that work well across several domains
- Foundational studies: Thus far, little progress has been made to enhance our understanding of hyper-heuristic approaches
- Distributed, agent-based and cooperative approaches: Since different low-level heuristics have different strengths and weakness, cooperation can allow synergies between them
- Multi-criteria, multi-objective and dynamic problems:  So far, hyper-heuristics have been mainly applied to single objective and static problems

# References: Hyper-heuristics


- E.K.Burke, G. Kendall, J.Newall, E.Hart, P.Ross & S.Schulenburg, Hyper-Heuristics: An Emerging Direction in Modern Search Technology, Handbook of Metaheuristics (eds. F.Glover & G.Kochenberger), pp 457 – 474, Kluwer, 2003.
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# References : Automated Heuristic Design

This a small sample of books, survey papers, and other journal papers


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# Section 3

# Heuristic Generation Methodologies

# Outline

- Introduction to this section
  - Hyper-Heuristic Definition
  - What's the Point?
- Case Study 1: SAT
- Case Study 2: Bin Packing
- Conclusion

# *Hyper-Heuristic Definition*

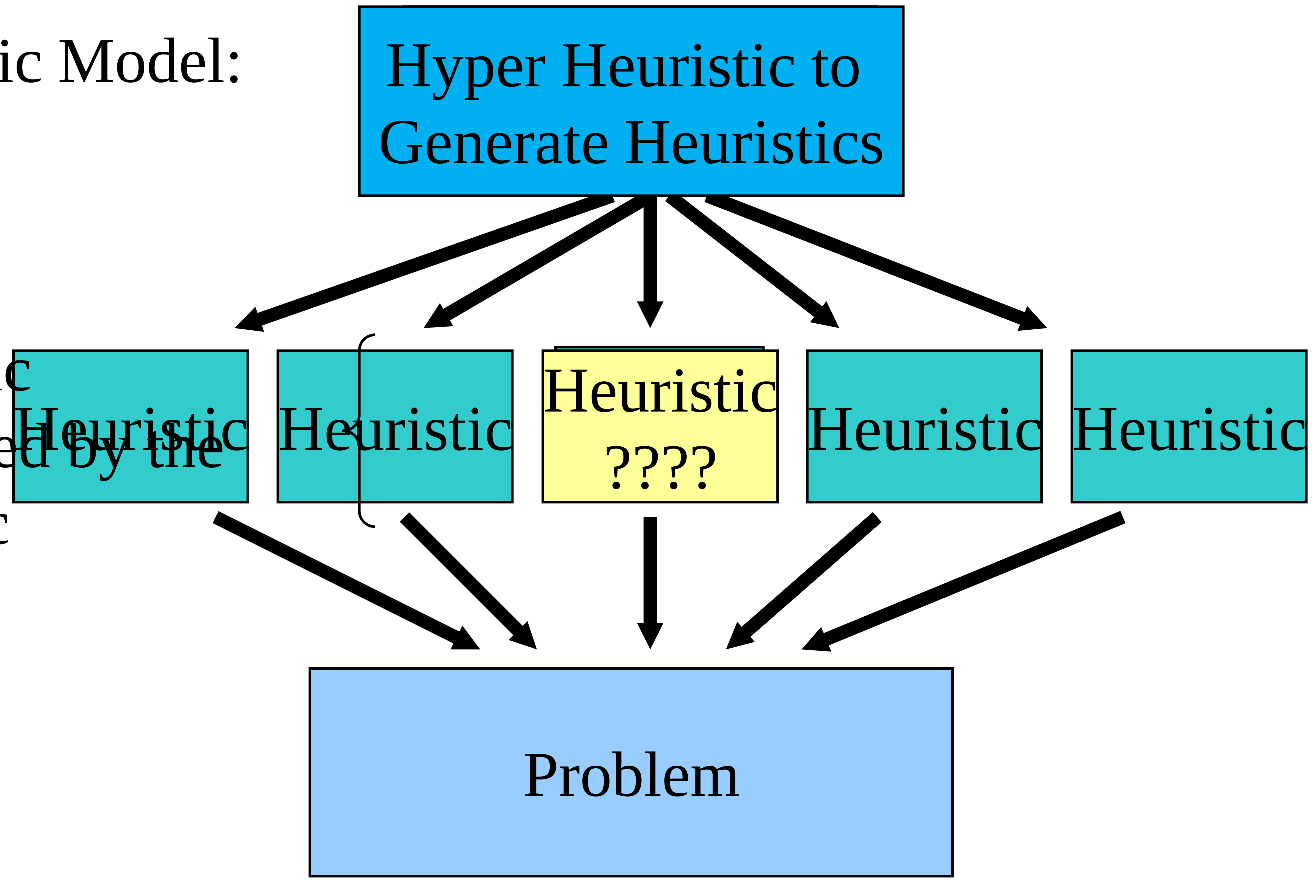
"A hyper-heuristic is an automated methodology for selecting or **generating** heuristics to solve hard computational search problems"

# Two Types of Hyper-Heuristic?

A Hyper Heuristic Model:

Heuristics defined by the user

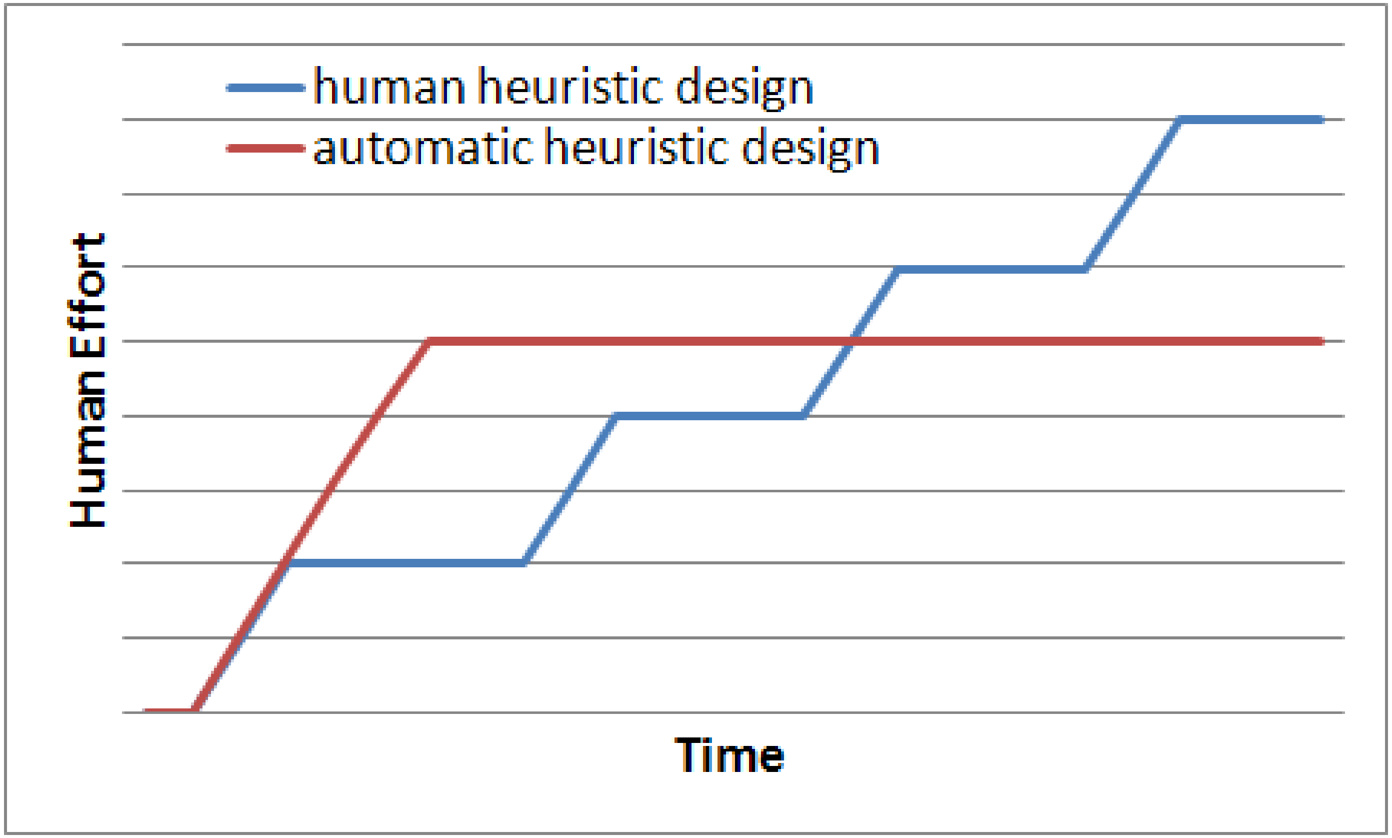
Domain-Specific Heuristic Defined by the Hyper-Heuristic



# What's the Point?

- We spend a lot of time testing, and fine tuning, solution methods.
- They are usually specialised to a particular problem instance set, with certain characteristics.
- Automating this creative process can potentially save time and/or effort.
- Humans still have a creative role in heuristic generation, but the idea is that more of the process is automated.

# What's the Point?



# Heuristic Generation Methodologies

## *Case Study 1*

# CASE STUDY 1

- Evolving Heuristics for SAT
- Bader-el-Den and Poli, 2007
- Based on Fukunaga, 2004, 2008
- SAT local search heuristics can be evolved from a set of components, obtained by analysing existing heuristics from the literature

# *Evolving Heuristics for SAT*

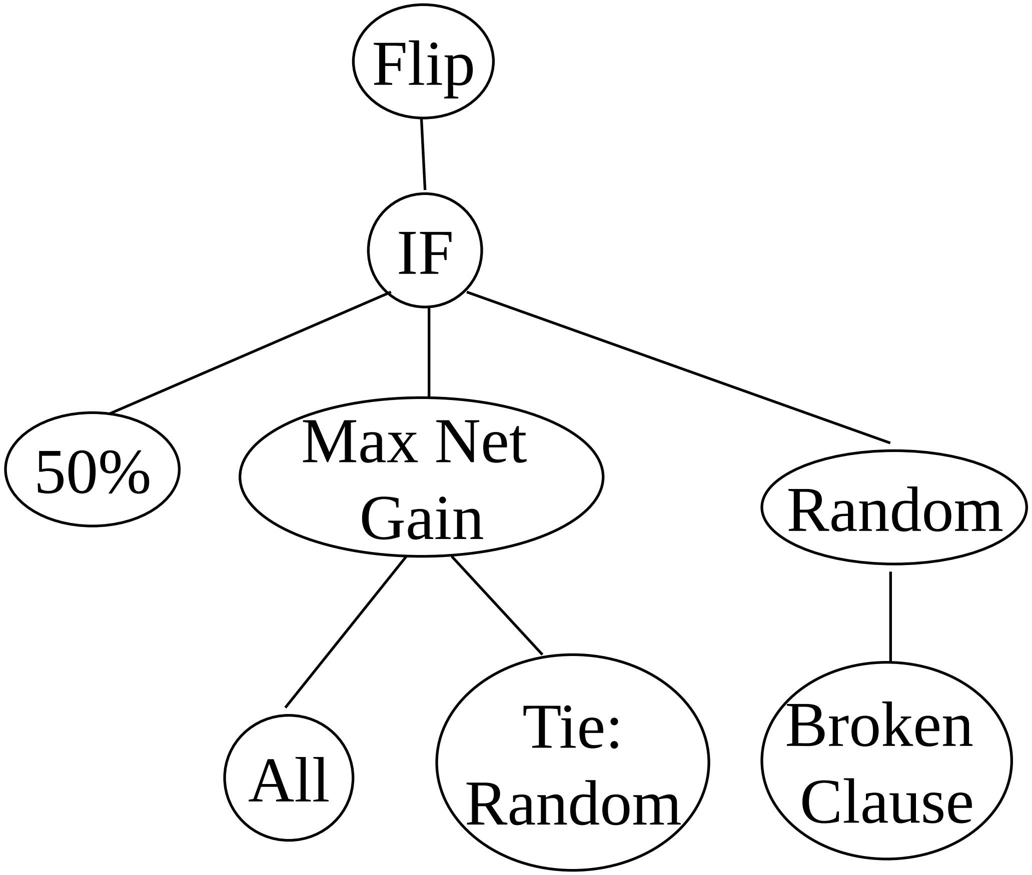
- Make a boolean expression true
- (¬A or B or C) AND (B or ¬C or E) AND (¬B or A or ¬D) AND (…) AND (…) …
- Hundreds/thousands of variables and clauses
- Local search heuristics iteratively choose a variable to flip.

# Existing Heuristics for SAT

- GSAT
  - Flip variable which removes the most broken clauses (highest 'net gain')
- HSAT
  - Same as GSAT, but break ties by choosing the variable that has remained 'unflipped' for the longest
- HARMONY
  - Pick random broken clause BC. Select the variable V in BC with highest net gain, unless V has been flipped most recently in BC. If so, select V with probability p. Otherwise, flip variable with 2nd highest net gain

# Existing Heuristics for SAT

- GWSAT
  - With probability 0.5, apply GSAT
  - Otherwise flip a random variable in a random broken clause.



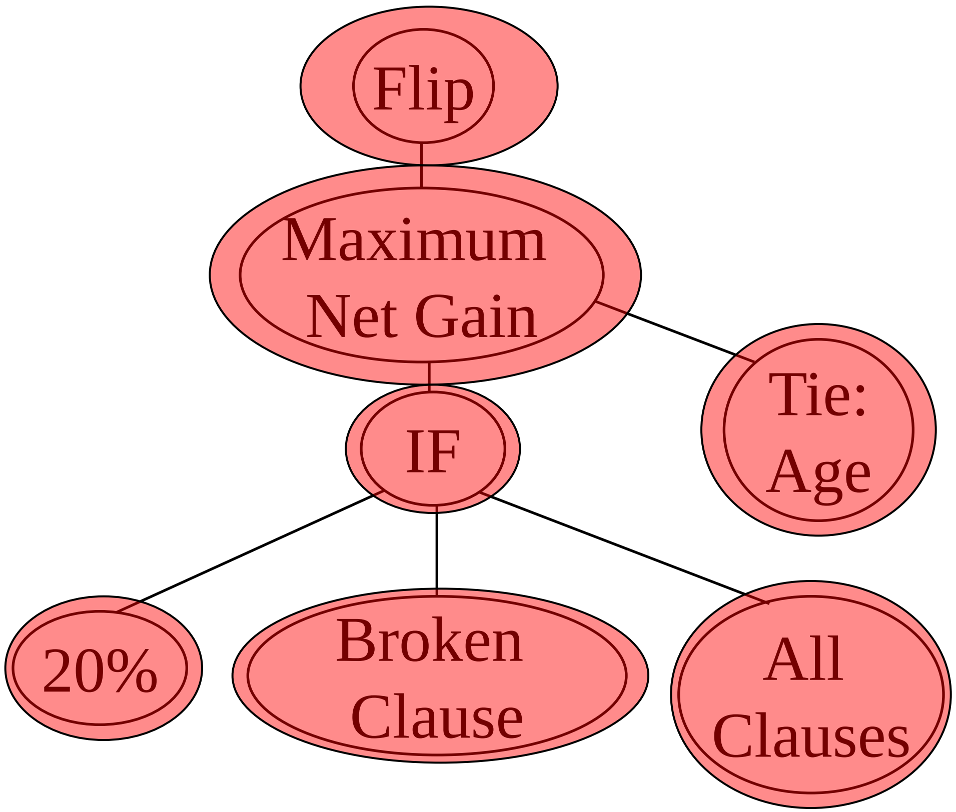
# Evolving New SAT Heuristics

- They define a grammar, which can represent many heuristics from the literature, and new heuristics

```
start →  FLIP v
v     →  RANDOM l
        | MAX_SCR l | MAX_SCR l, op
        | IFV prob, v, v
        | MIN_SCR l | MIN_SCR l, op
        | MAX_AGE l | MAX_AGE l, op
l     →  ALL | ALL_USC
        | RAND_USC | USC
        | IFL prob, l, l
        | SCR_Z l | SCR_Z l, op
op    →  TIE_RAND | TIE_AGE
        | TIE_SCR | NOT_ZERO_AGE
prob  →  20 | 40 | 50 |
         70 | 80 | 90
```

Taken from: Bader-El-Din and Poli, “Generating SAT local-search heuristics using a GP hyper-heuristic framework”, Proceedings of the 8th International Conference on Artificial Evolution. 2007. pp 37-49

# Evolving New SAT Heuristics




```
start → FLIP v
v     →  RANDOM l
        | MAX_SCR l | MAX_SCR l, op
        | IFV prob, v, v
        | MIN_SCR l | MIN_SCR l, op
        | MAX_AGE l | MAX_AGE l, op
l     →  ALL | ALL_USC
        | RAND_USC | USC
        | IFL prob, l, l
        | SCR_Z l | SCR_Z l, op
op    →  TIE_RAND | TIE_AGE
        | TIE_SCR | NOT_ZERO_AGE
prob  →  20 | 40 | 50 |
         70 | 80 | 90
```

# Lessons – Case Study 1

- Existing local search heuristics were broken down into components
- These heuristics return a variable to flip, not a value or 'score'
- Local search heuristics evolved here, rather than constructive heuristics

# Heuristic Generation Methodologies

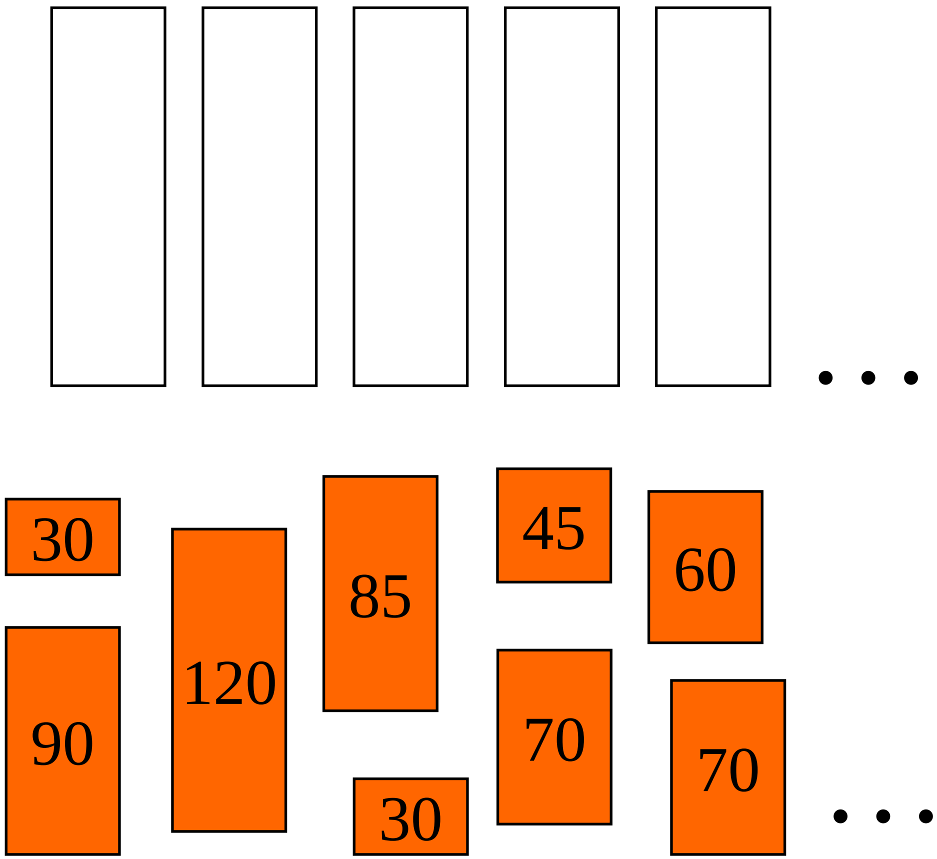
## *Case Study 2*

# CASE STUDY 2

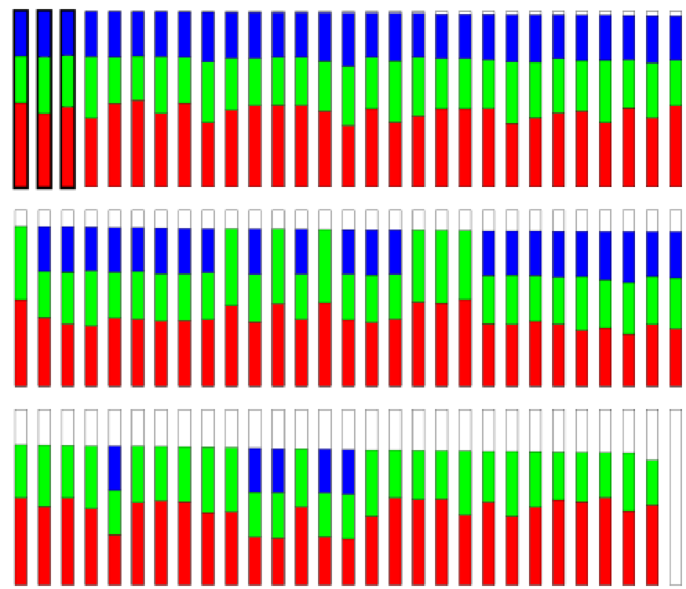
- One Dimensional Bin Packing
- Burke, Hyde, Kendall, and Woodward 2007
- Heuristics can be evolved that are specialised to different types of problems

- Extended to two dimensional packing heuristics in Burke, Hyde, Kendall, and Woodward 2010

# *The Bin Packing Problem*

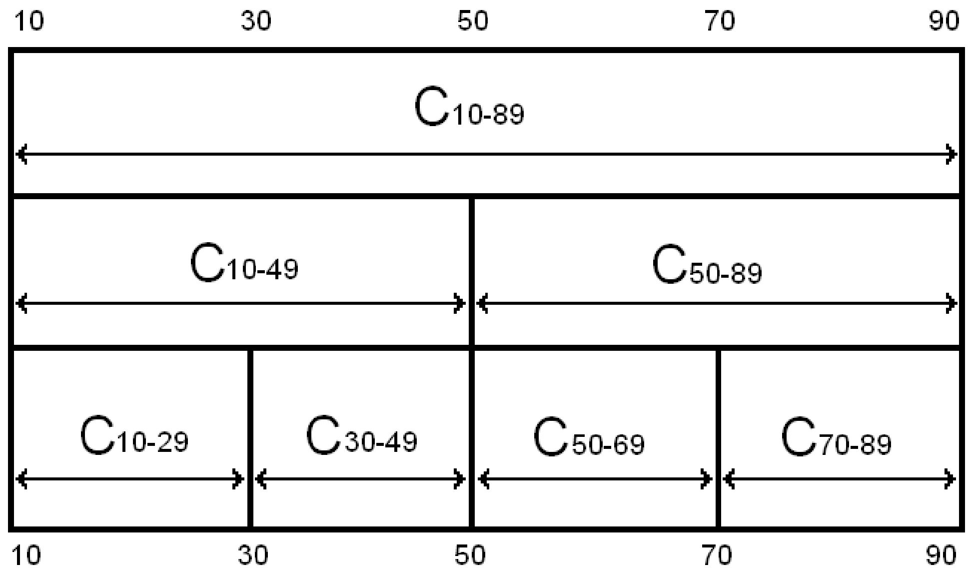



- Pack all the pieces into as few bins as possible



# The Bin Packing Problem Set

- Online
- 7 problem classes
- Bin Capacity 150
- 120 items



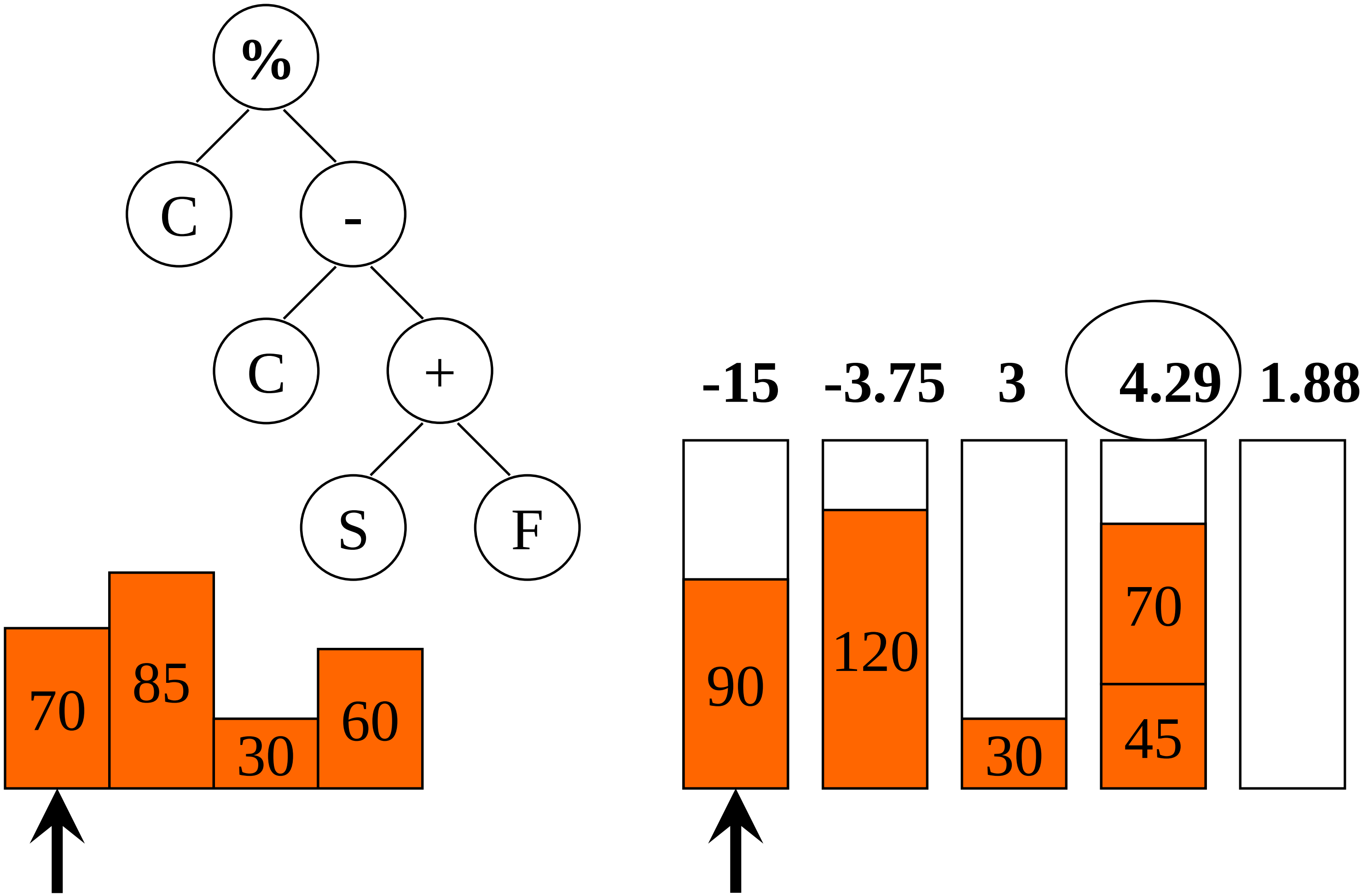

7 Training sets

7 Validation sets

# GP Parameters Outline

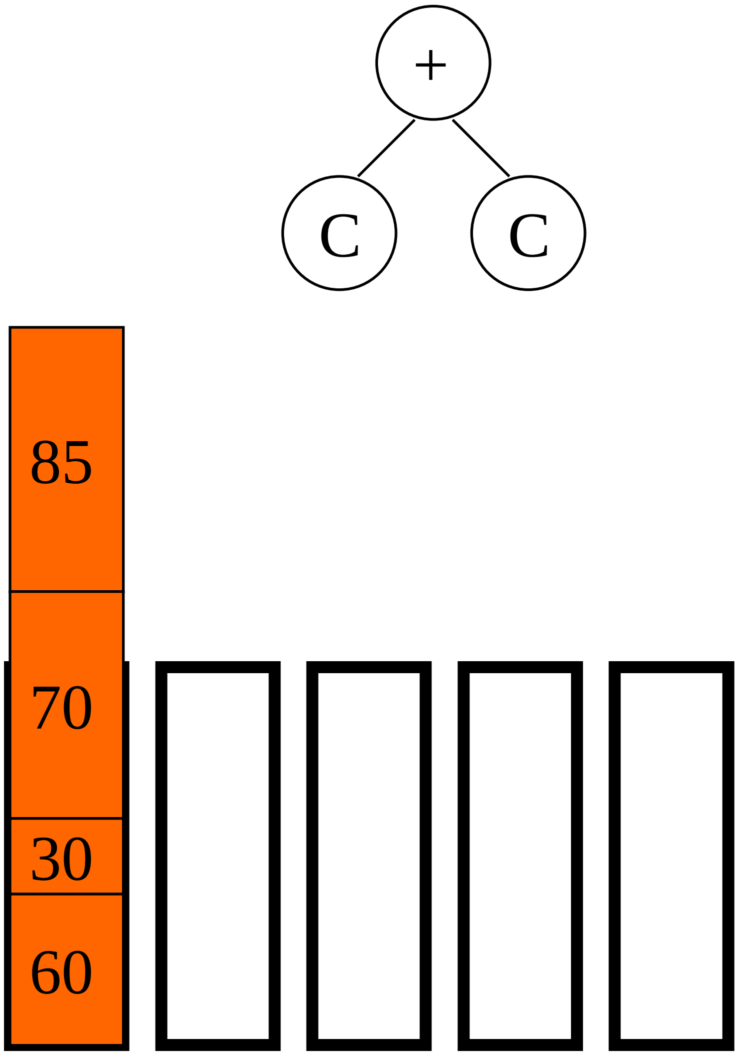
- 50 generations
- 90% crossover
- 10% reproduction
- Functions and terminals:
  - Bin Capacity — C
  - Bin Fullness — F
  - Piece Size — S
  - +, - , *, %, ≤
- 1000 population
- Fitness proportional selection

# *Evolving Bin Packing Heuristics*

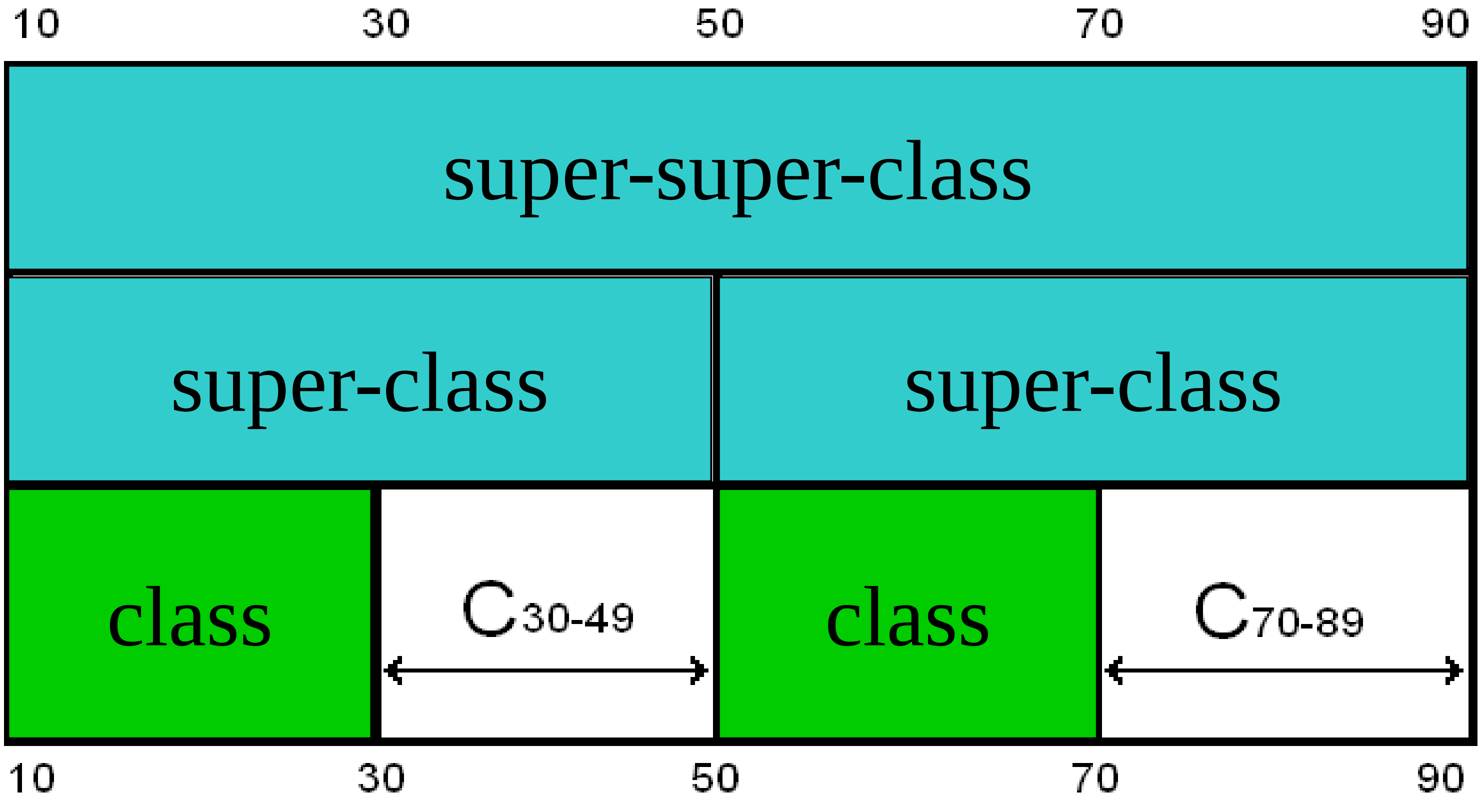


# *Illegal Heuristics*

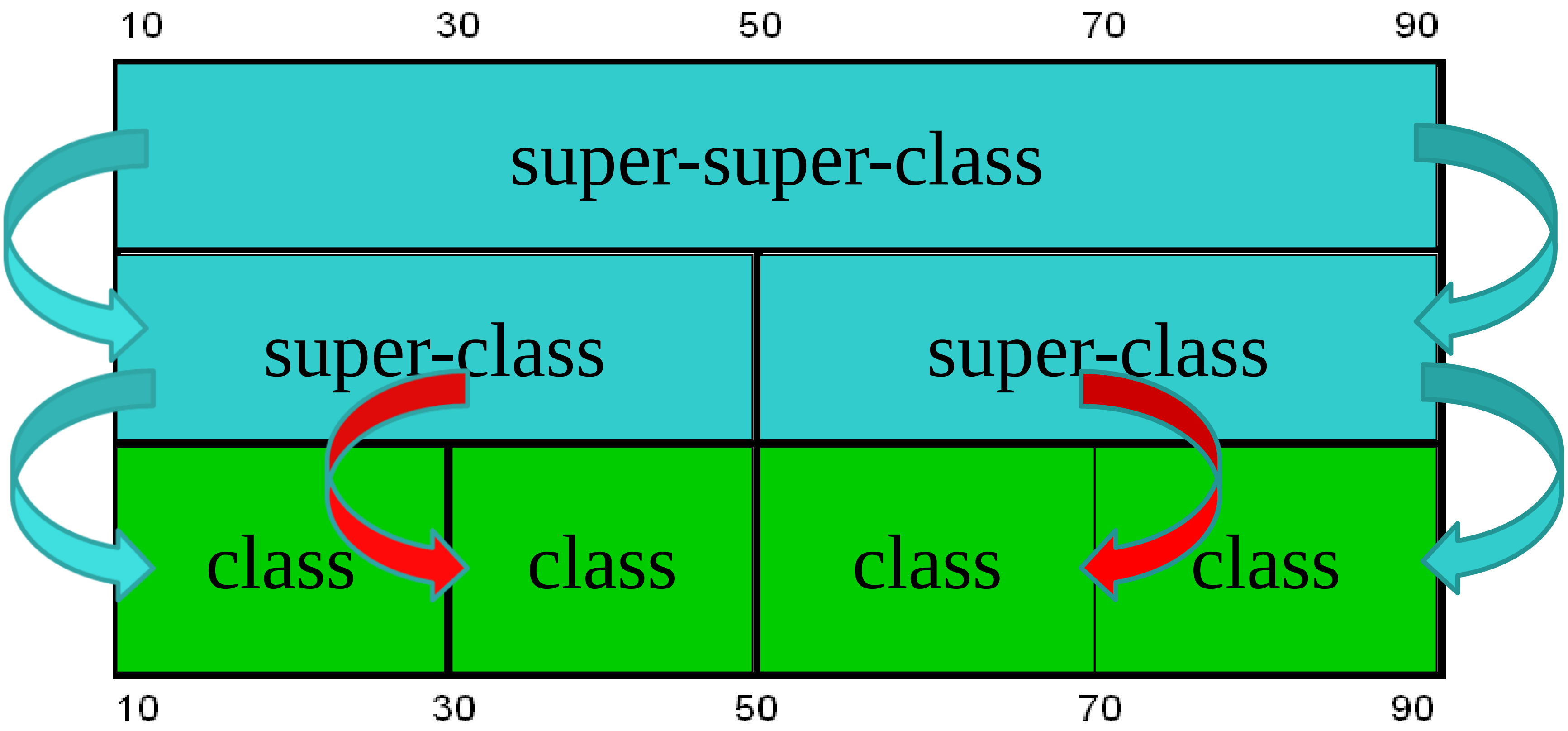
- Permitted
- High penalty
- The system evolves an understanding of the rules



# Results - Specialisation of Heuristics



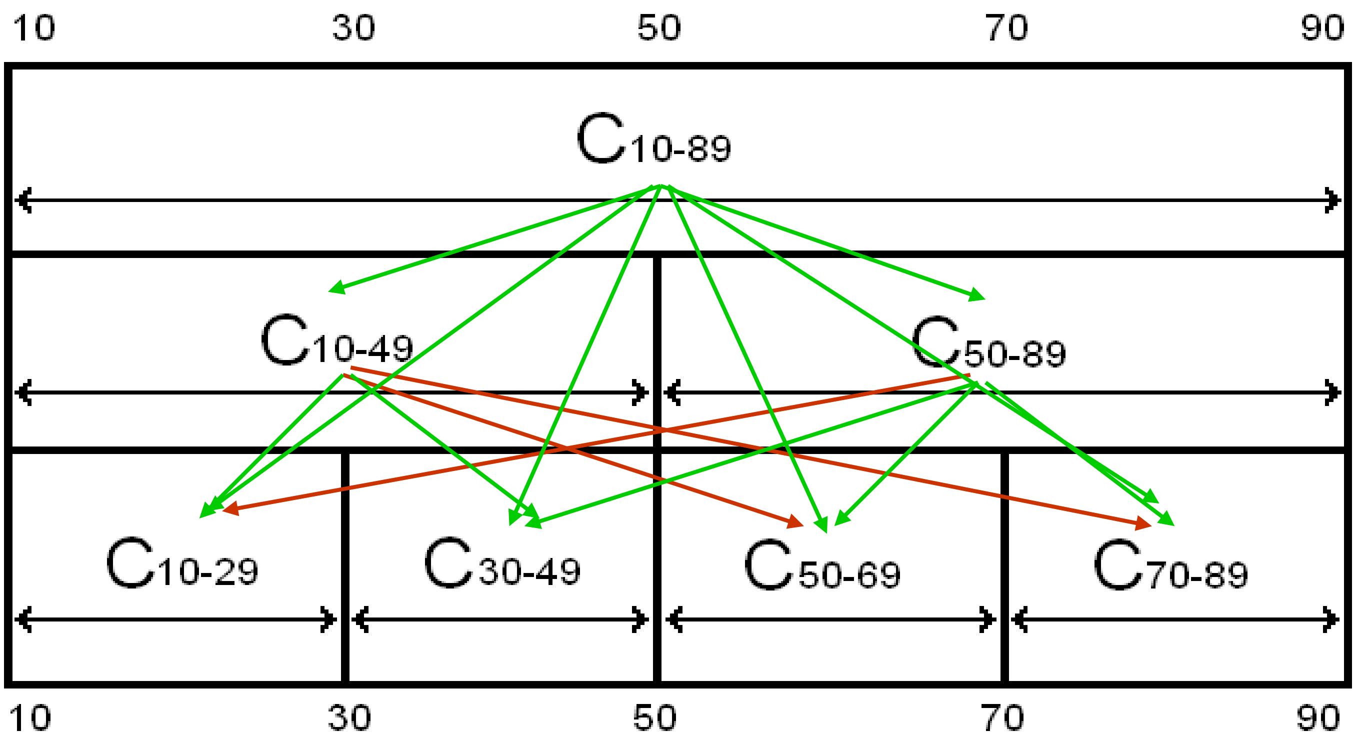
# Results - Specialisation of Heuristics



# Results - Robustness of Heuristics

= all legal results

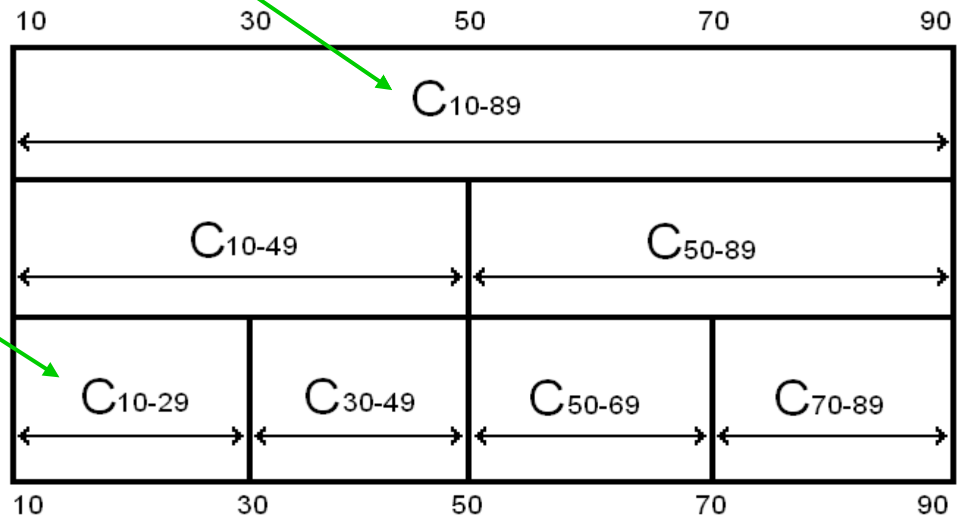
= some illegal results



# *Example of an evolved heuristic*

- Heuristic evolved on instances with the widest distribution

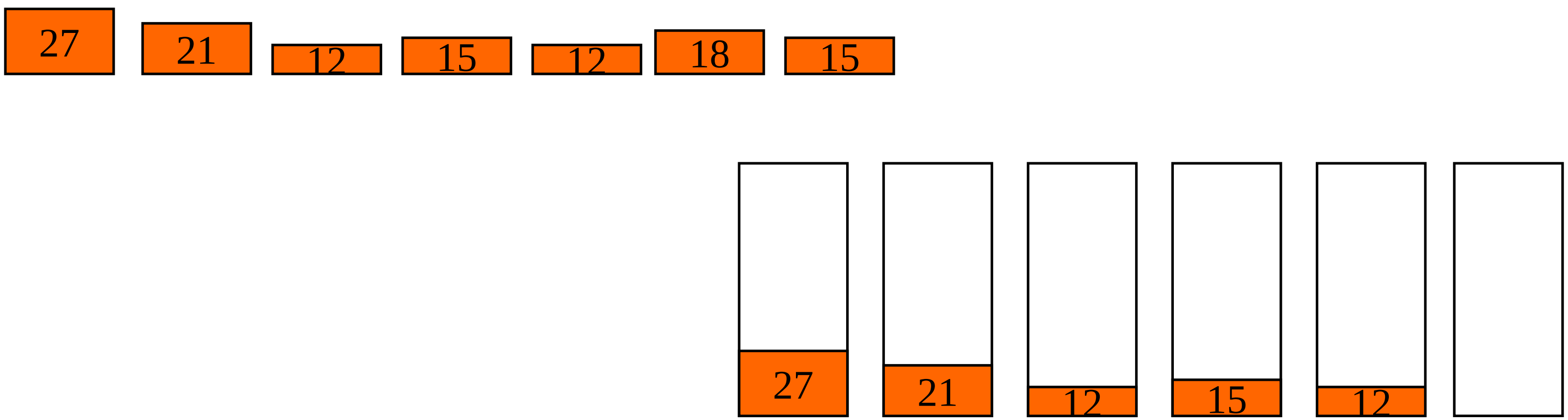
- Tested on instances with piece sizes between 10-29




- The heuristic performs very badly, by putting just one piece into each bin

# Example of an evolved heuristic

- The heuristic always scores the empty bin as the best




$$\frac{2S+F}{S+F} + \frac{C}{\left(\left(\frac{F}{C}\right) \leq (2C-F)\right) + (C-S-F)}$$

# *Lessons – Case Study 2*

- Heuristics can be **specialised** to specific types of sub problem
- Heuristics may not work at all on new instances if they contain different distributions of pieces
- The **training set must be carefully chosen** to ensure it represents every type of problem that the heuristic must solve in the future

# *Conclusion*

- Presented three case studies which highlight different research issues
- Humans will (always?) still have a role in heuristic generation
- The hyper-heuristic automates the process of combining elements that have been **chosen by humans**
- Our role moves from designing heuristics to **designing the search space** in which the best heuristic is likely to exist

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